AS 4022: Cosmology

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Online notes:
star-www.st-and.ac.uk/~hz4/cos/cos.html

take your own notes (including blackboard lectures

Look forward

Malcolm S. Longair's "Galaxy Formation" 2nd edition [Library]

Chpt 1-2,5-8: expanding metrics, energy density, curvature, distances

Chpt 4,11,15,20: DM, Structure growth, inflation

Chpt 9-10,13: Thermal History of Particle Reaction, Neutrinos, WIMPs

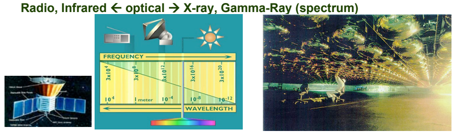
Text (intro): Andrew Liddle: Intro to Modern Cosmology
(advanced): John Peacock: Cosmological Physics
Web Lecture Notes: John Peacock, Ned Wright

Why Study Cosmology?

- **Fascinating questions:**
  - Birth, life, destiny of our Universe
  - Hot Big Bang --> (75% H, 25% He) observed in stars!
  - Formation of structure (galaxies …)
- **Technology -> much recent progress:**
  - Precision cosmology: uncertainties of 50% --> 2%
- **Deep mysteries remain:**
  - Dark Matter? Dark Energy? General Relativity wrong?
- **Stretches your mind:**
  - Curved expanding spaces, looking back in time, …

Observable Space-Time and Bands

See What is out there? In all Energy bands
Pupil → Galileo's Lens → 8m telescopes → square km arrays
Radio, Infrared ← optical → X-ray, Gamma-Ray (spectrum)

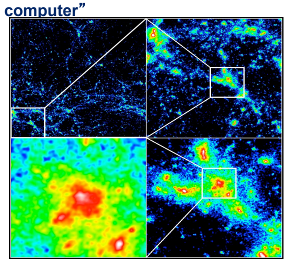


COBE satellites ← Ground → Underground DM detector

Know How were we created? XYZ & T ?
Us, CNO in Life, Sun, Milky Way, … further and further
→ first galaxy → first star → first Helium → first quark
Now → Billion years ago → first second → quantum origin

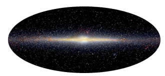
The Visible Cosmos: a hierarchy of structure and motion

"Cosmos in a computer"



Observe A Hierarchical Universe

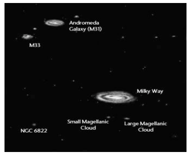
Planets
moving around stars;
Stars grouped together,
moving in a slow dance around the center of galaxies.



Cosmic Village

The Milky Way and Andromeda galaxies,
- along with about fifteen or sixteen smaller galaxies,
- form what's known as the Local Group of galaxies.

The Local Group
- sits near the outer edge of a supercluster, the Virgo cluster.
- the Milky Way and Andromeda are moving toward each other,
- the Local … e of the Virgo cluster, and



the entire Virgo cluster itself,
- is speeding toward a mass
- known only as "The Great Attractor."

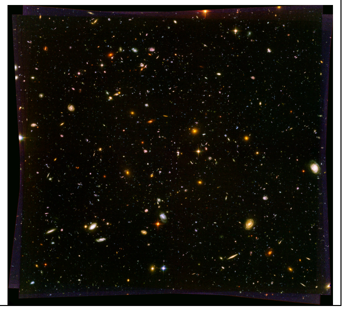
Hubble Deep Field:

At faint magnitudes, we see **thousands of Galaxies for every star** !

~10^{10} galaxies in the visible Universe

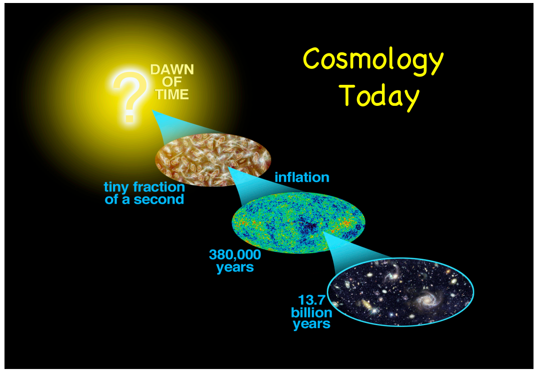
~10^{10} stars per galaxy

~10^{20} stars in the visible Universe

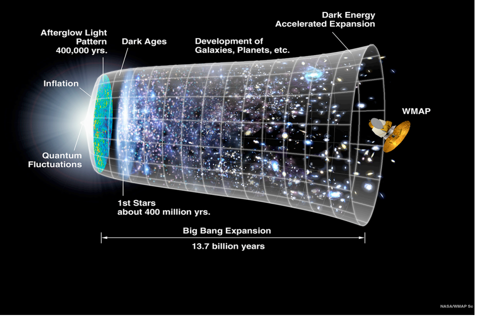


Galaxies themselves
- some 100 billion of them in the observable universe—
- form galaxy clusters bound by gravity as they journey through the void.

But the largest structures of all are superclusters,
- each containing thousands of galaxies
- and stretching many hundreds of millions of light years.
- are arranged in filament or sheet-like structures,
- between which are gigantic voids of seemingly empty space.



Accelerating/Decelerating Expansion



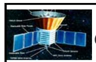
Introducing Gravity and DM (Key players)

These structures and their movements
- can't be explained purely by the expansion of the universe

must be guided by the gravitational pull of matter.

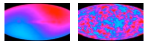
Visible matter is not enough

one more player into our hierarchical scenario: dark matter.



Cosmologists hope to answer these questions:

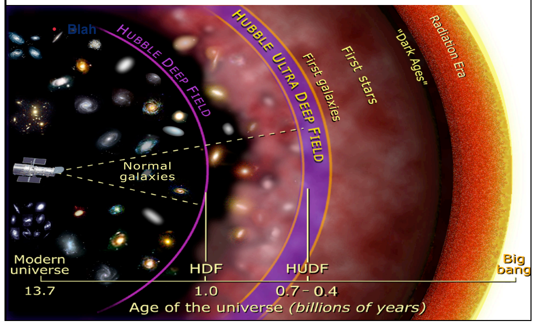
How old is the universe? H_0
Why was it so smooth? P(k), inflation



How did structures emerge from smooth? N-body
How did galaxies form? Hydro

Will the universe expand forever? Omega, Lamda
Or will it collapse upon itself like a bubble?

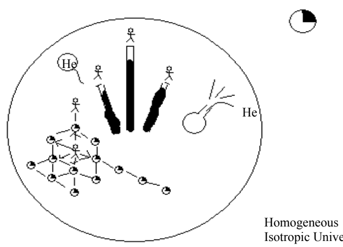
Looking Back in Time



main concepts in cosmology

Expansion & Metric
Cosmological Redshift
Energy density

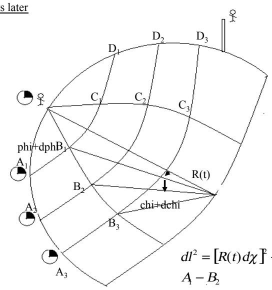
A few mins after New Year
Celebration at Trafalgar Square



Homogeneous
Isotropic Universe

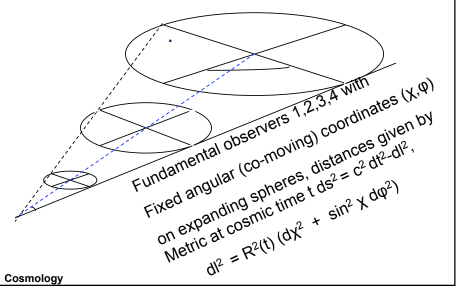
Walking ↔ Elevating ↔ Earth Radius Stretching $R(t)$

Feb 14 t=45 days later



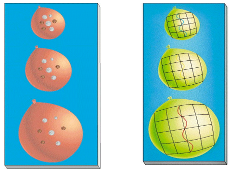
$$dl^2 = [R(t)d\chi]^2 + [R(t)\sin\chi d\phi]^2$$
$$A_1 - B_2$$

1st concept Metric: ant network on expanding sphere



Stretch of photon wavelength in expanding space

Emitted with intrinsic wavelength λ_0 from Galaxy A at time $t<t_{now}$ in smaller universe $R(t) < R_{now}$

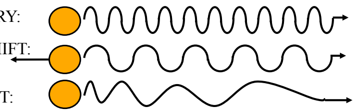


→ Received at Galaxy B now (t_{now}) with λ

$\lambda / \lambda_0 = R_{now} / R(t) = 1+z(t) > 1$

Redshift

- Expansion is a <u>stretching</u> of space.
- The more space there is between you and a galaxy, the faster it <u>appears</u> to be moving away.
- Expansion **stretches the wavelength of light**, causing a galaxy's spectrum to be REDSHIFTED:



REDSHIFT IS NOT THE SAME AS DOPPLER SHIFT

2nd main concept: Cosmological Redshift

The space/universe is expanding,

Galaxies (pegs on grid points) are receding from each other

As a photon travels through space, its wavelength becomes stretched gradually with time.

