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GRAVITATIONAL MICROLENSING BY THE GALACTIC HALO

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ABSTRACT

The massive halo of our Galaxy has an optical depth to gravitational microlensing $\tau \approx 10^{-6}$. If the halo is made of objects more massive than $\sim 10^{-8}\ M_{\odot}$, then any star in a nearby galaxy has a probability of 10^{-6} to be strongly microlensed at any time. The lensing events last ~ 2 hr if a typical "dark halo" object has a mass of $10^{-6}\ M_{\odot}$, and they last ~ 2 yr for objects of 100 $M_{\odot}$. Monitoring the brightness of a few million stars in the Magellanic Clouds over a time scale between 2 hr and 2 yr may lead to a discovery of "dark halo" objects in the mass range 10^{-6}–$10^{2}\ M_{\odot}$ or it may put strong upper limits on the number of such objects.
Subject headings: galaxies: Magellanic Clouds — gravitation — stars: variables

I. INTRODUCTION

The possibility of gravitational microlensing by stars in distant galaxies has been suggested and studied by many authors (Liebes 1964; Refsdal 1964; Chang and Refsdal 1979, 1984; Gott 1981, Young 1981; Vietri and Ostriker 1983; Nityananda and Ostriker 1984; Subramanian, Chitre, and Narasimaha 1985; Paczyński 1985). Unfortunately, in most cases the time scale of intensity changes of a distant quasar subject to microlensing by a solar mass star located at a cosmological distance is very long, and therefore it is not likely to be observed unless many lensed quasars are monitored for many years. If we want to make the time scale much shorter, we have to consider stars which are much closer to us, such as those in the halo of our own Galaxy. The price we pay for the shortened time scale is rather high: optical depth to gravitational lensing on known stars in the halo of our Galaxy is very small. However, most of the halo mass is believed to be, not in stars, but in some unknown form of "dark matter;" possibly black holes, Jupiters, snowballs, or some elementary particles. If the "dark matter" is made of massive objects, then it may give rise to gravitational lensing with an optical depth of $\sim 10^{-6}$, which is substantially higher than the optical depth for the known halo stars.

The aim of this paper is to present a simple model of microlensing by massive objects that might be present in the halo of our Galaxy. We calculate the probability of the effect, and we discuss some possible observations that may lead to a discovery of the effect or put interesting limits on the masses of individual objects that contribute to the mass of the halo.

II. A MODEL

Vietri and Ostriker (1983) and Nityananda and Ostriker (1984) introduced and developed a very useful concept of optical depth to gravitational microlensing. When optical depth is small, it gives a probability that one star strongly affects the intensity of a distant source of radiation. We consider here a case of a very small optical depth, and therefore gravitational microlensing is always due to just a single point with some mass M. We consider a flat space and a point source of radiation. The equation of gravitational lensing may be written in the deflector's plane as

$$r^2 - r_0 r - R_0^2 = 0 , \tag{1}$$

where the coordinate system is centered on the lensing point mass, the source is at r_0, the image is at r, and

$$R_0^2 = \frac{4GMD}{c^2} , \quad D = \frac{D_d D_{ds}}{D_s} , \tag{2}$$

and all symbols have their usual meaning. The quantity R_0 is the radius of the annular image that is formed when the source and the point mass are perfectly aligned.

The equation (1) has two solutions corresponding to the positions of two images:

$$r_{1,2} = [r_0 \pm (r_0^2 + 4R_0^2)^{1/2}]/2 . \tag{3}$$

Their amplifications are given by

$$A_{1,2} = \text{abs}\left(\frac{r_{1,2}}{r_0}\frac{dr_{1,2}}{dr_0}\right) = \text{abs}\left(\frac{r_{1,2}^4}{r_{1,2}^4 - R_0^4}\right) , \tag{4}$$

and their combined amplification is

$$A \equiv A_1 + A_2 = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}} , \quad u \equiv \frac{r_0}{R_0} . \tag{5}$$

If the optical depth is τ, then the probability that the source is found within a radius R_0 of some point mass is also τ. According to equation (5) the combined amplification of the two images is, in that case, larger than 1.34. Of course, the probability of a smaller amplification is larger, while the probability of a larger amplification is smaller. Let the probability that the amplification is larger than A be $p(A)$. The ratio $p(A)/\tau$ is given as

$$\frac{p(A)}{\tau} = \frac{r_0^2}{R_0^2} = u^2 , \tag{6}$$

and its variation with A is shown in Figure 1.

