MOND as “modified inertia”

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MOND in a nutshell

A theory of dynamics (gravity/inertia) involving a new constant a_0

'Correspondence principle': Standard limit ($a_0 \rightarrow 0$)

MOND limit: $a_0 \rightarrow \infty$: SI: $(t, \mathbf{r}) \rightarrow \lambda(t, \mathbf{r})$

Many theories that embody these tenets

Primary predictions follow from only the basic tenets:

$V_{circ} \rightarrow V_{\inf} = const.$; $V_{\inf}^4 = MGa_0$; RCs to a large extent; For DML systems $\sigma_{3d}^4/MGa_0 = A \sim 1$; enhanced stability of discs, central-surface-densities-relation.

Secondary predictions do not follow from the axioms, and are theory dependent: Exact RCs, dependence of MGa_0/σ^4 on dimensionless system parameters. Workings of the EFE, effects in the SS, workings of dynamical friction, etc.

No fully satisfactory underlying theory

Theory – directions

- Relativistic effective theories: Lensing, cosmology, structure
- "Microscopic" theories: Vacuum effects; Entropic gravity; Dipolar DM; Superfluid DM; MOND from a membrane-picture; etc.
- Nonrelativistic effective theories:
  - ▹ May point the way to relativistic and more fundamental ones.
  - ▹ May teach us about the possible variety in secondary predictions.

What is wrong with existing theories?

A single interpolating function of a single acceleration variable introduced already in the action.

a. Leads to a very restricted situation where all phenomena are described by the same single function of a single variable.

b. This is anything but what happens in other instances (e.g., relativity or QM vs Newtonian dynamics

The “modified-gravity” theories AQUAL/QUMOND are, at present, the NR MOND workhorses. But they are most probably not the final word.

Aims

My aim here is twofold

a. Acquaint you with some aspects of modified inertia

b. Demonstrate that there can be MOND theories that do not hinge on a sigle function of a single variable, and that can differ substantially from AQUAL/QUMOND on secondary predictions.

The issue is not MG vs. MI

There can also be a large variety among MG theories.

Example: Tripotential MOND theories

$$\mathcal{L} = -\frac{1}{8\pi G}\{2\vec{\nabla}(\phi - \epsilon\varphi)\cdot\vec{\nabla}\psi - a_0^2\mathcal{F}(x,y,z)\} + \rho(\frac{1}{2}\mathbf{v}^2 - \phi)$$

$$x \equiv (\vec{\nabla}\psi)^2/a_0^2, \quad y \equiv (\vec{\nabla}\varphi)^2/a_0^2, \quad z = 2\vec{\nabla}\psi\cdot\vec{\nabla}\varphi/a_0^2$$

$\mathbf{a} = -\vec{\nabla}\phi$ the MOND acceleration

$\Delta\psi = 4\pi G\rho$ is solved

φ gotten from an AQUAL equation, then ϕ from a Poisson

AQUAL and QUMOND are special cases

$$\text{Deep MOND (scale inv.)}: \quad \mathcal{F}(\lambda^{-4}x, \lambda^{-2}y, \lambda^{-3}z) = \lambda^{-3}\mathcal{F}(x,y,z)$$

$$\mathcal{F}(x,y,z) = x^{3/4}\mathcal{F}(1, y/x^{1/2}, z/x^{3/4}) = y^{3/2}\mathcal{F}(x/y^2, 1, z/y^{3/2})$$

For 1-D systems $g_M = g_N\nu(g_N/a_0), \mathcal{F} \Rightarrow \nu$ universal

Satisfies the same virial, two-body, $M - \sigma$ as AQUAL/QUMOND

What is (nonrelativistic) modified inertia?

Newtonian Lagrangian density : $\mathcal{L} = -\frac{1}{8\pi G}(\vec{\nabla}\phi)^2 - \rho\phi + \frac{1}{2}\rho\mathbf{v}^2$

$$\mathbf{a} = -\vec{\nabla}\phi, \qquad \nabla\phi = 4\pi G\rho$$

Modified gravity (AQUAL, QUMOND,...) modifies the first term.

“MI” modifies the last term to a functional of the trajectory

Possible “microscopic” origin of MI, perhaps of inertia itself

Mach’s principle, hidden inertia medium (the quantum vacuum?)

Need to modify the free actions of all degrees of freedom?