Photons wave-packets are like links between grid points

This redshift is defined by:

$$z \equiv \frac{\lambda - \lambda_o}{\lambda_o}$$

$$\frac{\lambda}{\lambda_o} = 1 + z$$

Galaxy Redshift Surveys

Large Scale Structure:

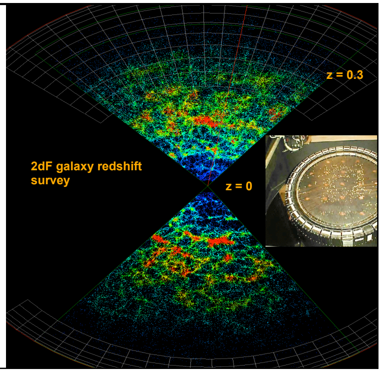
Empty **voids**

~50Mpc.

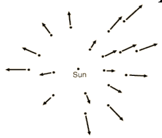
Galaxies are in

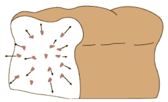
1. **Walls** between voids.
2. **Filaments** where walls intersect.
3. **Clusters** where filaments intersect.

Like Soap Bubbles !



Isotropic Expansion



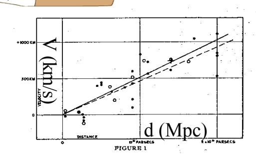


Hubble law :

$$V = H_0\, d$$

Hubble "constant":

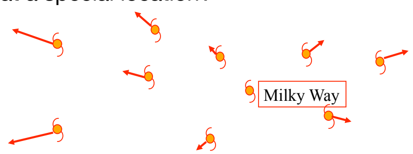
$$H_0 \approx 500 \text{ km s}^{-1}\text{Mpc}^{-1}$$



Why WRONG? Extinction by interstellar dust was not then known, giving incorrect distances.

Universal Expansion

Hubble's law appears to violate The Copernican Principle. Are we at a special location?



Is everything moving away from us?

Universal Expansion

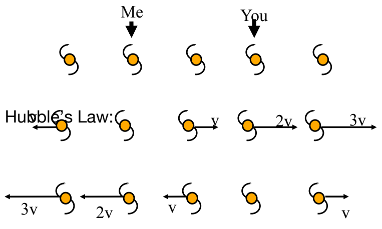
Q : What is so special about our location ?

A : Nothing !

According to Hubble's Law:

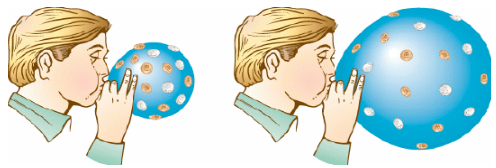
I see:

You see:



The Universal Expansion

- An observer in any galaxy sees all other galaxies moving away, with the same Hubble law.
- Expansion (or contraction) produces a <u>centre-less</u> but dynamic Universe.



E.g. Consider a quasar with redshift z=2. Since the time the light left the quasar the universe has expanded by a factor of 1+z=3. At the epoch when the light left the quasar,

- the distance between us and Virgo (presently 15Mpc) was 5Mpc.
- the CMB temperature then (presently 3K) was 9K.
- the quasar appears receding with Doppler speed V=2c.
- The quasar appears at a look-back "distance" d = [t(0)-t(z)] c, where the look-back time [t(0)-t(z)] ~ z/H_0 at small z<<1, H_0^{-1}=Taylor expansion coeff.

$$1+z = \frac{\lambda_{now}}{\lambda(t)} \quad \text{(wavelength)}$$

$$= \frac{R_{now}}{R(t)} \quad \text{(expansion factor)}$$

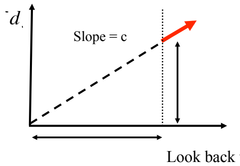
$$= \frac{T(t)}{T_{now}} \quad (\text{Photon Blackbody T } \propto 1/\lambda, \textit{why?})$$

Hubble Law → typical age (at z=1)

$$V = H_0 \, d$$

$$t_0 \approx \frac{d}{V} = \frac{1}{H_0} = \left(\frac{1\,\text{Mpc}}{72\,\text{km/s}}\right)\left(\frac{3\times10^{19}\,\text{km}}{\text{Mpc}}\right)\left(\frac{1\,\text{yr}}{3\times10^{7}\,\text{s}}\right)$$

$$\approx 13\times10^{9}\,\text{yr} = 13\ \text{Gyr.}$$



Convert H as a frequency Hertz, find an integer close to log10(H)? Multiple choices -30, -10, 0, 10, 30.

3rd concept: The changing rate of expansion

Newtonian Analogy:

Consider a sphere of radius R(t) ,

→ effective mass inside M = $4\pi\rho R^3/3$

if energy density inside is $\epsilon=\rho(t) c^2$.

On this expanding sphere, a test m

Kin.E. + Pot.E. = const Energy

→ $m (dR/dt)^2/2 - G m M/R = (-k/2) m c^2$

→ $(dR/dt)^2/(2c^2) - (4\pi G/3)\rho R^2/c^2 = -k/2$

Unitless k <0, =0, >0 → open-flat-close

Newtonian expansion satisfies

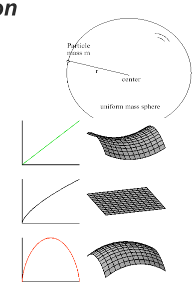
$H^2 = (dR/dt/R)^2 = (\rho + \rho_{cur}) (8\pi G/3)$

the cst k absorbed in "density"

$\rho_{cur}(t) = -k(cH_0^{-1}/R)^2 (3H_0^2/8\pi G)$

$\sim -k R^{-2} * cst$

Now $H = H_0$



4th Concept: The Energy density of Universe

The Universe is made up of three things:

- VACUUM
- MATTER
- PHOTONS (radiation fields)

The total energy density of the universe is made up of the sum of the energy density of these three components.

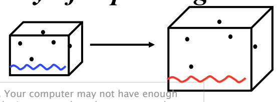
$$\epsilon(t) = \epsilon_{vac} + \epsilon_{matter} + \epsilon_{rad}$$

From t=0 to t=10^9 years the universe has expanded by R(t).

Energy Density of expanding box

volume R^3

N particles



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Cold gas or Cold DM ($p << mc$)

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Radiation: ($m = 0$)

Hot neutrino: ($p >> mc > 0$)

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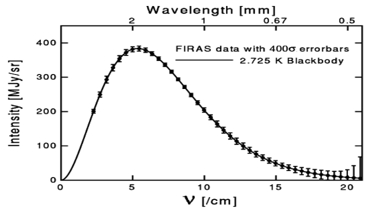
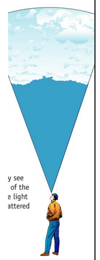
COBE spectrum of CMB





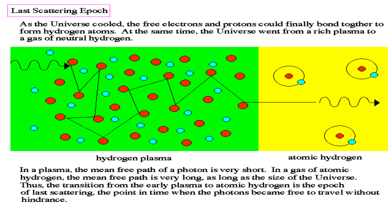
A perfect Blackbody !

No spectral lines -- strong test of Big Bang.
Expansion preserves the blackbody spectrum.

$T(z) = T_0 (1+z)$ $T_0 \sim 3000$ K $z \sim 1100$

Acronyms in Cosmology

- **Cosmic Background Radiation (CBR)**
  - **Or CMB (microwave because of present temperature 3K)**
    - Tutorial: Argue **about 10^5 photons fit in a 10cmx10cmx10cm microwave oven. [Hint: $3kT = h c / \lambda$]**



Cosmic Neutrino Background:

neutrinos (Hot DM) decouple from electrons (due to very weak interaction) while still hot (relativistic 0.5 Mev ~ $kT > mc^2$ ~ 0.02-2 eV)

Presently there are 3 x 113 neutrinos and 452 CMB photons per cm^3 . Details depend on

Neutrinos have 3 species of spin-1/2 fermions while photons are 1 species of spin-1 bosons

Neutrinos are a wee bit colder, 1.95K vs. 2.7K for photons [during freeze-out of electron-positions, more photons created]

Initially mass doesn't matter in hot universe

relativistic (comparable to photon number density ~ $R^{-3} \sim T^3$),

frequent collisions with other species to be in thermal equilibrium and cools with photon bath.

Photon numbers (approximately) conserved, so is the number of relativistic massive particles

Concept: Particle-Freeze-Out?

Freeze-out of equilibrium means NO LONGER in thermal equilibrium.

Freeze-out temperature means a species of particles have the SAME TEMPERATURE as radiation up to this point, then they bifurcate.