When a point mass passes between the observer and the source, the apparent intensity of the source varies in proportion to the variation of the combined amplification of the two microimages. This may be easily calculated with equation (5). The variation of a combined intensity with time is shown in Figure 2 for 12 values of the impact parameter d: $d/R_0 = 0.1$,

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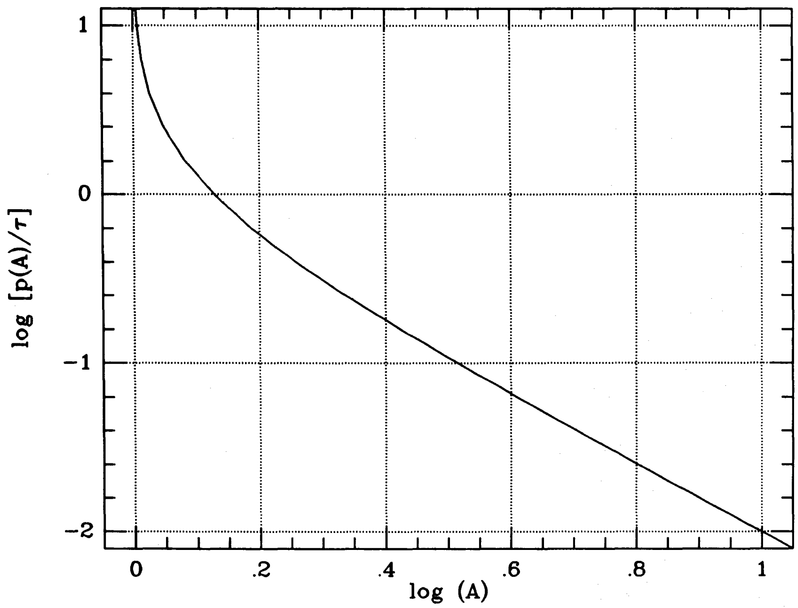

FIG. 1.—Variation of the ratio $p(A)/\tau$ with amplification A. The value $p(A)$ is the probability that a particular source is amplified by a factor larger than A as a result of gravitational microlensing, τ is the optical depth to microlensing, $p(A) = \tau$ for $A \geq 1.34$.