Perhaps then also the Einstein-Hilbert action

MOND needs to define an inertial frame with respect to which accelerations are measured (quantum vacuum?)

Here treated as an effective theory that bypasses these deep questions

Who is afraid of modified inertia?

Much less activity on MI than on MG; why?

Developers: It seems more difficult to construct MI theories under the standard requirement (symmetries, conservation laws, etc.)

Users: No full-fledged theory or model; existing models are harder to solve; so fewer predictions (RCs), hard to simulate, etc.

But these by no means argue against MI

Physics can develop from easy to hard (for us):

GR is much harder than Newtonian gravity, QM much harder than classical mechanics, Standard Model

Don’t reject MI because you do not feel comfortable with it!!

Don’t see MI as a threat to your existing work!!

MG simulations are still very useful: may capture the main features

Special relativity as modified inertia

$$\mathbf{F} = \frac{md(\gamma \mathbf{v})}{dt} = m\gamma\left(1 + \gamma^2 \frac{\mathbf{v} \otimes \mathbf{v}}{c^2}\right)\mathbf{a} = m\overset{\leftrightarrow}{\mu}\left(\frac{\mathbf{v}}{c}\right)\mathbf{a} \Rightarrow$$

$$\overset{\leftrightarrow}{\nu}\left(\frac{\mathbf{v}}{c}\right)\mathbf{F} \equiv \gamma^{-1}\left(1 - \frac{\mathbf{v} \otimes \mathbf{v}}{c^2}\right)\mathbf{F} = m\mathbf{a}.$$

$\overset{\leftrightarrow}{\nu}^{-1}\left(\frac{\mathbf{v}}{c}\right) = \overset{\leftrightarrow}{\mu}\left(\frac{\mathbf{v}}{c}\right) \equiv \gamma(1 + \gamma^2 \frac{\mathbf{v} \otimes \mathbf{v}}{c^2})$ is an “interpolation tensor”.

Circular orbits: $\overset{\leftrightarrow}{\mu} = \gamma$;   Linear acceleration: $\overset{\leftrightarrow}{\mu} = \gamma^3$

The Lorentz factor itself is an “interpolating function”; e.g., in time dilations, etc.

Some lessons

- The interpolating functions are not introduced “by hand” (follow from Lorentz invariance).

- Different interpolating functions connect force and acceleration for different phenomena: $\overset{\leftrightarrow}{\mu}$, Lorentz factor (V/c), GR (MG) Strong gravity $\rightarrow$ Newtonian gravity around a BH (MG/Rc^2)

- No **acceleration** field is defined, only a $\frac{d\mathbf{P}}{dt}$ field.

Quantum mechanics as modified inertia

“First quantization”: Leaves force fields intact; modifies the theory governing particle motion.

$$\mathbf{r}(t) \quad \Rightarrow \quad \psi(\mathbf{r}, t)$$

Newtonian dynamics $\quad\Rightarrow\quad$ Schrödinger equation

$|\psi|^2$ as the probability

Spin and spin-statistics; etc.

Next, modify all actions field quantization, quantum field theory.

Some lessons

- MOND could be much more drastic than just changing the field equation or EoM.

  E.g., degrees of freedom may be different

- Interpolating functions are not introduced at the fundamental level, but different IFs emerge for different phenomena.

  BB function: Rayleigh-Jean ($\hbar \ll kT/\nu$) $\Leftrightarrow$ Wien ($\hbar \gg kT/\nu$)

  Atoms ($L/\hbar$), quantization in a box ($pl/\hbar$)

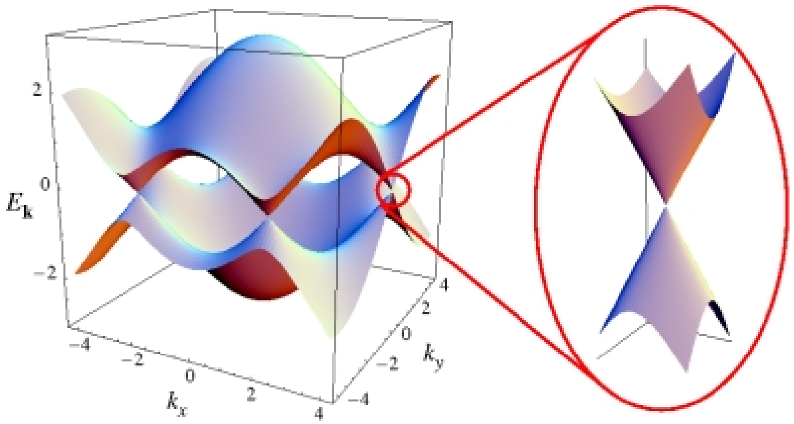
  Specific heat of solids, $C_v \propto \mu(\alpha T/\hbar) \Leftrightarrow \mu(x \gg 1) = const.$ (classical Dulong-Petit limit).

