Observational Cosmology example solutions

P12

$$1+z = \frac{R_{now}}{R(t)} \; ; \; z=2, \; R_{Now} = 15\,\text{Mpc}$$

$$R(t) = \frac{R_{now}}{1+z} = \frac{15\,\text{Mpc}}{3} = 5\,\text{Mpc}$$

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$$1+z = \frac{T(t)}{T_{now}} \; ; \; z=2, \; T_{now} = 3\text{K}$$

$$T(t) = T_{now}(1+z) = 3\text{K} \times (1+2) = 9\text{K}$$

P24

We are told $\rightarrow$ $3kT = \dfrac{hc}{\lambda}$

degrees of freedom of photon 

$$\lambda = \frac{hc}{3kT} = \frac{(6.63\times10^{-34}\,\text{Js})\times(3\times10^{8}\,\text{ms}^{-1})}{3\,(1.38\times10^{-23}\,\text{JK}^{-1})(3\text{K})}$$

$$= \underline{\underline{1.3\,\text{mm}}}$$

Volume of microwave oven $= (100\,\text{mm})^3$

Number of photons inside $= \left(\dfrac{100\,\text{mm}}{1.3\,\text{mm}}\right)^3$

$$\approx \underline{\underline{4.5\times10^{5}\ \text{photons}}}$$

P24 cont

$$m = \gamma m_o = \frac{m_o}{1-\frac{v^2}{c^2}} \quad ; \; m = 1.1, \; m_o = 1.0$$

$$\frac{m_o}{m} = 1 - \frac{v^2}{c^2} \Rightarrow \frac{1}{1.1} = 1 - \frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} = 1 - \frac{1}{1.1} = 1 - 0.91$$

$$= 0.09$$

$$\frac{v}{c} = 0.3$$

so relativistic effects small.

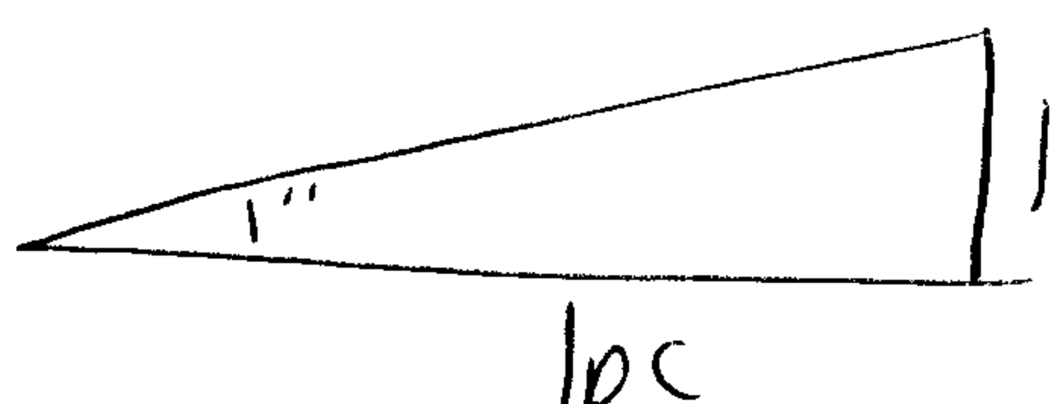
$$kT \sim 0.1 mc^2$$

$$mc^2 = 10 \times (1.38 \times 10^{-23} \, J K^{-1})(10^{14} K) = 1.38 \times 10^{-8} J$$

$$= \frac{1.38 \times 10^{-8} J}{1.6 \times 10^{-19} C} = 8.6 \times 10^{10} eV = 86 GeV$$

$\gg$ proton

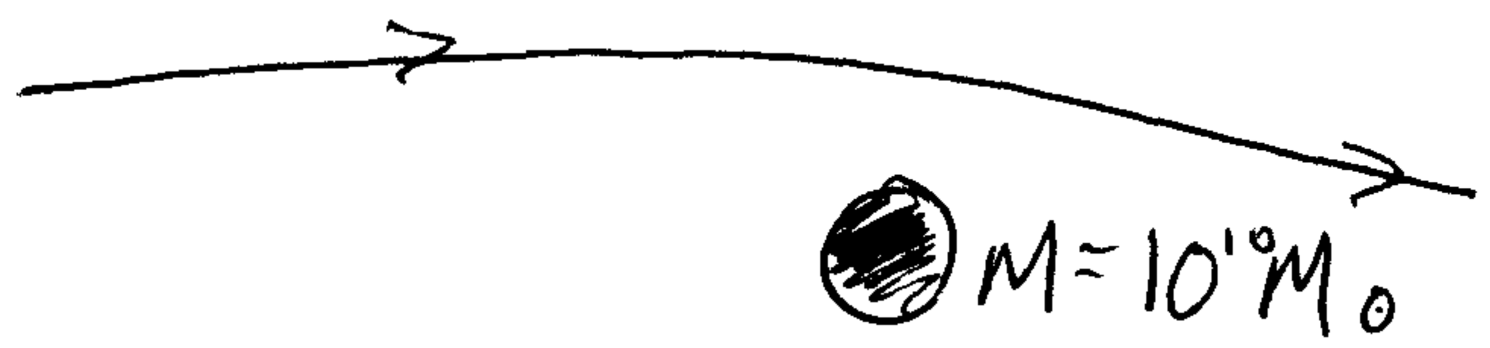
P26



$$1AU = \frac{1}{206265} pc$$

1Kpc of parallax means distance of $\Rightarrow 1 \times 206265 \times 10^3 = 200 Mpc$