Decouple = switch off the reaction chain

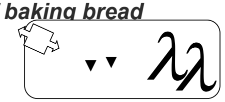
= insulation = Freeze-out

a massive particle

CDM/WIMPs: Cold Dark Matter, weakly-interact massive particles

- **If DM decoupled from photons at kT ~ 10^{14}K ~ 0.04 mc^2**
  - Then that dark particles were
    - **non-relativistic (v/c << 1), hence "cold".**
    - **And massive ($m >> m_{proton}$ =1 GeV)**

Eq. of State for Expansion & analogy of baking bread



Vacuum~air holes in bread

Matter ~nuts in bread

Photons ~words painted

Verify expansion doesn't change N_{hole}, N_{proton}, N_{photon}

No Change with rest energy of a proton, changes energy of a photon

$$\varepsilon(t) = \rho_{eff}(t) c^2$$

$$\frac{\varepsilon(t)}{c^2} = \rho_{eff}(t)$$

VACUUM ENERGY: $\rho = \text{constant} \quad \Rightarrow \mathrm{E}_{vac} \propto R^3$

MATTER: $\rho R^3 = \text{constant}, \quad \Rightarrow m \approx \text{constant}$

RADIATION:number of photons N_{ph} = constant

Wavelength stretches : $\lambda \sim R$

$$\Rightarrow n_{ph} \approx \frac{N_{ph}}{R^3} \qquad \text{Photons:} E = h\upsilon = \frac{hc}{\lambda} \sim \frac{1}{R}$$

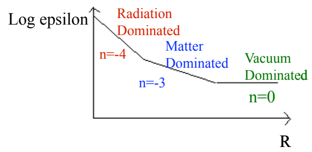
$$\Rightarrow \varepsilon_{ph} \sim n_{ph} \times \frac{hc}{\lambda} \sim \frac{1}{R^4}$$

Total Energy Density $rho\, c^2$ =epsilon

is given by:

$$\varepsilon \propto \varepsilon_{vac} + \varepsilon_{matter} + \varepsilon_{ph}$$

$$\propto R^0 \qquad \propto R^{-3} \qquad \propto R^{-4}$$



Tutorial: Typical scaling of expansion

$H^2 = (dR/dt)^2/R^2 = 8\pi G\,(\rho_{cur} + \rho_m + \rho_r + \rho_v)/3$

Assume domination by a component $\rho \sim R^{-n}$

Show Typical Solutions Are

$\rho \propto R^{-n} \propto t^{-2}$

$n = 2$(curvature constant dominate)

$n = 3$(*matter* dominate)

$n = 4$(*radiation* dominate)

$n \sim 0$(vaccum dominate) : $\ln(R) \sim t$

Argue also $H = (2/n)\, t^{-1} \sim t^{-1}$. Important thing is scaling!

Tutorial: Eternal Static (R=cst) and flat (k=0) Universe

Einstein introduced Λ to enable an eternal static universe.

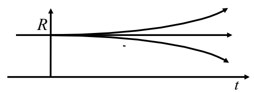
$$\dot{R}^2 = \left(\frac{8\pi G \rho + \Lambda}{3}\right) R^2 - k c^2$$

$$\ddot{R} = 0 \quad \rightarrow \quad \Lambda = \frac{3 k c^2}{R^2} - 8\pi G \rho$$

Einstein's biggest blunder. (Or, maybe not.)

Static models unstable.

Fine tuning.



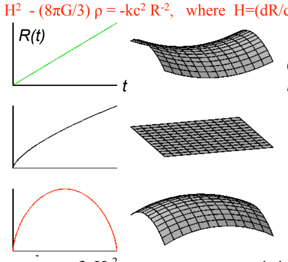
Density - Evolution - Geometry

$H^2 - (8\pi G/3)\,\rho = -kc^2 R^{-2}$, where $H = (dR/dt/R)$

$\rho < \rho_c$

$\rho = \rho_c$

$\rho > \rho_c$



Open *k = -1*

Flat *k = 0*

Closed *k = +1*

$$\rho_c \equiv \frac{3 H_0^2}{8\pi G} \approx 10^{-26}\,\text{kg m}^{-3} \approx \frac{1.4 \times 10^{11}\,\text{Msun}}{(\text{Mpc})^3}$$

E.g.,: Empty Universe without vacuum

$$\dot{R}^2 = \left(\frac{8\pi G\,\rho + \Lambda}{3} \right) R^2 - k\,c^2$$

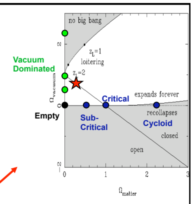
Set $\rho = 0, \quad \Lambda = 0.$ Then $\dot{R}^2 = -k\,c^2$

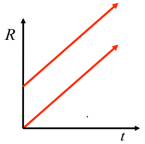
$\rightarrow \quad k = -1$ (negative curvature)

$\dot{R} = c, \quad R = c\,t$

$$H \equiv \frac{\dot{R}}{R} = \frac{1}{t}$$

$$\text{age}: \quad t_0 = \frac{R_0}{c} = \frac{1}{H_0}$$





Negative curvature drives rapid expansion/flattening

Four Pillars of Hot Big Bang

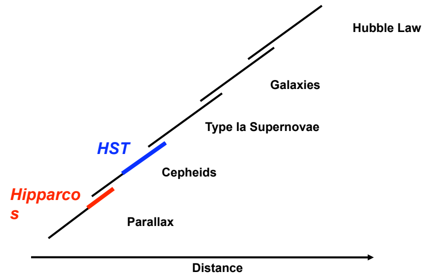
Galaxies moving apart from each other
- Redshift or receding from each other
- Universe was smaller.

Helium production outside stars
- Universe was hot, at least 3x10⁹K to fuse 4H → He, to overcome a potential barrier of 1MeV.

Nearly Uniform Radiation 3K Background (CMB)
- Universe has cooled, hence expanded by at least a factor 10^9. Photons ($3K \sim 10^{-4}$eV) are only 10^{-3} of baryon energy density, so photon-to-proton number ratio ~ 10^{-3}(GeV/10^{-4}eV) ~ 10^9

Missing mass in galaxies and clusters (Cold DM)
- Cluster potential well is deeper than the potential due to baryons.
- CMB fluctuations: photons climb out of random potentials of DM.
- If 1/10 of the matter density in 1GeV protons, 9/10 in dark particles of e.g. 9GeV, then dark-to-proton number density ratio ~ 1

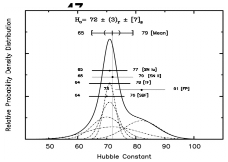
Cosmic Distance Ladder

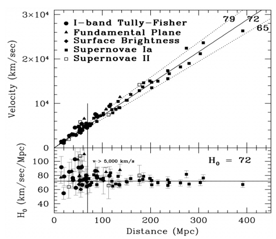


H_0 from the HST Key Project

$$H_0 \approx 72 \pm 3 \pm 7 \quad \text{km s}^{-1}\ \text{Mpc}^{-1}$$

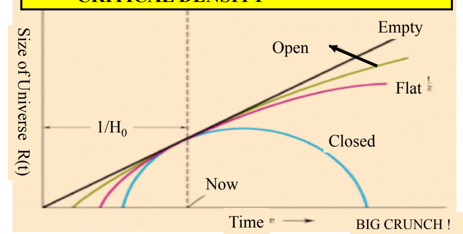
Freedman, et al. 2001 ApJ 553, 47





Re-collapse or Eternal Expansion ?

Inflation => expect FLAT GEOMETRY CRITICAL DENSITY



Hubble Parameter Evolution -- H(z)

$$H^2 \equiv \left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k\,c^2}{R^2}$$

$$\frac{H^2}{H_0^2} = \Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda - \frac{k\,c^2}{H_0^2 R_0^2} x^2$$

$$\text{evaluate at } x = 1 \quad \rightarrow \quad 1 = \Omega_0 - \frac{k\,c^2}{H_0^2 R_0^2}$$

Dimensionless Friedmann Equation:

$$\frac{H^2}{H_0^2} = \Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda + (1 - \Omega_0)\, x^2$$

$$x = 1 + z = R_0 / R$$

$$\rho_c = \frac{3\,H_0^2}{8\pi G}$$

$$\Omega_M \equiv \frac{\rho_M}{\rho_c}, \quad \Omega_R \equiv \frac{\rho_R}{\rho_c}$$

$$\Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda}{3\,H_0^2}$$

$$\Omega_0 \equiv \Omega_M + \Omega_R + \Omega_\Lambda$$

Curvature Radius today:

$$R_0 = \frac{c}{H_0} \sqrt{\frac{k}{\Omega_0 - 1}} \quad \rightarrow \begin{cases} k = +1 & \Omega_0 > 1 \\ k = 0 & \Omega_0 = 1 \\ k = -1 & \Omega_0 < 1 \end{cases}$$

Density determines Geometry

Possible Universes

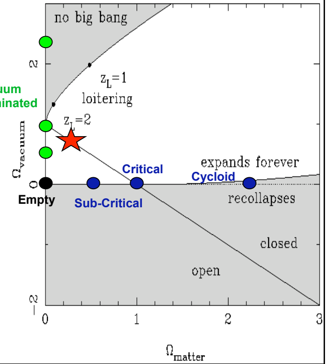
$$H_0 \approx 70 \frac{\text{km/s}}{\text{Mpc}}$$

