AS 4022: Cosmology

HS Zhao and K Horne

Online notes:

star-www.st-and.ac.uk/~hz4/cos/cos.html

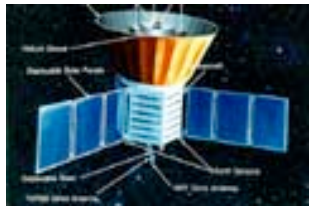
Handouts in Library

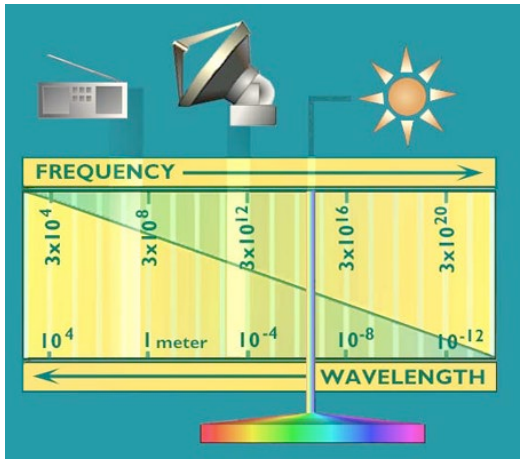
Summary sheet of key results (from John Peacock)

take your own notes (including blackboard lectures)

Observable Space-Time and Bands

- **See What is out there? In all Energy bands**
  - **Pupil → Galileo's Lens → 8m telescopes → square km arrays**
  - **Radio, Infrared ← optical → X-ray, Gamma-Ray (spectrum)**