P26 cont



We know the gravitational constant G is

$$G = 4.42 \times 10^{-3} \frac{kms^{-1} \cdot pc^2}{M_\odot \cdot Myr}$$

we want it in $pc, M_\odot$ & km/s

$$\frac{pc}{Myr} \approx \frac{3.1 \times 10^{16} m}{3.1 \times 10^{13} yr} = 1 kms^{-1}$$

$$G = 4.42 \times 10^{-3} \frac{(kms^{-1})^2 pc}{M_\odot}$$

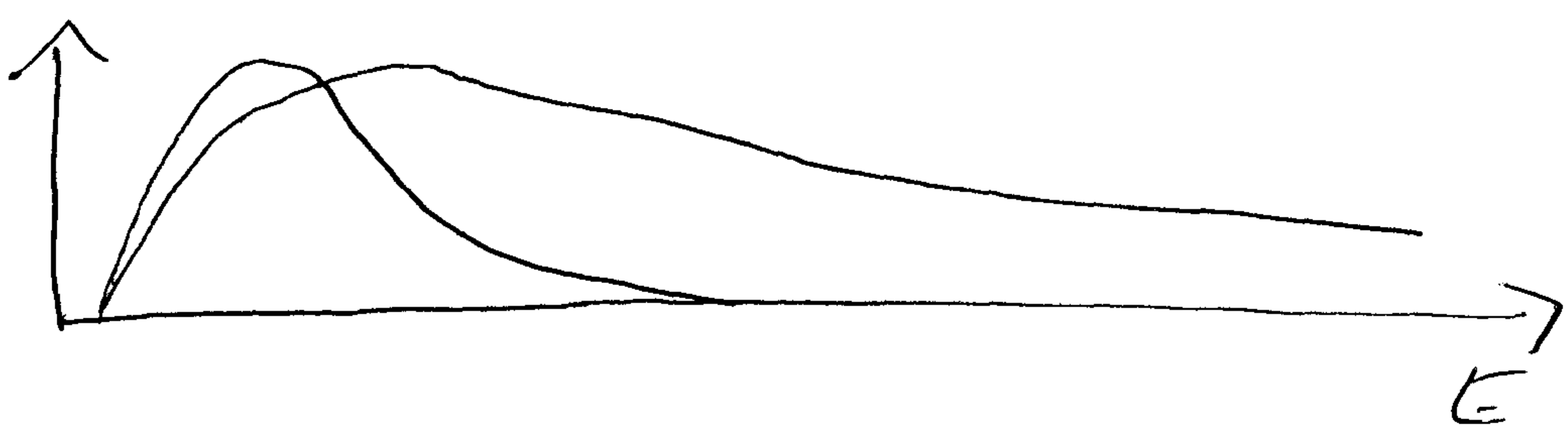
$$\theta \sim \frac{4GM}{bc^2} \sim \frac{4 \times 4.42 \times 10^{-3} \frac{(kms^{-1})^2 pc}{M_\odot} \times 10^{10} M_\odot}{10^3 pc \times (3 \times 10^5 kms^{-1})^2}$$

$$\sim \frac{4 \times 4.42 \times 10^{-3} \times 10^{10}}{10^3 \times (3 \times 10^5)^2} \text{ radians} \sim 1.77 \times 10^{-5} \text{ rad}$$

$$= 1.77 \times 10^{-5} \times 206265''$$

$$= 3.6 \text{ arc secs}$$

SZ effect. If effect is $\propto E$ then higher energy photons recieve more energy meaning BB gets stretched with increasing E



P27 $\frac{M_{DM}}{M_{pr}} = 10$; $\frac{N_{DM} M_{DM}}{N_{pr} M_{pr}} = 10 \Rightarrow \frac{N_{DM}}{N_{pr}} = 10 \frac{M_{pr}}{M_{DM}}$

$= 1$

$$\frac{N_{ph}}{N_{pr}} \propto \frac{\varepsilon_{ph}/\varepsilon_{tot} \times E_{pr}}{E_{ph}} \approx \frac{10^{-4} E_{pr}}{E_{ph}} \sim 10^{9}$$

P54 $$C_s^2 = \frac{\partial P/\partial R}{\partial \rho/\partial R}$$

$P = \frac{c^2}{3}\rho_r$ since matter term is unimportant

$P \propto \frac{c^2}{3} R^{-4}$

$\rho = \rho_r + \rho_m \propto R^{-4} + R^{-3}$

$\frac{\partial P}{\partial R} = -\frac{4}{3} c^2 R^{-5}$ $\quad \frac{\partial \rho}{\partial R} = -4R^{-5} - 3R^{-4}$