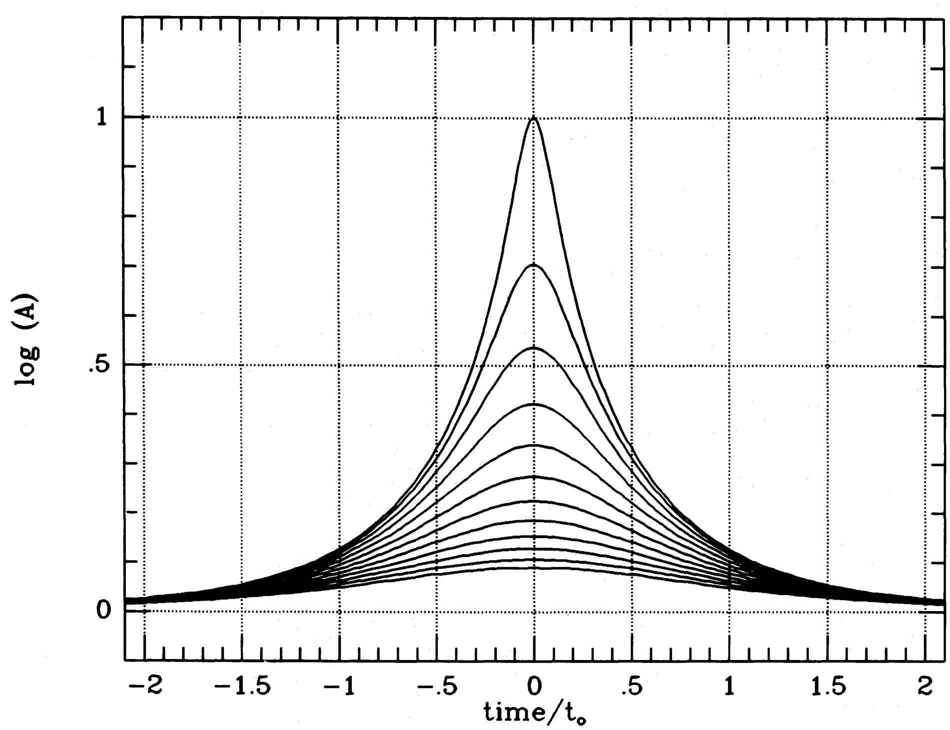

FIG. 2.—Time variation of the amplification due to gravitational microlensing for events with the impact parameter d/R_0 equal 0.1, 0.2, ..., 1.1, 1.2. The largest amplitude corresponds to the smallest impact parameter. The unit of time is given as $t_0 \equiv R_0/v$, where R_0 is the radius of ringlike image formed when the source, the lensing mass, and the observer are perfectly aligned (see eq. [2] and [16]) and v is the relative tangential velocity of the lensing object.

0.2, ..., 1.1, 1.2. The intensity varies more for passages with a smaller impact parameters.

If the source and the observer are stationary and all the point masses move along straight lines with the same tangential velocity v, then the average time interval between the interactions with an impact parameter smaller than d is

$$\langle \Delta t \rangle = \frac{\pi R_0^2}{2vd\tau} = \frac{t_0}{\tau}\frac{\pi R_0}{2d}, \quad t_0 \equiv \frac{R_0}{v}. \tag{7}$$

The optical depth to gravitational microlensing is equal to the ratio of surface mass density of microlensing matter to the critical mass density defined as

$$\sum \text{crit} \equiv \frac{c^2}{4\pi GD}. \tag{8}$$

If the mass density varies along the line of sight, then we have an integral expression for the optical depth

$$\tau = \int_0^{D_s} \frac{4\pi GD}{c^2}\rho(D_d)dD_d, \tag{9}$$

where $\rho(D_d)$ is the average density of microlensing matter at the distance D_d from the observer.

We adopt a simple isothermal model for the mass distribution in the "dark" halo of our Galaxy, with mass $M(R) \propto R$ and density $\rho(R) \propto R^{-2}$.

$$M(R) = \frac{V_{\rm rot}^2}{G}R, \quad \rho(R) = \frac{1}{4\pi R^2}\frac{dM}{dR} = \frac{V_{\rm rot}^2}{4\pi GR^2}, \tag{10}$$

where R is the distance from the center of our Galaxy and $V_{\rm rot}$ is the circular rotational velocity in the gravitational field of the halo. We adopt $V_{\rm rot} = 210$ km s^{-1} = constant.

Let the source of radiation be located in the sky at an angular distance α from the Galactic center. The density of "dark" halo matter along that line of sight may be calculated as

$$\rho(Dd) = \frac{\rho_0}{1 + d_d^2 - 2d_d \cos\alpha}, \quad d_d \equiv \frac{D_d}{R_{\rm GC}}, \quad \rho_0 = \frac{V_{\rm rot}^2}{4\pi GR_{\rm GC}^2}, \tag{11}$$

where ρ_0 is the density of "dark" halo matter near the Sun, the $R_{\rm GC} = 10$ kpc is the adopted distance from the Sun to the Galactic center.