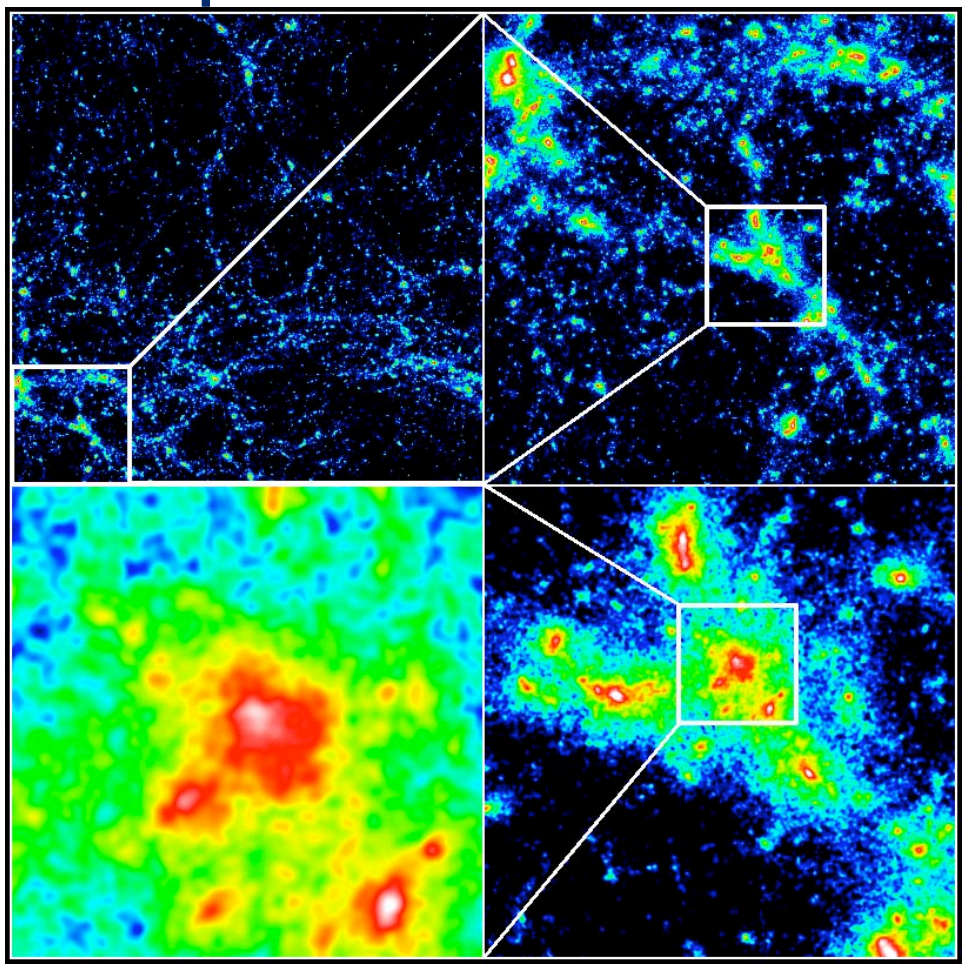




  - **COBE satellites ← Ground → Underground DM detector**
- **Know How were we created? XYZ & T ?**
  - **Us, CNO in Life, Sun, Milky Way, … further and further**
  - **→ first galaxy → first star → first Helium → first quark**
  - **Now → Billion years ago → first second → quantum origin**

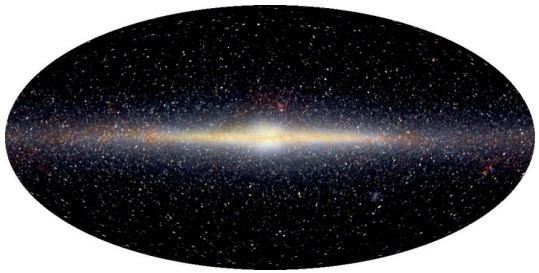
The Visible Cosmos: a hierarchy of structure and motion

- **“Cosmos in a computer”**



Observe A Hierarchical Universe

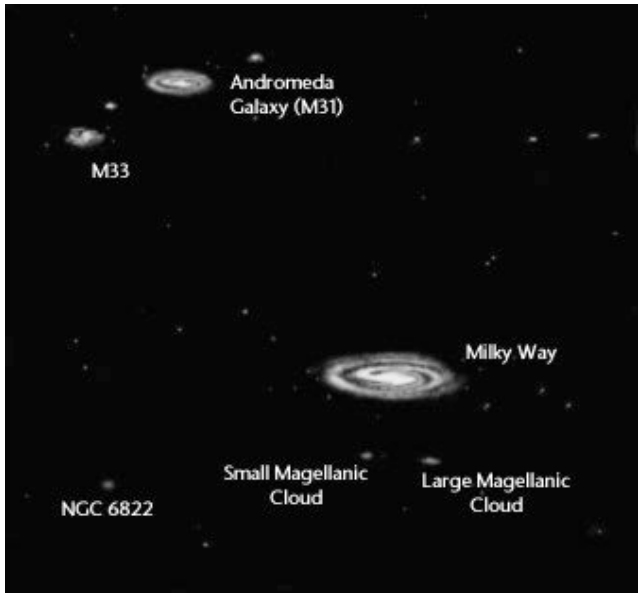
- **Planets**
  - **moving around stars;**
- **Stars grouped together,**
  - **moving in a slow dance around the center of galaxies.**



- **Galaxies themselves**
  - some 100 billion of them in the observable universe—
  - form galaxy clusters bound by gravity as they journey through the void.
- **But the largest structures of all are superclusters,**
  - each containing thousands of galaxies
  - and stretching many hundreds of millions of light years.
  - are arranged in filament or sheet-like structures,
  - between which are gigantic voids of seemingly empty space.

Cosmic Village

- **The Milky Way and Andromeda galaxies,**
  - **along with about fifteen or sixteen smaller galaxies,**
  - **form what's known as the Local Group of galaxies.**
- **The Local Group**
  - **sits near the outer edge of a supercluster, the Virgo cluster.**
  - **the Milky Way and Andromeda are moving toward each other,**
  - **the Local Group is falling into the middle of the Virgo cluster, and**



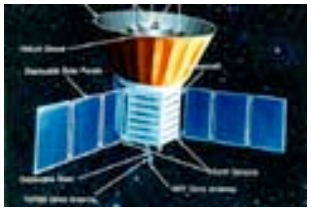
- **the entire Virgo cluster itself,**
  - **is speeding toward a mass**
  - **known only as "The Great Attractor."**