$\Omega_M \sim 0.3$

$\Omega_\Lambda \sim 0.7$

$\Omega_R \sim 8\times10^{-5}$

$\Omega = 1.0$



Precision Cosmology

$h = 71 \pm 3$ expanding

$\Omega = 1.02 \pm 0.02$ flat

$\Omega_b = 0.044 \pm 0.004$ baryons

$\Omega_M = 0.27 \pm 0.04$ Dark Matter

$\Omega_\Lambda = 0.73 \pm 0.04$ Dark Energy

$t_0 = 13.7 \pm 0.2 \times 10^9$ yr now

$t_* = 180^{+220}_{-80} \times 10^6$ yr $\quad z_* = 20^{+10}_{-5}$ reionisation

$t_R = 379 \pm 1 \times 10^3$ yr $\quad z_R = 1090 \pm 1$ recombination

(From the WMAP 1-year data analysis)

Cosmology Milestones

- 1925 Galaxy redshifts $\lambda = \lambda_0 \; (1+z) \qquad V = c\,z$
  - Isotropic expansion. (Hubble law $V = H_0 d$)
  - Finite age. ($t_0 = 13 \times 10^9$ yr)
- 1965 Cosmic Microwave Background (CMB)
  - Isotropic blackbody. $T_0 = 2.7$ K
  - Hot Big Bang $T = T_0 (\, 1 + z \,)$
- 1925 General Relativity Cosmology Models :
  - Radiation era: $R \sim t^{1/2} \quad T \sim t^{-1/2}$
  - Matter era: $R \sim t^{2/3} \quad T \sim t^{-2/3}$
- 1975 Big Bang Nucleosynthesis (BBN)
  - light elements (^{1}H … ^{7}Li) $\quad t \sim 3$ min $\quad T \sim 10^9$ K
  - primordial abundances (75% H, 25% He) as observed!

Tutorial: 3 Eras: radiation-matter-vacuum

radiation : $\rho_R \propto R^{-4}$

matter : $\rho_M \propto R^{-3}$

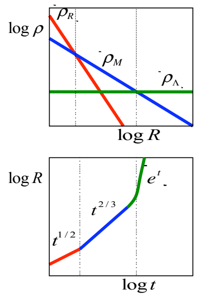
vacuum: $\rho_\Lambda = \text{const}$

$$a \equiv \frac{R}{R_0} = \frac{1}{1+z}$$

$$\rho = \frac{\rho_{R,0}}{a^4} + \frac{\rho_{M,0}}{a^3} + \rho_\Lambda$$

$\rho_R = \rho_M$ at $a \sim 10^{-4}$ $\quad t \sim 10^4$ yr

$\rho_M = \rho_\Lambda$ at $a \sim 0.7$ $\quad t \sim 10^{10}$ yr



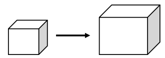
Presently vacuum is twice the density of matter.

5th concept: Equation of State w

Equation of state :

$$\rho \propto R^{-n} \qquad n = 3(1+w)$$

$$w \equiv \frac{\text{pressure}}{\text{energy density}} = \frac{p}{\rho c^2} = \frac{n}{3} - 1$$



Radiation : $(n = 4, w = 1/3)$

$$p_R = \frac{1}{3} \rho_R \; c^2$$

Matter : $(n = 3, w = 0)$

$$p_M \sim \rho_M \; c_S^{\,2} << \rho_M \; c^2$$

Vacuum : $(n = 0, w = -1)$

$$p_\Lambda = -\rho_\Lambda \; c^2$$

Negative Pressure ! ?

$$d[energy] = work$$

$$d[\rho\, c^2 R^3] = -p\; d[R^3]$$

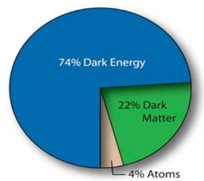
$$\rho c^2 \,(\, 3 R^2 dR \,) + R^3 \, c^2 d\rho = -p \,(\, 3 R^2 dR \,)$$

$$1 + \frac{R\, d\rho}{3\, \rho\, dR} = -\frac{p}{\rho c^2} \equiv -w$$

$$w = -\frac{1}{3} \frac{d[\ln \rho]}{d[\ln R]} - 1$$

$$w = \frac{n}{3} - 1$$

Current Mysteries from Observations



Dark Matter ?
Holds Galaxies together
Triggers Galaxy formation

Dark Energy ?
Drives Cosmic Acceleration
and negative w.

Modified Gravity ?
General Relativity wrong ?

Density Parameters

critical density : density parameters (today) :

$$\rho_c \equiv \frac{3\,H_0^{\,2}}{8\pi\,G} \qquad \Omega_R \equiv \frac{\rho_R}{\rho_c} \quad \Omega_M \equiv \frac{\rho_M}{\rho_c} \quad \Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda}{3\,H_0^{\,2}}$$

total density parameter today :

$$\Omega_0 \equiv \Omega_R + \Omega_M + \Omega_\Lambda$$

density at a past/future epoch in units of today' s critical density :

$$\Omega \equiv \frac{\rho}{\rho_c} = \sum_w \Omega_w x^{3(1+w)} = \Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda \qquad x \equiv 1 + z = R_0 / R$$

in units of critical density at the past/future epoch :

$$\Omega(x) \equiv \frac{8\pi G\rho}{3H^2} = \frac{H_0^{\,2}}{H^2}\sum_w \Omega_w x^{3(1+w)} = \frac{\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda}{\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda + (1-\Omega_0) x^2}$$

Note: radiation dominates at high z, can be neglected at lower z.

Key Points

- Scaling Relation among
  - Redshift: z,
  - expansion factor: R
    - Distance between galaxies
  - Temperature of CMB: T
    - Wavelength of CMB photons: lambda
- Metric of an expanding 2D+time universe
  - Fundamental observers
    - Galaxies on grid points with fixed angular coordinates
- Energy density in
  - vacuum, matter, photon
  - How they evolve with R or z
- If confused, recall the analogies of
  - balloon, bread, a network on red giant star, microwave oven

Sample a wide range of topics Theoretical and Observational

Universe of uniform density
- Metrics ds, Scale R(t) and Redshift
- EoS for mix of vacuum, photon, matter, geometry, distances

Thermal history
- Freeze-out of particles,
- Neutrinos, CDM wimps
- Nucleo-synthesis He/D/H

Structure formation
- Inflation and origin of perturbations
- Growth of linear perturbation
- Relation to CMB peaks, sound horizon

Quest of H0 (obs.)
- Applications of expansion models
- Distances Ladders

Cosmic Background
- COBE/MAP/PLANCK etc.
- Parameters of cosmos

Quest for Omega (obs.)
- Galaxy and SNe surveys
- Luminosity Functions

(thanks to slides from K. Horne)

Hongsheng Zhao

6^{th} concept: Distances in Non-Euclidean Curved Space

How Does Curvature affect Distance Measurements ?

Is the universe very curved?

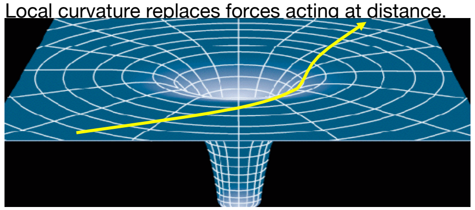
Geodesics

Gravity = curvature of space-time by matter/energy.

Freely-falling bodies follow **geodesic trajectories**.

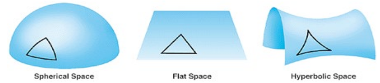
Shortest possible path in curved space-time.

Local curvature replaces forces acting at distance.



Is our Universe Curved?

	Closed	Flat	Open

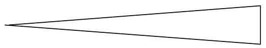


	Closed	Flat	Open
Curvature:	+	0	--
Sum of angles of triangle:	> 180°	= 180°	< 180°
Circumference of circle:	< 2 p r	= 2 p r	> 2 p r
Parallel lines:	converge	remain parallel	diverge
Size:	finite	infinite	infinite
Edge:	no	no	no

Distance Methods

- **Standard Rulers ==> Angular Size Distances**

$\theta = \dfrac{l}{D}$

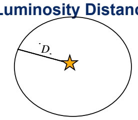


$D_A = \dfrac{l}{\theta}$

(for small angles << 1 radian)

- **Standard Candles ==> Luminosity Distances**

$F = \dfrac{\text{energy/time}}{\text{area}} = \dfrac{L}{4\pi D^2}$



$D_L = \left(\dfrac{L}{4\pi F} \right)^{1/2}$

- **Light Travel Time**

$t = \dfrac{\text{distance}}{\text{velocity}} = \dfrac{2D}{c}$

$D_t = \dfrac{c}{2t}$

(e.g. within solar system)

Olber's Paradox

Why is the sky dark at night ?

Flux from all stars in the sky :

$$F = \int n_* \, F_* \, d(\text{Vol}) = \int_0^{\chi_{\max}} n_* \left(\frac{L_*}{A(\chi)} \right) (\, A(\chi)\; R\; d\chi\,)$$

$$= n_* \, L_* \, R \, \chi_{\max}$$

$$\Rightarrow \infty \quad \text{for flat space}, \; R \to \infty.$$

A dark sky may imply :

(1) an edge (we don't observe one)

(2) a curved space (finite size)

(3) expansion ($R(t)$ => finite age, redshift)

Minkowski Spacetime Metric

$ds^2 = -c^2 dt^2 + dl^2$

$d\tau^2 = dt^2 - \dfrac{dl^2}{c^2} = dt^2 \left(1 - \dfrac{1}{c^2} \left(\dfrac{dl}{dt} \right)^2 \right)$

Time-like intervals:
$ds^2 < 0$, $d\tau^2 > 0$
Inside light cone.
Causally connected.