The most important number in this problem is the optical depth of the Galactic halo to gravitational microlensing, since this will determine the probability of the effect. We calculated the optical depth in the direction of four nearby galaxies in which individual stars are easily resolved. Those stars may be used as almost pointlike background sources. The linear distances to LMC, SMC, M31, and M33 were adopted at D_s = 50, 60, 650, and 730 kpc, respectively, following Hodge (1981). The angular distances of those galaxies from the Galactic center were taken as $\alpha = 82°$, 69°, 119°, and 127°, respectively. The extent of "dark" Galactic halo along the line of sight, D_h, was adopted as a free parameter. Equations (9) and (11) may be combined to obtain

$$\tau = \frac{\tau_0}{x_s}\int_0^{x_h} \frac{(x_s - x)x}{1 + x^2 - 2x\cos\alpha}dx, \tag{12a}$$

$$\tau_0 \equiv \frac{V_{\rm rot}^2}{c^2} \approx 5 \times 10^{-7}, \tag{12b}$$

$$x \equiv \frac{D_d}{R_{\rm GC}}, \quad x_s \equiv \frac{D_s}{R_{\rm GC}}, \quad x_h \equiv \frac{D_h}{R_{\rm GC}}, \tag{12c}$$

The integral in equation (12a) may be evaluated analytically:

$$\begin{aligned}\frac{\tau}{\tau_0} = &-\frac{x_h}{x_s} + \left(\frac{1}{2} - \frac{\cos\alpha}{x_s}\right)\ln(x_h^2 - 2x_h\cos\alpha + 1)\\ &+ \frac{1 - 2\cos^2\alpha + x_s\cos\alpha}{x_s\sin\alpha}\\ &+ \arctan\left[\left(\frac{x_h - \cos\alpha}{\sin\alpha}\right) + \frac{\pi}{2} - \alpha\right].\end{aligned} \tag{13}$$

The variation of optical depth with adopted extent of the Galactic halo was calculated with equation (13), and it is shown in Figure 3. In all cases the optical depth turns out to be $\sim 10^{-6}$, which means there is a probability of $\sim 10^{-6}$ that any star in SMC, LMC, M31, or M33 is strongly gravitationally microlensed by a "dark" object in the Galactic halo.

III. DISCUSSION

We have just found that in any nearby galaxy one star out of a million is strongly microlensed by a "dark" object located in the halo of our Galaxy. This is true, provided the "dark" objects are massive enough to enhance significantly the brightness of a real star, which is not exactly a point source, as assumed in the previous chapter of this paper. We shall estimate the lower limit to the mass of a "dark" object that can still strongly enhance intensity of a star which has a small but finite angular size.

The maximum amplification is obtained when the source, the lensing point mass, and the observer are perfectly aligned. In this case, a source, which is a circular disk with a radius r_0 (as projected at the deflector's plane), forms a ringlike image with the inner and outer radii given by equation (3). Notice that $r_0 = R_{\rm star} D_d/D_s$, where $R_{\rm star}$ is the radius of a star located at the distance D_s. In the simplest case of a source with a uniform surface brightness, the total intensity is amplified by a factor A, given as

$$A = \frac{\pi r_1^2 - \pi r_2^2}{\pi r_0^2} = \left(1 + 4\frac{R_0^2}{r_0^2}\right)^{1/2}. \tag{14}$$

The maximum amplification is large, provided $r_0 < R_0$, i.e., provided the angular size of the source is smaller than the angular size of the ringlike, gravitationally lensed image. We may combine this conclusion with equation (2) to estimate the minimum mass of the lensing object:

$$\begin{aligned}M_{\rm min} &\approx \frac{c^2}{4GD}(R_0^2)_{R_0=r_0} = \frac{c^2}{4GD}\left(R_{\rm star}\frac{D_d}{D_s}\right)^2\\ &\approx \frac{c^2D_d}{4G}\left(\frac{R_{\rm star}}{D_s}\right)^2\\ &\approx 3 \times 10^{-9}M_\odot\left(\frac{D_d}{10\ \text{kpc}}\right)\left(\frac{R_{\rm star}}{R_\odot}\right)^2\left(\frac{100\ \text{kpc}}{D_s}\right)^2,\\ &\text{for} \quad D_s \gg D_d.\end{aligned} \tag{15}$$