$= -4R^{-5}\left(1 + \frac{3}{4}R\right)$

$$C_s^2 = \frac{-\frac{4}{3}c^2 R^{-5}}{-4R^{-5}(1+\frac{3}{4}R)} = \frac{c^2/3}{1+Q} \quad \text{where } Q = \frac{3\rho_m}{4\rho_r}$$

during radiation dominated era $\rho_r \gg \rho_m$ $Q \to 0$

$C_s = \frac{c}{\sqrt{3}}$

at equality $\rho_m = \rho_r \Rightarrow Q = \frac{3}{4}$, $C_s = \frac{c}{\sqrt{5.25}}$

P62 Matter radiation equality occurs at around $z = 10^3$

so between $z = 10$ & 10^2 universe is matter dominated

$$\delta \propto R \propto (1+z)$$

$$\delta(z) \propto (1+z)$$

$$\frac{\delta(100)}{\delta(10)} = \frac{101}{11} \sim 9$$

between $z = 10^5$ & 10^6 universe is radiation dominated

$$\delta \propto R^2 \propto (1+z^2) \propto z^2$$

$$\delta(z) \propto z^2$$

$$\frac{\delta(10^6)}{\delta(10^5)} = \frac{10^{12}}{10^{10}} = 100$$

P92 $$x_c = \frac{(1^{1-q} - t_i^{1-q})}{1-q} \qquad q = \frac{2}{3} \text{ so } 1 - \frac{2}{3} = \frac{1}{3}$$

$$x_c = \frac{1 - t_i^{1/3}}{1/3} = 3 - 3t_i^{1/3}$$ $t_i < 1$ so x_c is finite

$$R = 1\,\text{lightyr} \left(\frac{t}{1\text{yr}}\right)^{2/3}$$

$$\dot{R} = 10^{-2}\,\text{ms}^{-1} \left(\frac{t}{1\text{yr}}\right)^{-1/3}$$

P92 cont

For relativistic $\dot{R} = \frac{c}{10}$

$$\Rightarrow \frac{c}{10} = 10^{-2}\,\mathrm{ms^{-1}}\left(\frac{t}{1\,\mathrm{yr}}\right)^{-1/3}$$

$$\Rightarrow \frac{3\times10^{8}\,\mathrm{ms^{-1}}}{10^{-1}\,\mathrm{ms^{-1}}} = \left(\frac{t}{1\,\mathrm{yr}}\right)^{-1/3} \Rightarrow t = 1\,\mathrm{yr}\times\left(3\times10^{9}\right)^{-3}$$

$$= 3.7\times10^{-29}\,\mathrm{yr} \approx 10^{-21}\,\mathrm{s}$$

The distance travelled is $\int_0^{1\,\mathrm{yr}} 10^{-2}\,\mathrm{ms^{-1}}\left(\frac{t}{1\,\mathrm{yr}}\right)^{-1/3} dt$

$$= \left[\frac{10^{-2}\,\mathrm{ms^{-1}}\times 1\,\mathrm{yr}}{\frac{2}{3}}\right]\times\left[\frac{t^{2/3}}{1\,\mathrm{yr}}\right]_0^{1\,\mathrm{yr}}$$

$$= \frac{10^{-2}\times 1\,\mathrm{yr}}{2/3} = \frac{10^{-2}\,\mathrm{ms^{-1}}\times3.1\times10^{7}\,\mathrm{s}}{2/3}$$

$$= 4.5\times10^{5}\,\mathrm{m}$$

The fraction travelled relativistically is $\left(\frac{3.7\times10^{-29}\,\mathrm{yr}}{1\,\mathrm{yr}}\right)^{2/3}$

$$\sim 10^{-19}!$$

P93

$R \propto t^{2/3}$

So from $\dfrac{\varepsilon_K(R)}{\varepsilon(R)} = \dfrac{\varepsilon_K(R_i)}{\varepsilon(R_i)}\left(\dfrac{R}{R_i}\right)^{\overset{1}{m}-2}$

0.1

we get after inflation $\dfrac{\varepsilon_K(R)}{\varepsilon(R)} = 0.1\left(10^{40}\right)^{-1}$

$= \dfrac{0.1}{10^{40}} = 10^{-41}$

Then for normal expansion

$$\frac{\varepsilon_K(R)}{\varepsilon(R)} = 10^{-41}\left(\frac{3\times10^{7}}{1}\right)^{2} \approx 10^{-41}\times10^{15} = \underline{10^{-26}}$$

P94

$$\frac{P}{\rho c^2} = \frac{(n-3)}{3}$$

for matter dominated $n=3$ $P=0$

for radiation " $n=4$ $P=\dfrac{\rho c^2}{3}$

for vacuum " $n=0$ $P=-\rho c^2$