Proper time (moving clock):

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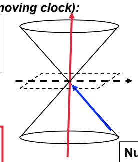


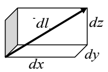
Space-like intervals:
$ds^2 > 0$, $d\tau^2 < 0$
Outside light cone.
Causally disconnected.

World line of massive particle at rest.

Null intervals light cone:
v = c, $ds^2 = 0$

Photons arrive from our past light cone.

Flat Space: Euclidean Geometry



Cartesian coordinates :

1 D: $dl^2 = dx^2$

2 D: $dl^2 = dx^2 + dy^2$

3 D: $dl^2 = dx^2 + dy^2 + dz^2$

4 D: $dl^2 = dw^2 + dx^2 + dy^2 + dz^2$

Metric tensor : coordinates - > distance

$$dl^2 = (\, dx \;\; dy \;\; dz\,) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}$$

Orthogonal coordinates
<--> diagonal metric

$g_{xx} = g_{yy} = g_{zz} = 1$

$g_{xy} = g_{xz} = g_{yz} = 0$

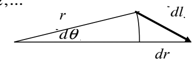
symmetric : $g_{ij} = g_{ji}$

Summation convention :

$$dl^2 = g_{ij}\, dx^i\, dx^j \equiv \sum_i \sum_j g_{ij}\, dx^i\, dx^j$$

Polar Coordinates

Radial coordinate r, angles $\phi, \theta, \alpha, \ldots$



1 D: $dl^2 = dr^2$

2 D: $dl^2 = dr^2 + r^2\, d\theta^2$

3 D: $dl^2 = dr^2 + r^2\, (\, d\theta^2 + \sin^2\theta \; d\phi^2)$

4 D: $dl^2 = dr^2 + r^2\, [\, d\theta^2 + \sin^2\theta \; (d\phi^2 + \sin^2\phi \; d\alpha^2)\,]$

$dl^2 = dr^2 + r^2\, d\psi^2$ generic angle : $d\psi^2 = d\theta^2 + \sin^2\theta\, d\phi^2 + \ldots$

$$dl^2 = (\, dr \;\; d\theta \;\; d\phi\,) \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2\theta \end{pmatrix} \begin{pmatrix} dr \\ d\theta \\ d\phi \end{pmatrix}$$

$g_{rr} = ?$ $g_{r\theta} = ?$

$g_{\theta\theta} = ?$

$g_{\phi\phi} = ?$

$g_{\alpha\alpha} = ?$

metric of space embedded in Sphere of radius R

R = radius of curvature

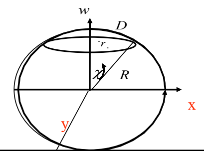
1−D: $R^2 = x^2$ — 0 - D 2 points

2−D: $R^2 = x^2 + y^2$ — 1 - D circle

3−D: $R^2 = x^2 + y^2 + z^2$ — 2 - D surface of 3 - sphere

4−D: $R^2 = x^2 + y^2 + z^2 + w^2$ — 3 - D surface of 4 - sphere

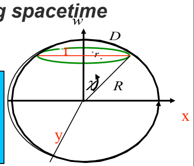
?



7th Concept: Robertson-Walker metric

uniformly curved, evolving spacetime

$$ds^2 = -c^2 dt^2 + R^2(t)\,(d\chi^2 + S_k^2(\chi)\,d\psi^2)$$



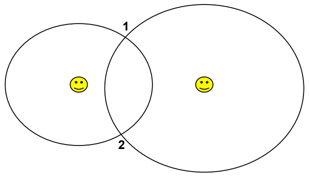
$$S_k(\chi) = \begin{cases} \sin\chi & (k=+1) \quad \text{closed} \\ \chi & (k=0) \quad \text{flat} \\ \sinh\chi & (k=-1) \quad \text{open} \end{cases}$$

$$d\psi^2 \equiv d\theta^2 + \sin^2\theta\, d\phi^2$$
$$a(t) \equiv R(t)/R_0$$
$$R_0 \equiv R(t_0)$$

radial distance $= D(t) = R(t)\,\chi$

circumference $= 2\pi\, r(t)$ $\quad r(t) = \ldots = R(t)\,S_k(\chi)$

Cosmological Principle (assumed) +
Isotropy (observed)
=> Homogeneity



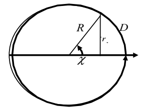
$\rho_1 = \rho_2$ otherise not isotropic for equidistant fidos

Distances-Redshift relation

- We observe the **redshift** : $z \equiv \dfrac{\lambda - \lambda_0}{\lambda_0} = \dfrac{\lambda}{\lambda_0} - 1$ $\quad \lambda = \text{observed},$ $\quad \lambda_0 = \text{emitted (rest)}$
- Hence we know the **expansion factor:** $x \equiv 1 + z = \dfrac{\lambda}{\lambda_0} = \dfrac{\lambda(t_0)}{\lambda(t)} = \dfrac{R(t_0)}{R(t)} = \dfrac{R_0}{R(t)}$
- Need the time of light emitted $\quad t(z) = ?$
- Need coordinate of the source $\quad \chi(z) = ?$
- Need them as functions of $\quad H_0 \quad \Omega_M \quad \Omega_\Lambda$
- Distances

$D(t,\chi) = R(t)\,\chi \qquad D_A = r_0(\chi)/(1+z)$

$r(t,\chi) = R(t)\,S_k(\chi) \qquad D_L = r_0(\chi)\,(1+z)$



- E.g. **D_L is 4 x D_A for an object at z=1.**

Tutorial: Time -- Redshift relation

$$x = 1 + z = \frac{R_0}{R}$$
$$\frac{dx}{dt} = -\frac{R_0}{R^2}\frac{dR}{dt}$$
$$= -\frac{R_0}{R}\frac{\dot R}{R}$$
$$= -x\,H(x)$$

Memorise this derivation!

Hubble parameter : $\quad H \equiv \dfrac{\dot R}{R}$

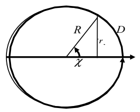
$$\therefore\ dt = \frac{-dx}{x\,H(x)} = \frac{-dz}{(1+z)\,H(z)}$$

Tutorial:
Time and Distance vs Redshift

$$\frac{d}{dt}\left(x = 1 + z = \frac{R_0}{R}\right) \rightarrow \quad dt = \frac{-dx}{x\,H(x)}$$

Look - back time :

$$t(z) = \int_t^{t_0} dt = \int_{1+z}^{1} \frac{-dx}{x\,H(x)} = \int_1^{1+z} \frac{dx}{x\,H(x)}$$



Age : $\quad t_0 = t(z \rightarrow \infty)$

Distance : $\quad D = R\,\chi \qquad r = R\,S_k(\chi)$

$$\chi(z) = \int d\chi = \int_t^{t_0} \frac{c\,dt}{R(t)} = \frac{c}{R_0}\int_1^{1+z} \frac{R_0}{R(t)}\frac{dx}{x\,H(x)} = \frac{c}{R_0}\int_1^{1+z} \frac{dx}{H(x)}$$

Horizon : $\quad \chi_H = \chi(z \rightarrow \infty)$

Need to know *R(t)*, or R_0 and *H(x)*.

Einstein's General Relativity

- ***1. Spacetime geometry tells matter how to move***
  - **gravity = effect of curved spacetime**
  - **free particles follow geodesic trajectories**
    - ***ds²* < 0 v < c time-like massive particles**
    - ***ds²* = 0 v = c null massless particles (photons)**
    - ***ds²* > 0 v > c space-like tachyons (not observed)**
- ***2. Matter (+energy) tells spacetime how to curve***
  - **Einstein field equations**
    - **nonlinear**
    - **second-order derivatives of metric**

with respect to space/time coordinates

8^{th} concept: Einstein Field Equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R^{\alpha}{}_{\alpha}\, g_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu} - \Lambda\, g_{\mu\nu}$$

$g_{\mu\nu}$ = spacetime metric ($ds^2 = g_{\mu\nu}\, dx^{\mu}\, dx^{\nu}$)

$G_{\mu\nu}$ = Einstein tensor (spacetime curvature)

$R_{\mu\nu}$ = Ricci curvature tensor

$R^{\alpha}{}_{\alpha}$ = Ricci curvature scalar

G = Netwon's gravitational constant

$T_{\mu\nu}$ = energy - momentum tensor

Λ = cosmological constant

Homogeneous perfect fluid

density ρ pressure p

Einstein field equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^2}\begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} - \Lambda \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

---> Friedmann equations :

$$\dot{R}^2 = \left(\frac{8\pi G\,\rho + \Lambda}{3}\right) R^2 - k\,c^2 \qquad \text{energy}$$

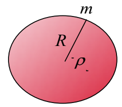
$$\ddot{R} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3p)\, R + \frac{\Lambda}{3} R \qquad \text{momentum}$$

Note: energy density and pressure decelerate, Λ accelerates.