- Different system characteristics with the dimensions of $\hbar$ may enter.

- Very difficult to solve many-particle systems, even with simple potentials, such as Coulomb's.

Instances of modified or acquired inertia in physics

Higgs, Electrons in solids, acoustic analogues



Dirac cones in graphene.

Modified inertia in the context of MOND

(a) $$\mu(a/a_0)\mathbf{a} = -\vec{\nabla}\phi \quad ??$$

(b) $$L_K = \frac{1}{2}m\mu(a/a_0)\mathbf{v}^2 \quad ??$$

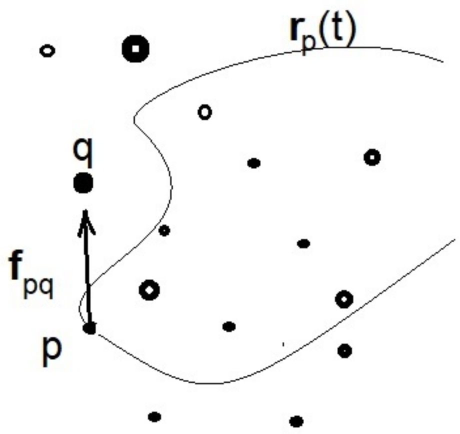
- Both give $\mu(a/a_0)a = a_N$ for rotation curves
- (a) has Galilei invariance, (b) does not
- (b) has a conserved momentum, (a) does not (no action)
- Both still involve one function of one variable

Some lower-tier phenomena where perhaps MI $\neq$ MG

- Stronger EFE on outer galactic discs:
  - ▹ Declining rotation curve (due to LSS)
  - ▹ Inducement of the warps by satellites
- Galaxy-field effects in the solar system may be absent in MI.
- Stronger quenching EFE on vertical dynamics and wide-binary dynamics.
- Dynamical friction?
- Gravitational waves
  - ▹ In MI it is more natural for the path followed by GW (gravitons) to be the same as that for photons.
  - ▹ We would not necessarily expect additional gravitational DoFs or modes of metric propagation.

(Toy) models

Predictions not necessarily generic to MI; demonstrate possible differences from **existing** MG theories



$$\mathbf{F}_p(t) = \sum_{q \neq p} \mathbf{f}_{pq}[\mathbf{r}_p(t), \mathbf{r}_q(t)]$$

$$\hat{\mathbf{a}}_N(\omega) = m^{-1}\hat{\mathbf{F}}(\omega) \qquad \Rightarrow \qquad \hat{\mathbf{a}}(\omega)\mathcal{I}[\{\hat{\mathbf{r}}\}, \omega, a_0] = m^{-1}\hat{\mathbf{F}}(\omega) = \hat{\mathbf{a}}_N(\omega)$$

The "inertia functional"

$$\mathcal{I} \xrightarrow{a_0 \to 0} 1; \qquad \mathcal{I} \xrightarrow{a_0 \to \infty} \mathcal{A}[\{\hat{\mathbf{r}}\}, \omega]/a_0 \quad (scale-invariant\ gravity)$$

Conservation laws

$$\frac{d\mathbf{P}}{dt} = \mathbf{F}(t) \;\Leftrightarrow\; i\omega\hat{\mathbf{P}}(\omega) = \hat{\mathbf{F}}(\omega)$$

$$\hat{\mathbf{P}}(\omega) = m\hat{\mathbf{v}}(\omega)\mathcal{I}[\{\hat{\mathbf{r}}\}, \omega, a_0]$$

Isolated system: $\sum_p \mathbf{F}_p = 0 \quad \Rightarrow \quad d(\sum_p \mathbf{P}_p)/dt = 0.$

$$dE_k/dt = \mathbf{v}(t)\cdot\mathbf{F}(t) \;\Rightarrow\; \hat{E}_k(\omega) = \frac{m}{2\pi}\int \frac{\omega'}{\omega}\hat{\mathbf{v}}(\omega-\omega')\cdot\hat{\mathbf{v}}(\omega')\mathcal{I}[\{\hat{\mathbf{r}}\}, \omega', a_0]d\omega'$$

If the interbody forces are derivable from potentials, $\phi_{pq}(\mathbf{r}_{pq})$ the total energy $\phi + E_k$ is conserved ($E_k = \sum_p E_p^k$).