Introducing Gravity and DM (Key players)

- **These structures and their movements**
  - **can't be explained purely by the expansion of the universe**
- **must be guided by the gravitational pull of matter.**

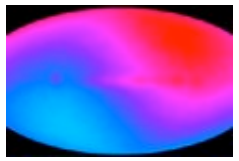
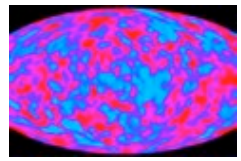
- **Visible matter is not enough**

- **one more player into our hierarchical scenario:**
- **dark matter.**

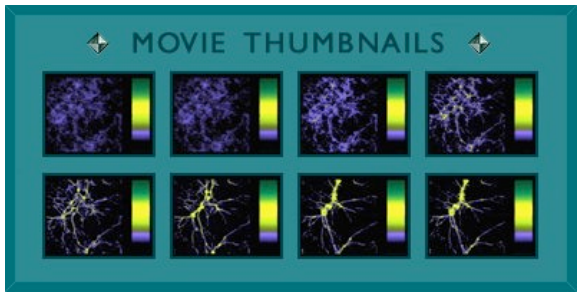


Cosmologists hope to answer these questions:

- **How old is the universe? H_0**
- **Why was it so smooth? P(k), inflation**

- **How did structures emerge from smooth? N-body**
- **How did galaxies form? Hydro**



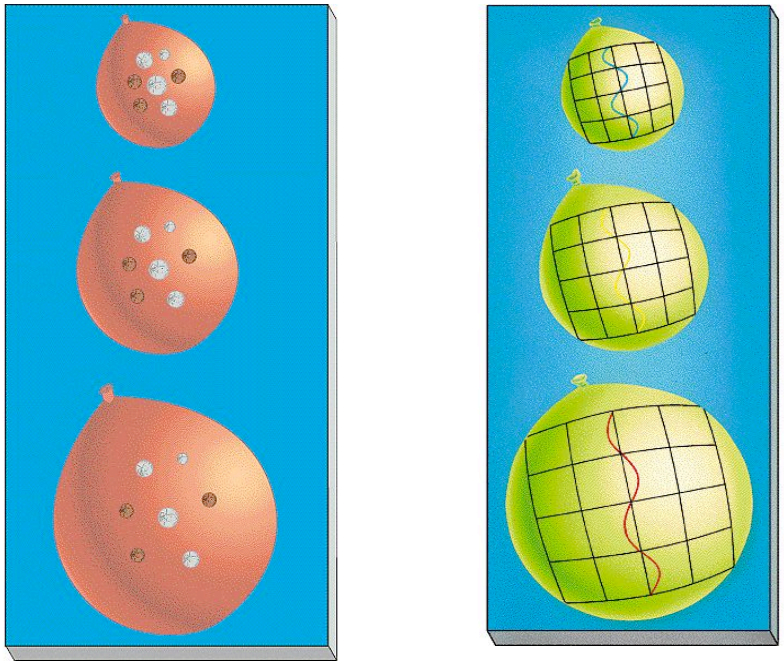
- **Will the universe expand forever? Omega, Lamda**
- **Or will it collapse upon itself like a bubble?**

1st main concept in cosmology

- **Cosmological Redshift**

Stretch of photon wavelength in expanding space

- **Emitted with intrinsic wavelength λ_0 from Galaxy A at time $t<t_{now}$ in smaller universe $R(t) < R_{now}$**



- **→ Received at Galaxy B now (t_{now}) with λ**
- **$\lambda / \lambda_0 = R_{now} / R(t) = 1+z(t) > 1$**