Reading: Newtonian Analogy

$$E = \frac{m}{2}\dot{R}^2 - \frac{G\,M\,m}{R} \qquad M = \frac{4\pi}{3} R^3 \rho$$

$$\dot{R}^2 = \frac{8\pi\, G}{3}\rho\, R^2 + \frac{2E}{m}$$

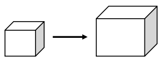


Friedmann equation:

$$\dot{R}^2 = \left(\frac{8\pi\, G\, \rho + \Lambda}{3}\right) R^2 - k\, c^2$$

same equation if $\rho \rightarrow \rho + \dfrac{\Lambda}{8\pi\ \mathrm{G}}, \quad \dfrac{2E}{m} \rightarrow -k\, c^2$

Reading: Local Conservation of Energy



$$d[energy] = work$$

$$d[\rho\, c^2 R^3] = -p\, d[R^3]$$

$$\dot{\rho}\, c^2 R^3 + \rho\, c^2\,(3\,R^2 \dot{R}) = -p\,(3\,R^2 \dot{R})$$

$$\dot{\rho} = -3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{R}}{R} \qquad p = p(\rho) = \text{equation of state}$$

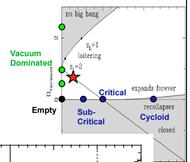
$$\text{Friedmann 1}: \dot{R}^2 = \frac{8\pi G}{3}\rho\, R^2 + \frac{\Lambda}{3} R^2 - k\, c^2$$

$$(2\,\dot{R}\,\ddot{R}) = \frac{8\pi G}{3}(\dot{\rho}\, R^2 + 2\,R\,\dot{R}\,\rho\ ) + \frac{\Lambda}{3}(2\,R\,\dot{R})$$

$$\ddot{R} = \frac{8\pi G}{3}\left(\frac{\dot{\rho}\, R^2}{2\,\dot{R}} + R\,\rho\right) + \frac{\Lambda}{3} R$$

$$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3\,p}{c^2}\right) R + \frac{\Lambda}{3} R = \text{Friedmann 2}$$

Critical Universe
(Einstein - de Sitter)



$$\Omega_{\mathrm{M}} \equiv \frac{\rho_M}{\rho_c} = 1$$

$$\Omega_R = \Omega_\Lambda = 0 \quad \rightarrow \quad k = 0\ \ (\text{ flat })$$

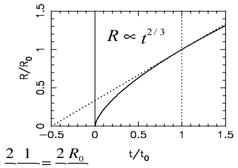
$$\rho = \frac{3\,H_0{}^2}{8\pi\, G}\left(\frac{R_0}{R}\right)^3$$

$$\dot{R}^2 = \frac{8\pi\, G}{3}\rho\, R^2 = \frac{H_0{}^2\, R_0{}^3}{R}$$

$$dR\; R^{1/2} = H_0\; R_0{}^{3/2}\; dt$$

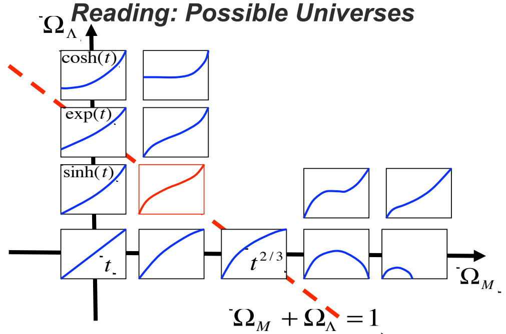
$$\frac{2}{3} R^{3/2} = H_0\; R_0{}^{3/2}\; t$$

$$\frac{R}{R_0} = \left(\frac{t}{t_0}\right)^{2/3}, \qquad \text{age}: \quad t_0 = \frac{2}{3}\frac{1}{H_0} = \frac{2}{3}\frac{R_0}{c}$$



Matter decelerates expansion.

Reading: Possible Universes



$$\Omega_M + \Omega_\Lambda = 1$$

Reading: Rad. => Matter =>Vacu.

$$\Omega(z) = \Omega_R\, x^4 + \Omega_M\, x^3 + \Omega_\Lambda = \sum_i \Omega_i\, x^{3(w_i+1)}$$

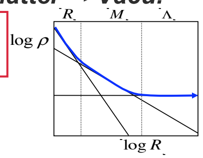
$$x \equiv 1+z = R_0/R \equiv a^{-1} \qquad w \equiv p/\rho c^2$$

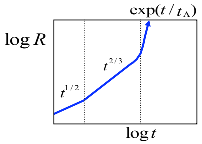
$$\Omega_R x^4 = \Omega_M x^3 \quad \rightarrow \quad x = \Omega_M/\Omega_R$$

$$\rightarrow \quad z = \left(\frac{\Omega_M}{\Omega_R}\right) - 1 = \frac{0.3}{8.4\times10^{-5}} \approx 3600$$

$$\Omega_M x^3 = \Omega_\Lambda \rightarrow \quad x^3 = \Omega_\Lambda/\Omega_M$$

$$\rightarrow \quad z = \left(\frac{\Omega_\Lambda}{\Omega_M}\right)^{\frac{1}{3}} - 1 = \left(\frac{0.7}{0.3}\right)^{\frac{1}{3}} - 1 \approx 0.33$$





Tutorial: What observations justify the "Concordance" Parameters?

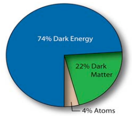
$$H_0 \equiv 100\, h\, \frac{\text{km/s}}{\text{Mpc}} \approx 70 \frac{\text{km/s}}{\text{Mpc}} \qquad h \approx 0.7$$

$$\Omega_R \approx 4.2\times10^{-5}\, h^{-2} \approx 8.4\times10^{-5} \; (CMB\ photons + neutrinos)$$

$$\Omega_B \sim 0.02\, h^{-2} \sim 0.04 \quad (baryons)$$

$$\Omega_M \sim 0.3 \quad (Dark\ Matter)$$

$$\Omega_\Lambda \sim 0.7 \quad (Dark\ Energy)$$

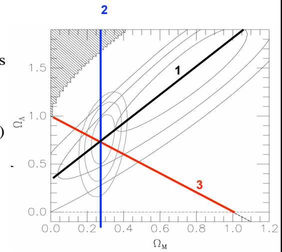


$$\Omega_0 \equiv \Omega_R + \Omega_M + \Omega_\Lambda = 1.0 \quad \rightarrow \quad Flat\ Geometry$$

9th Concept: "Concordance" Model

Three main constraints:

1. Supernova Hubble Diagram
2. Galaxy Counts + M/L ratios
   $\Omega_M \sim 0.3$
3. Flat Geometry
   (inflation, CMB fluctuations)
   $\Omega_0 = \Omega_M + \Omega_\Lambda = 1$



concordance model

$H_0 \approx 72 \quad \Omega_M \approx 0.3 \quad \Omega_\Lambda \approx 0.7$

Deceleration parameter

$$a(t) \equiv \frac{R(t)}{R_0} = 1 + H_0\, (t-t_0) - \frac{q_0}{2} H_0^{\,2}\, (t-t_0)^2 + \ldots$$

Friedmann momentum equation :

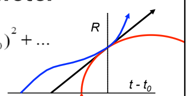
$$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) R + \frac{\Lambda}{3} R$$

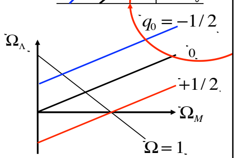
$$\frac{\ddot{R}}{H_0^{\,2} R} = -\frac{4\pi G}{3 H_0^{\,2}} \rho (1+3w) + \frac{\Lambda}{3 H_0^{\,2}}$$

$\rho, p > 0$ decelerate, $\Lambda > 0$ accelerates

Equation of state : $p = \sum_i w_i\, \rho_i\, c^2$

$$w_R = \frac{1}{3} \quad w_M = 0 \quad w_\Lambda = -1$$





$$q_0 = -\left(\frac{\ddot{R}}{R\, H^2}\right)_0 = \sum_i \left(\frac{1+3\, w_i}{2}\right) \Omega_i = \Omega_R + \frac{\Omega_M}{2} - \Omega_\Lambda$$

- $q_0 > 0$ => **deceleration**
- $q_0 = 0$ => *constant velocity*
- $q_0 < 0$ => **acceleration**

Tutorial: Observable Distances

angular diameter distance :

$$\theta = \frac{l}{D_A} \qquad D_A = \frac{r_0}{(1+z)} = \frac{c\, z}{H_0}\left(1 - \frac{q_0+3}{2} z + \ldots\right)$$

luminosity distance :

$$F = \frac{L}{4\pi\, D_L^{\,2}} \qquad D_L = r_0\,(1+z) = \frac{c\, z}{H_0}\left(1 + \frac{1-q_0}{2} z + \ldots\right)$$

deceleration parameter :

$$q_0 = \frac{\Omega_M}{2} - \Omega_\Lambda$$

Verify these low-z expansions.

q0 from Hubble Diagram

$$m = M + 5 \log\left(\frac{D_L(z)}{\text{Mpc}}\right) + 25 + A + K(z)$$

$m =$ apparent mag

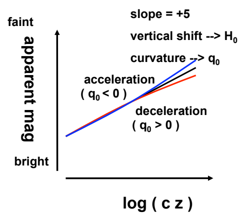
$M =$ absolute mag

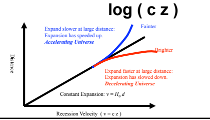
$A =$ extinction (dust in galaxies)