Similarly, we define the angular momentum

For high-acceleration trajectories, for which $\mathcal{I} \to 1$, $\mathbf{P}(t)$, $E_k(t)$, and $\mathbf{J}(t)$ reduce to the standard expressions, $m\mathbf{v}(t)$, $(1/2)m\mathbf{v}^2(t)$, and $m\mathbf{r}(t) \times \mathbf{v}(t)$, respectively.

"Center of mass"

$$\hat{\mathbf{R}}(\omega) = \frac{\sum_p M_p \hat{\mathbf{r}}_p(\omega)}{\sum_p M_p}, \qquad M_p(\omega) \equiv m_p \mathcal{I}[\{\hat{\mathbf{r}}_p\}, \omega, a_0]$$

For an isolated system $\quad \dfrac{d^2R(t)}{dt^2} = 0$

Reduced two-body problem in the deep-MOND regime

$$\bar{m}\hat{\mathbf{a}}_{12}(\omega)\frac{\mathcal{A}_{12}(\omega)}{a_0} = \hat{\mathbf{F}}(\omega), \qquad \bar{m} = \frac{m_1 m_2}{(m_1^{1/2} + m_2^{1/2})^2}$$

$$\hat{\mathbf{a}}_{12}(\omega) = (1+\alpha^{-1})\hat{\mathbf{a}}_1(\omega), \quad \mathcal{A}_{12}(\omega) = (1+\alpha^{-1})\mathcal{A}_1(\omega), \quad \alpha = (m_2/m_1)^{1/2}$$

Gravitating 2-body on a circular orbit

$$\Delta V_{12}^4 = (q_1^{1/2} + q_2^{1/2})^2 MGa_0$$

Coefficient is different from that in AQUAL/QUMOND

Harmonic oscillator (e.g., a constant-density sphere)

Harmonic motion with amplitude-dependent frequency

Uniqueness of solutions given “initial” position and velocity.

Subclass

$$\mathcal{I}[\{\hat{\mathbf{r}}\}, \omega, a_0] = \bar{\mu}\left[\frac{\mathcal{A}_1(\omega)}{a_0}, \frac{\mathcal{A}_2(\omega)}{a_0}, ...\right]$$

$$\hat{\mathbf{a}}(\omega)\bar{\mu}\left[\frac{\mathcal{A}_1(\omega)}{a_0}, ...\right] = \hat{\mathbf{a}}_N(\omega)$$

Different "interpolation functions" for different phenomena

Deep-MOND limit ($a_0 \to \infty$):

Scale invariance $\Rightarrow$ $\bar{\mu}(x_1, x_2)$ becomes homogeneous of degree 1.

$$\bar{\mu} \to \frac{\mathcal{A}_1}{a_0} K(\mathcal{A}2/\mathcal{A}_1)$$

Examples

$$\mathcal{A}_1(\omega) \propto \int \theta_1\left(\frac{\omega'}{\omega}\right)[\hat{\mathbf{r}}(\omega')\cdot\hat{\mathbf{r}}^*(\omega')]^{1/2}\omega'^2 d\omega'$$

$$\mathcal{A}_2(\omega) \propto \int \theta_2\left(\frac{\omega'}{\omega}\right)|\hat{\mathbf{r}}(\omega')\cdot\hat{\mathbf{r}}(\omega')|^{1/2}\omega'^2 d\omega'$$

$$\mathbf{r}(t) = \frac{1}{\sqrt{2}}(\mathbf{r}_0 e^{i\omega_0 t} + \mathbf{r}_0^* e^{-i\omega_0 t}) \quad \Rightarrow \quad \hat{\mathbf{r}}(\omega) = [\mathbf{r}_0\delta(\omega-\omega_0) + \mathbf{r}_0^*\delta(\omega+\omega_0)]$$

$$\mathbf{r}^2(t) = \mathbf{r}_0\cdot\mathbf{r}_0^* + |\mathbf{r}_0\cdot\mathbf{r}_0|\cos(2\omega_0 t + \varphi_0)$$

Separation of frequencies? $[\theta(x) \propto \delta(x-1)]$ No EFE!