A typical distance to a "dark halo" object may be $D_d \approx 10$ kpc. For the adopted distances to LMC, SMC, M31, and M33 and for a solar radius star we obtain the minimum mass of a "dark halo" object capable of microlensing: 1.2×10^{-8}, 0.8×10^{-8}, 7×10^{-11}, and 6×10^{-11} $M_\odot$, respectively. These

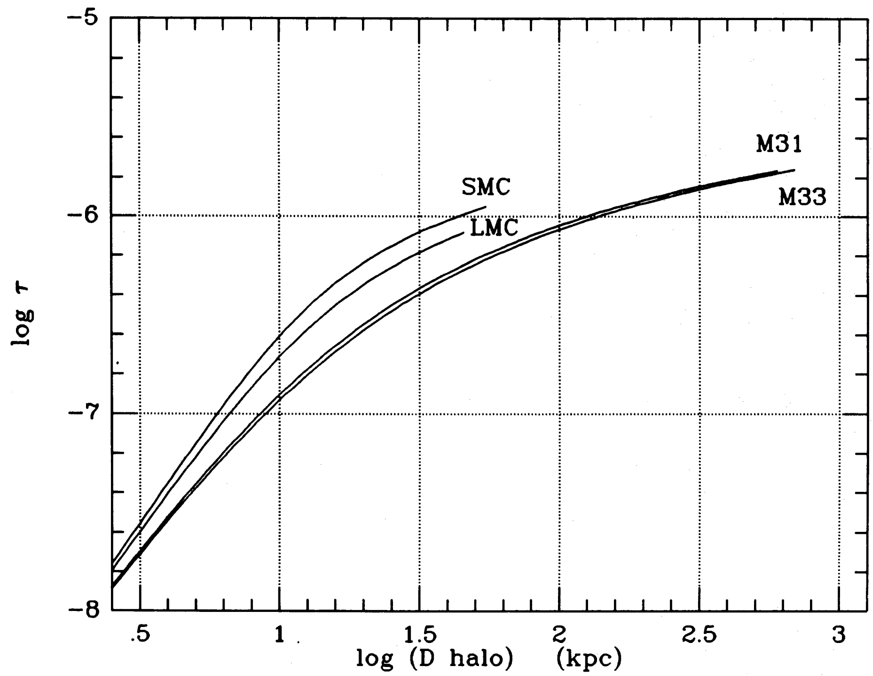

FIG. 3.—Variation of the optical depth to gravitational microlensing, with the extent of the dark halo of our Galaxy is shown for four nearby galaxies, in which individual stars are easily observable.

are very low masses indeed, and we have a potential here to discover objects as small as asteroids, only 1000 km in diameter, if they make up the massive halo of our Galaxy. Of course, this lower mass limit becomes higher when the source is a giant star, as indicated by equation (15).

The radius of a ringlike image that is formed when the source, the lensing mass, and the observer are perfectly aligned is given by equation (2). Its numerical value is

$$R_0 \approx 1.4 \times 10^{14}\ \mathrm{cm} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D_d}{10\ \mathrm{kpc}}\right)^{1/2}, \quad \text{for } D_s \gg D_d . \tag{16}$$

This is much larger than the likely radius of a "dark" halo object if it is a black hole, a star, a planet, or an asteroid.

Let us estimate now the expected time scale for the intensity variation of a microlensed star. That time scale is given as t_0 (see eq. [7] and Fig. 2). For a "dark" lensing object located at 10 kpc and moving with a tangential velocity of ~ 200 km s^{-1} this time scale is given as

$$t_0 \equiv \frac{R_0}{v} \approx 6 \times 10^6 s \left(\frac{M}{M_\odot}\right)^{1/2} \approx 0.2\ \mathrm{yr} \left(\frac{M}{M_\odot}\right)^{1/2} . \tag{17}$$

The brightness of a solar radius star located in the Magellanic Clouds may vary on a time scale as short as 10 minutes, because of microlensing by a $10^{-8}\ M_\odot$ "dark halo" object, while a similar star in M31 or M33 may vary on even shorter time scale of 1 minute, because of microlensing by a $10^{-10}\ M_\odot$ "dark halo" object, the lowest masses capable of microlensing. A solar mass "dark halo" object would microlens on a time scale of ~ 2 months.