$K(z) =$ K correction

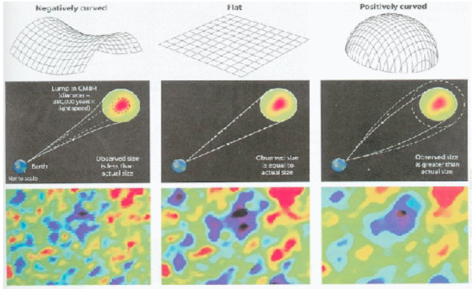
(accounts for redshift of spectra relative to observed bandpass)

$$D_L(z) = \frac{c\, z}{H_0}\left(1 + \frac{1-q_0}{2} z + \ldots\right)$$

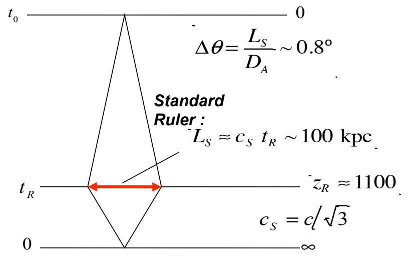




CMB Angular scale --> Geometry



10th Concept: Sound Horizon at z = 1100



Sound Horizon at z = 1100

$$L_S(t_R) = \frac{c_S}{(1+z)} \int_{1+z}^{\infty} \frac{dx}{H(x)} \approx \frac{c_S}{(1+z)\,H_0} \int_{1+z}^{\infty} \frac{dx}{\sqrt{x^4\,\Omega_R + x^3\,\Omega_M}}$$

$$= \frac{c_S}{(1+z)\,H_0\sqrt{\Omega_R}} \int_{1+z}^{\infty} \frac{dx}{\sqrt{x^3(x+x_0)}} \qquad x_0 \equiv \frac{\Omega_M}{\Omega_R} \approx 3500\left(\frac{\Omega_M}{0.3}\right)$$

$$= \frac{c_S}{(1+z)\,H_0\sqrt{\Omega_R}} \left(-\frac{2}{x_0}\sqrt{1+\frac{x_0}{x}}\right)_{1+z}^{\infty}$$

$$= \frac{2c_S}{(1+z)\,H_0\sqrt{\Omega_M\,x_0}} \left(\sqrt{1+\frac{x_0}{1+z}} - 1\right) \qquad c_S = \frac{c}{\sqrt{3}}$$

$$= \frac{c}{H_0} \frac{2\left(\sqrt{4.6}-1\right)}{1100\sqrt{3\times 0.3\times 3500}}$$

$$= 3.4\times 10^{-5}\frac{c}{H_0} \approx 110\left(\frac{0.7}{h}\right)\left(\frac{0.3}{\Omega_M}\right)^{1/2} \text{kpc}$$

Expands by factor 1 + z = 1100 to ~120 Mpc today.

Reading: Angular scale → *flatness*

sound horizon :

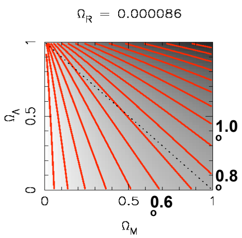
$$L_S(z) = \frac{1}{1+z}\int_{1+z}^{\infty} \frac{c_S\,dx}{H(x)}$$

angular diameter distance :

$$D_A(z) = \frac{R_0\,S_K(\chi)}{1+z} \qquad \chi = \int_t^{t_0} \frac{c\,dt}{R(t)} = \frac{c}{R_0}\int_1^{1+z} \frac{dx}{H(x)}$$

angular scale

$$\theta = \frac{L_S(z)}{D_A(z)} = \frac{\int_{1+z}^{\infty} \frac{c_S\,dx}{H(x)}}{R_0\,S_k\left(\frac{c}{R_0}\int_1^{1+z}\frac{dx}{H(x)}\right)}$$

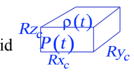


Angular scale depends mainly on the curvature.

Gives theta~ 0.8° for flat geometry, *Omega =1*

Evolution of Sound Speed

Expand a box of fluid



Sound Speed $C_S^2 = \frac{\partial P/\partial(vol)}{\partial\rho/\partial(vol)} = \frac{\partial P/\partial R}{\partial\rho/\partial R}$

$$Vol = R^3(t)\, x_c y_c z_c \propto R^3(t)$$

Coupled radiation-baryon relativistic fluid

	Radiation	Matter
Where fluid density $\rho(t)=$	ρ_r	ρ_m
Fluid pressure $P(t)=$	$\frac{c^2}{3}\rho_r$	$\frac{\rho_m}{\mu}\cdot KT_m$

Matter number density — Random motion energy Non-Relativistic IDEAL GAS

Note $\rho_r \propto R^{-4}$

$\rho_m \propto R^{-3}$ Neglect $\frac{1}{\mu}KT_m << c^2$

Tutorial: Show $C_s^2 = c^2/3\,/(1+Q)$, $Q = (3\,\rho_m)/(4\,\rho_r)$, ➔ C_s drops

from c/sqrt(3) at radiation-dominated era

to c/sqrt(Y) at dark-matter-radiation equality

11^{th} Concept: Inflation in Early Universe

Consider universe goes through a phase with

$\rho(t) \sim R(t)^{-n}$

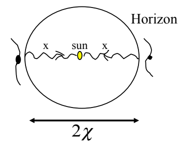
$R(t) \sim t^{q}$ *where* q=2/n

Problems with normal expansion theory (n=2,3,4):

- What is the state of the universe at t→0? Pure E&M field (radiation) or exotic scalar field?
- Why is the initial universe so precisely flat?
- What makes the universe homogeneous/similar in opposite directions of horizon?

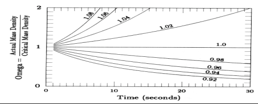
Solutions: Inflation, i.e., n=0 or n<2

- Maybe the horizon can be pushed to infinity?
- Maybe there is no horizon?
- Maybe everything was in Causal contact at early times?



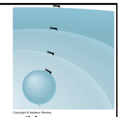
Why are these two galaxies so similar without communicating yet?

$$\frac{\varepsilon_K(z)}{\varepsilon\ (z)} = \frac{\varepsilon_K(0)\times R^{-2}}{\varepsilon(0)\times R^{-n}} \sim R^{n-2} \sim 0 \text{ at } t=0$$



Why is the curvature term so small (universe so flat) at early universe if radiation dominates n=4 >2?

Inflation broadens Horizon



Light signal travelling with speed c on an expanding sphere R(t), e.g., a fake universe R(t)=1 lightyr (t/1yr)^q

Emitted from time t_i

By time t=1yr will spread across (co-moving coordinate) angle c_c

Horizon in co-moving coordinates

$$x_c = \int_{t_i}^{1} \frac{\mathrm{cdt}}{\mathrm{R(t)}} = \int_{t_i}^{1} \frac{\mathrm{cdt}}{\mathrm{t}^q} = \frac{(1^{1-q} - t_i^{1-q})}{(1-q)}$$

Normally $x_c < \frac{1}{(1-q)}$ is finite if q=2/n<1

(e.g., n=3 matter-dominate or n=4 photon-dominate)

INFLATION phase $x_c = \frac{(t_i^{1-q} - 1)}{(q-1)}$ can be very large for very small t_i if q=2/n>1

(e.g., $t_i = 0.01, q = 2, x_c = 99 >> \pi$, Inflation allows we see everywhere)

Tutorial: Inflation dilutes any initial curvature of a quantum universe

$\frac{\varepsilon_K(R)}{\varepsilon\ (R)} = \frac{\varepsilon_K(R_i)}{\varepsilon(R_i)}\left(\frac{R}{R_i}\right)^{n-2} \sim 0$ (for n<2) sometime after R>>R_i

even if initially the universe is curvature-dominated $\frac{\varepsilon_K(R_i)}{\varepsilon(R_i)} = 1$

E.g.

If a toy universe starts with $\frac{\varepsilon_K(R_i)}{\varepsilon(R_i)} = 0.1$ inflates from $t_i=10^{-40}$sec to t_f=1sec with n=1,

and then expand normally with n=4 to t=1 year,

SHOW at this time the universe is far from curvature-dominated.

Exotic Pressure drives Inflation

$$P = -\frac{d(\rho c^2 R^3)}{d(R^3)}$$

=>

$$\frac{\rho}{3} + \frac{P}{c^2} = -\frac{d(\rho R^2)}{3RdR} = \frac{n-2}{3}\rho \quad \text{if } \rho \sim R^{-n}$$

=>

$P/\rho c^2$=(n-3)/3

Inflation $n<2$ requires exotic (negative) pressure,

define w=$P/\rho c^2$, then w = (n-3)/3<0,

Verify negligble pressure for cosmic dust (matter),

Verify for radiation P=ρc^2 / 3

Verify for vaccum P=-ρc^2

List of keys

Scaling relations among

- Redshift z, wavelength, temperature, cosmic time, energy density, number density, sound speed
- Definition formulae for pressure, sound speed, horizon
- Metrics in simple 2D universe.