ω independence of $\mathcal{A}$? $[\theta(x) = Const.]$ Wrong CoM motion!

$$\text{For correct CoM motion}: \quad \theta(x) \xrightarrow{x\to\infty} 0$$

[Normalize $\theta(1) = 1$]

Rotation curves

Exact circular motion in an axisymmetric field $\Rightarrow \quad \mathcal{A}_2 = 0$. Only one frequency; so only $\theta(1)$ enters.

Only acceleration parameter is $\mathcal{A}_1 \propto V^2/R$.

Normalize $\mathcal{A}_1$ so that

$$\bar{\mu}(\mathcal{A}_1/a_0, \mathcal{A}_2 = 0) = \mu(V^2/Ra_0)$$

μ it that appearing in rotation-curve analysis.

RCs inform us only to a small extent on the theory in general: Here, only dependence on $\mathcal{A}_1$, and no information on $\theta(x)$

Rotation curves – effects of vertical motions

Vertical-motions can modify the prediction

$$\mu\left(\frac{V^2}{Ra_0}\right) \quad \Rightarrow \quad \sim \mu\left[\frac{V^2}{Ra_0} + \frac{\omega_z^2 z_0}{a_0}\theta\left(\frac{\omega_z R}{V}\right)\right]; \quad \text{Note} \quad \mathcal{A}_2 \neq 0!!$$

Lowers V in inner parts; lowers Q parameter, etc. May lead to somewhat different rotational V for different populations