The expected frequency of microlensing events may be estimated with equation (7). If we are interested in events during which the image intensity varies by a factor of 1.34 or more, i.e., the impact parameter for the lensing encounter is $d < R_0$, then, for the optical depth of 10^{-6}, the average time interval between the events will be

$$\langle \Delta t \rangle \approx \frac{1}{N} \frac{t_0}{\tau} \frac{\pi}{2} \frac{R_0}{d} \approx \frac{1.5 \times 10^6}{N} t_0 , \tag{18}$$

where N is the number of stars that are monitored for light variations. Notice that if we could detect brightness variations smaller than 0.3 mag, then the encounters with an impact parameter $d/R_0 > 1$ would be detectable and the frequency of events would be somewhat higher. If we specify the threshold of detectability as a variation of brightness by a factor A, then we may use equation (5) to calculate the impact parameter $u = d/R_0$ and then calculate the frequency of events with equation (18).

If the observing program was to last for 2 yr, then microlensing events due to a 100 $M_\odot$ "dark halo" object would be detectable. The shortest timescales for the variation of brightness of faint stars that might be easily detectable are probably ~ 100 minutes or so, corresponding to the "dark halo" objects of $10^{-6}\ M_\odot$. We conclude that an observing program aimed at monitoring the brightness of a few million stars over a period of 2 yr could put interesting constraints on the masses of objects that make the mass of the Galactic halo. Such a program could even detect the effect of microlensing due to the "dark halo" objects if their masses are in the range $10^{-6} < M/M_\odot < 10^2$. A single event may give only a rough estimate of a "dark object," since its distance and its velocity within the Galactic halo may be estimated only statistically. To learn about the distribution of masses of the "dark objects" we have to observe a large number of lensing events.

For the program to be successful, the brightness of a few million stars in one of the nearby galaxies has to be monitored on a time scale from hours to years. The data-processing aspect of the suggested observational program seems formidable. However, there are already many projects at various observatories that involve automatic scanning of photographic plates and automatic processing of the data. According to Freeman (1985), the idea of monitoring the brightness of a million stars may be feasible even now. Certainly, a lot of new intrinsically variable stars would be discovered first, and this is a very interesting project in its own right. It is known that most stars do not vary much, and once the variables are recognized, the rest may be looked at for possible gravitational lensing events. If a candidate for such event is found, then it may be looked at more carefully in order to verify its identity. If the variations are very rapid, then lensing may resolve the stellar disk and lead to complicated light curves and spectral variations due to limb darkening of the lensed star. If the time scale for variability is more than a few days and if the target star is a main-sequence dwarf, then a point-source approximation is excellent and there should be no spectral or color changes throughout the event. A good coverage of light variations may allow a detailed comparison with one of the expected light curves, as shown in Figure 2. If this closer look finds an event that is a good candidate for lensing, then very accurate photometry may allow a very precise comparison even with the low-amplitude decay tail and may discriminate between a lensing event and intrinsic variability.

Gravitational lensing will change somewhat the apparent position of the lensed star. Unfortunately, this effect is very small, the angular displacement is roughly equal to $(r_g/D)^{1/2}$, where $r_g = GM/c^2$ is a gravitational radius of the lensing point mass M and D is the effective distance to the lens (see [2]). For a solar-mass lens at 10 kpc the angle is only 1 mas.

It is clear that the observational project is not simple, but one of its by-products, a systematic discovery of a large number of variable stars in a nearby galaxy, is attractive, even if no lensing events are discovered.

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