Describe in words the concepts of

- Fundamental observers
- thermal decoupling
  - Fixed number to photon ratio after
  - Hot and Cold DM.
- gravitational growth.

Expectations

Remember basic concepts (or analogies)

See list

Can apply various scaling relations to do some of the short questions at the lectures.

See list

Relax.

thermal history, structure formation, DM counting are advanced subjects, just be able to recite the big picture.

Some formulae to remember

$$R(t)\, d\chi = c\, dt, \quad c\, dt = \frac{-c\, dz}{(1+z)\, H(z)}, \quad D_A = \frac{R_0}{(1+z)} \cdot S_k(\chi), \quad D_L = (1+z)\, R_0 \cdot S_k(\chi)$$

$$\rho_i = \frac{3 H_0^2}{8 \pi G} \Omega_i (1+z)^{n_i}, \quad H(z)^2 = \Sigma_{i=m,r,v,c} \frac{8 \pi G \rho_i}{3};$$

$$c_s^2 = \frac{dP}{d\rho}, \quad P = w \rho c^2, \quad P = -\frac{d(\rho c^2 R^3)}{d(R^3)};$$

What have we learned?

Where are we heading?

Inflation as origin of perturbation, flatness, horizon.

Sound speed of gas before/after decoupling, and sound horizon.

Topics Next:

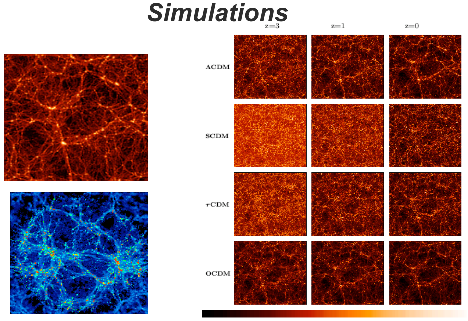
Growth of [bankruptcy of uniform universe]

Density Perturbations (how galaxies form by N-body simulations)

peculiar velocity (how galaxies move and merge)

CMB fluctuations (temperature variation in CMB)

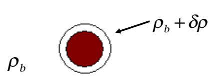
Reading: Supercomputer Simulations

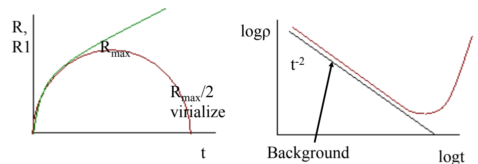


Non-linear Collapse of an Overdense Sphere

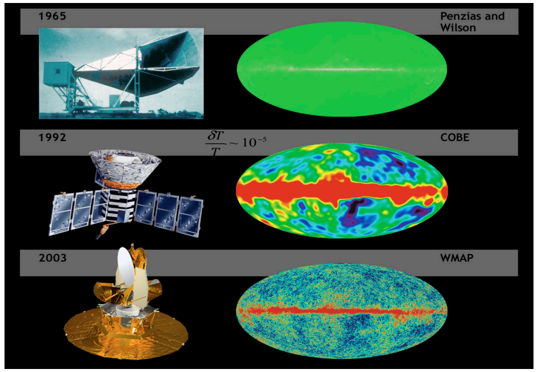
An overdense sphere is a very useful non linear model as it behaves in exactly the same way as a closed sub-universe.

Any spherically symmetric perturbation will clearly evolve at a given radius in the same way as a uniform sphere containing the same amount of mass.





$$\rho_b = \frac{1}{6\pi G t^2}$$

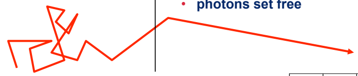


Recombination Epoch (z~1100)

ionised plasma --> neutral gas

- Redshift z > 1100
- Temp T > 3000 K
- H ionised
- electron -- photon Thompson scattering

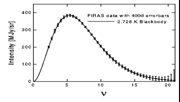
- z < 1100
- T < 3000 K
- H recombined
- almost no electrons
- neutral atoms
- photons set free



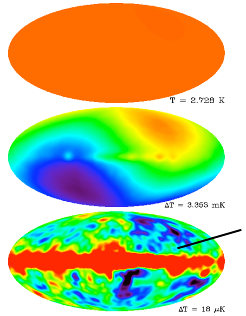
e - scattering optical depth

$$\tau(z) \approx \left(\frac{z}{1080}\right)^{13}$$

thin surface of last scattering



Cosmic Microwave Background



Almost isotropic

$T = 2.728$ K

Dipole anisotropy

$$\frac{V}{c} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T} \approx 10^{-3}$$

Our velocity:

$V \approx 600$ km/s

Milky Way sources

$+$ anisotropies $\frac{\Delta T}{T} \sim 10^{-5}$

Theory of CMB Fluctuations

Linear theory of structure growth predicts that the perturbations:

δ_D in dark matter $\frac{\delta\rho_D}{\rho_D}$

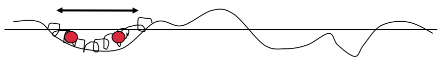
δ_B in baryons $\frac{\delta\rho_B}{\rho_B}$

δ_r in radiation $\frac{\delta\rho_r}{\rho_r}$ Or $\tilde{\delta}_r = \frac{3}{4}\delta_r = \frac{\delta n_\gamma}{n_\gamma}$

will follow a set of coupled Harmonic Oscillator equations.

Acoustic Oscillations

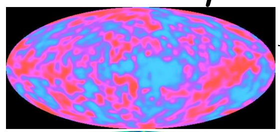
$\lambda \approx \theta\, D_A$



Dark Matter potential wells - many sizes.
photon-electron-baryon fluid
fluid falls into DM wells
photon pressure pushes it out again
oscillations starting at t = 0 (post-inflation)
stopping at z = 1100 (recombination)

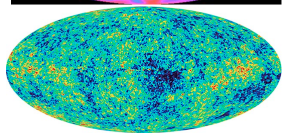
CMB Anisotropies

COBE 1994



$\frac{\Delta T}{T} \sim 10^{-5}$

WMAP 2004



$\Delta\theta \sim 1^{\circ}$

Snapshot of Universe at z = 1100
Seeds that later form galaxies.

Resonant Oscillations

size of potential well λ

oscillation period $P \approx \frac{\lambda}{c_s}$

sound speed $c_s = \frac{c}{\sqrt{3}}$

temperature oscillations

$\Delta T(t) = \Delta T(0) \cos(2\pi\, t/P)$

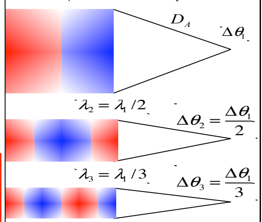
$\max|\Delta T|$ at $t = \frac{nP}{2} \sim \frac{n\lambda}{2c_s}$

angular size

$\Delta\theta_n = \frac{\lambda_n}{D_A} = \frac{\Delta\theta_1}{n}$ $\quad \Delta\theta_1 \approx \frac{2c_s t}{D_A} \sim 0.8°$

$z = \infty$ $\quad z = 1100$ $\quad z = 0$

$t \approx P_1/2$ $\quad \lambda_1 \approx 2c_s t$



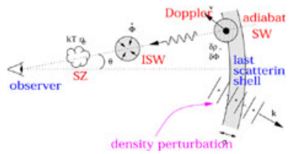
Smaller wells oscillate faster.

The observed temperature also depends on how fast the Baryon Fluid is moving.

Velocity Field $\nabla v = -\frac{d\delta_B}{dt}$

$$\left(\frac{\Delta T}{T}\right)_{obs} = \frac{\delta_B}{3} + \frac{\psi}{c^2} \pm \frac{v}{c}$$

Doppler Term



Why Analogies in Cosmology

Help you memorizing

Cosmology calls for knowledge of many areas of physics. Analogies help to you memorize how things move and change in a mind-boggling expanding 4D metric.

Help you reason, avoid “more equations, more confusions”.

If unsure about equations, e.g. at exams, the analogies *help you recall* the right scaling relations, and get the big picture right.

Years after the lectures,

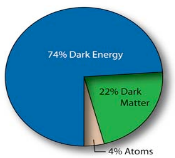
Analogies go a long way

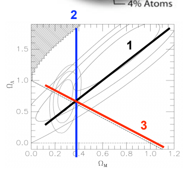
Dark Matter Evidences?

Galaxies: (r ~ 20 Kpc)
Flat Rotation V ~ 200 km/s
Galaxy Clusters: (r ~ 200 Kpc)
Galaxy velocities V ~ 1000 km/s
X-ray Gas T ~ 10^8 K
Giant Arcs

$\Omega_M \sim 0.3$

$\Omega_b \approx 0.04$





Dark Matter Candidates

- **MACHOS = Massive Compact Halo Objects**
  - Black holes
  - Brown Dwarfs
  - Loose planets
  - Ruled out by gravitational lensing experiments.
- **WIMPS = Weakly Interacting Massive Particles**
  - Massive neutrinos
  - Supersymmetry partners
  - Might be found soon by Large Hadron Collider in CERN

Quasars Lensed by Galaxies