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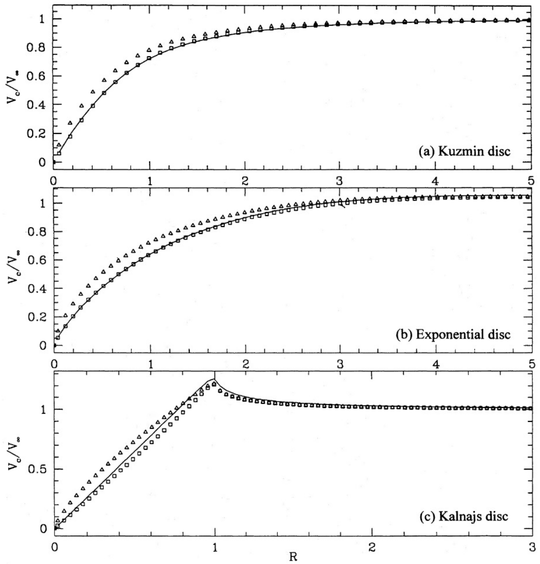
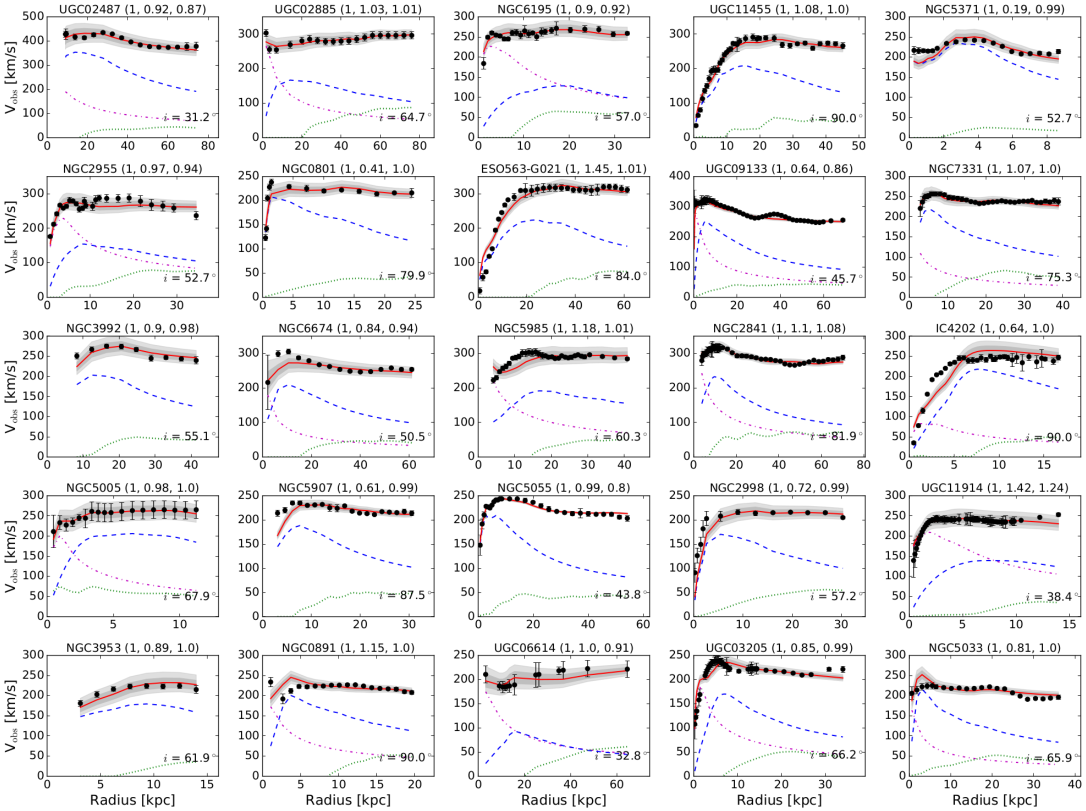
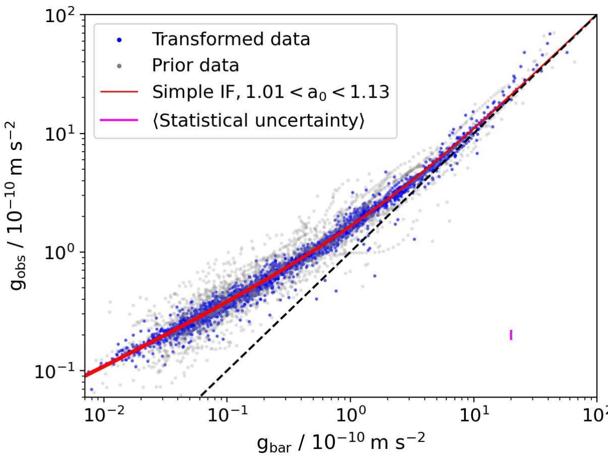
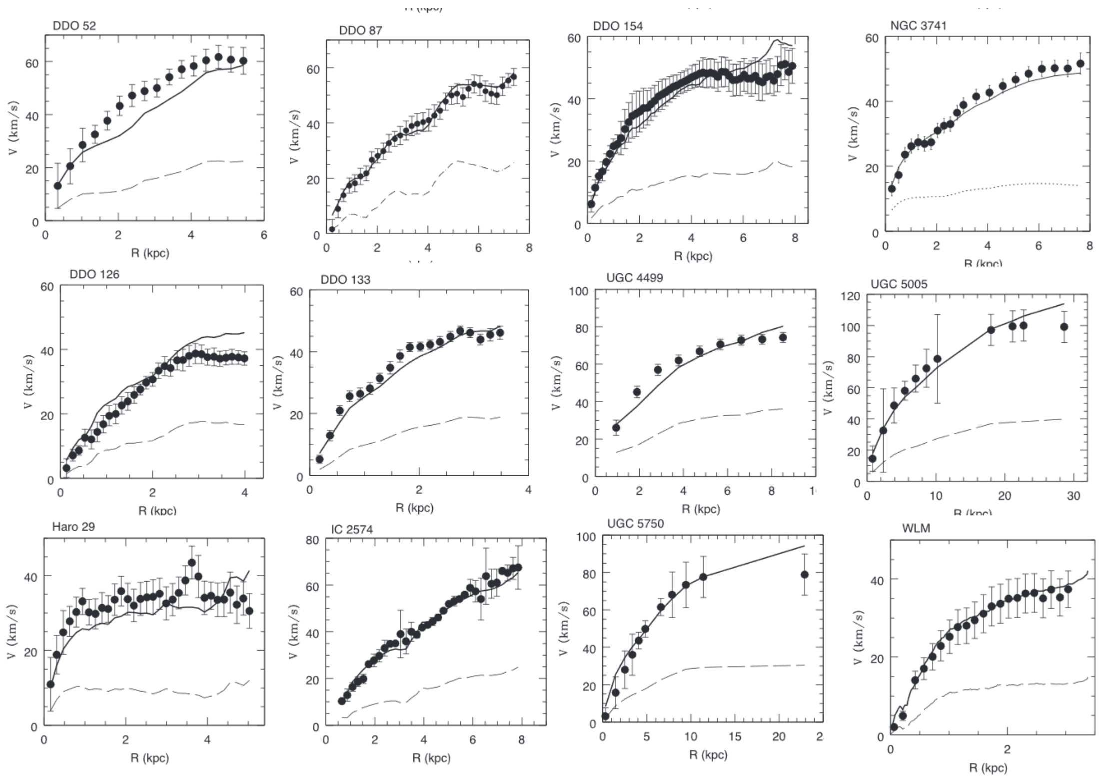


Figure 3. The rotation curves for the first three disc models of Fig. 1: the line is the exact curve; triangles and squares mark the curves calculated, respectively, by the algebraic relation and by the improved approximation.

Rotation curves tests







Sanders 2019

Vertical motions in a disc

Again, in the ‘simple’ case of only $\mathcal{A}_1$ (but note that here $\mathcal{A}_2 \neq 0$)

$$\mu[g_z + g_r\theta(\omega_r/\omega_z)] \stackrel{?}{=} \mu[g_z + g_r\theta(0)] \stackrel{?}{\approx} \mu[g_r\theta(0)]$$

Instead of $\mu[g_r]$

System falling in a "constant" acceleration field

A system with typical internal radius r_i, frequencies ω_i, and accelerations g_i, is falling in an external (MOND) acceleration field of typical size r_o, frequencies ω_o, and acceleration g_o

CoM motion, Galaxy effects in the solar system, EFE, z-motions,

Constant external field (simple model keeping only $\mathcal{A}_1$):

In space: $\frac{dg_o}{dr_o} = 0 \;\; [g_o \left(\frac{dg_o}{dr_o}\right)^{-1} \gg r_i]$

In time: $\omega_o = 0 \quad [\omega_o \ll \omega_i]$

"External motion": $\mu[g_o + g_i\theta(\omega_i/\omega_o)] \Rightarrow \;\; \mu[g_o] \quad \theta(\infty) = 0$

"Internal motions": $\mu[g_i + g_o\theta(\omega_o/\omega_i)] \Rightarrow \;\; \mu[g_i + g_o\theta(0)]$

$[a_0 = 1]$

Center-of-mass motion

“External motion”: $\mu[g_o + g_i\theta(\omega_i/\omega_o)]$

$\omega_o \ll \omega_i$. [For a constant field ($\omega_o = 0$), $\theta = 0$]

g_i can be much larger than g_o

For a finite ω_o, if, e.g., $\theta(x) \stackrel{x\to\infty}{\longrightarrow} 1/x^2 \quad \Rightarrow \qquad \delta g_o/g_o \sim \hat{\mu}(g_o)(r_i/r_o)$

All constituents share the same infall acceleration g_o.

Galaxy effect on the inner solar system

$$a_0 \sim g_o \ll g_i, \quad \omega_o \ll \omega_i$$

"Internal motions": $\mu[g_i + g_o\theta(\omega_o/\omega_i)] = \mu[g_i + g_o\theta(0)]$

The effect is proportional to $1 - \mu(g_i/a_0 \ggg 1) \lll 1$

In AQUAL and QUMOND what appears is $\mu(g_o/a_0)$, not $\mu(x \ggg 1)$

Other solar-system effects?

External-field effect

Unlike in AQUAL/QUMOND the μ (or ν) that enters RCs is not expected to determine the EFE through $\mu(g_{ex}/a_0)$.

Another function altogether (withe the same asymptotes, which follow from the basic tenets, or at least a different variable lik $\alpha g_{ex}/a_0$

$$r_o \gg r_i, \quad g_o \gtrsim g_i$$

Sub-tidal: $g_o(r_i/r_o) \ll g_i \quad \Rightarrow \quad \omega_o \ll \omega_i$ ("adiabatic")

"Internal motions": $\mu[g_i + g_o\theta(\omega_o/\omega_i)] = \mu[g_i + g_o\theta(0)]$

In MG, g_o enters μ with strength g_o, here, as $\theta(0)g_o > g_o$

$\theta(x)$ is decreasing, and $\theta(1) = 1$, so we expect at least $\theta(0) \sim$ a few

Many open questions

Matters of principle

- Uniqueness given two “initial” conditions
- Non-gravitational forces
- Fusion and fission of bodies
- Causality

Practical

- Difficult to treat many (most) many-body problems.

But also much promise