1st main concept: Cosmological Redshift

- **The space/universe is expanding,**
  - **Galaxies (pegs on grid points) are receding from each other**
- **As a photon travels through space, its wavelength becomes stretched gradually with time.**
  - **Photons wave-packets are like links between grid points**
- **This redshift is defined by:**

$$z \equiv \frac{\lambda - \lambda_o}{\lambda_o}$$

$$\frac{\lambda}{\lambda_o} = 1 + z$$

- **E.g. Consider a quasar with redshift z=2. Since the time the light left the quasar the universe has expanded by a factor of 1+z=3. At the epoch when the light left the quasar,**
  - **What was the distance between us and Virgo (presently 15Mpc)?**
  - **What was the CMB temperature then (presently 3K)?**

$$1+z=\frac{\lambda_{now}}{\lambda(t)} \quad \text{(wavelength)}$$

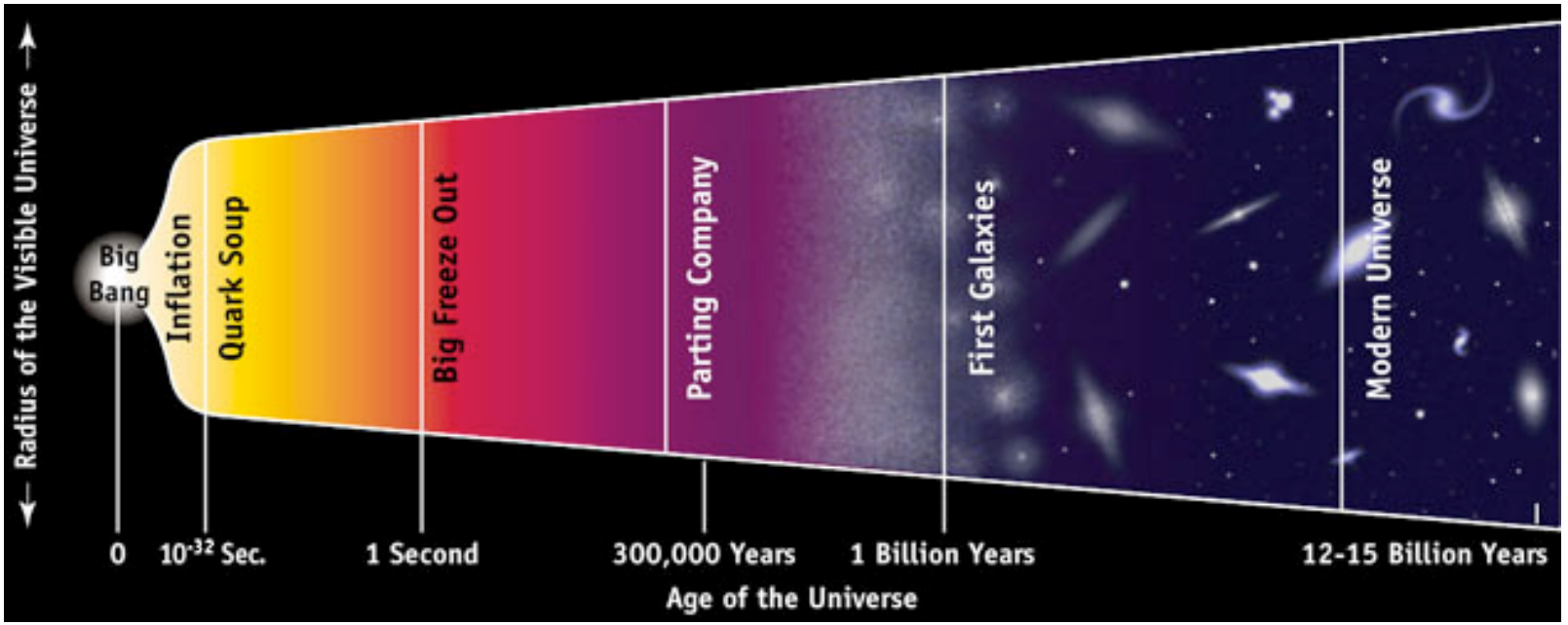
$$=\frac{R_{now}}{R(t)} \quad \text{(expansion factor)}$$

$$=\frac{T(t)}{T_{now}} \quad \text{(Photon Blackbody T } \propto 1/\lambda, why?)$$

Lec 2

Cosmic Timeline

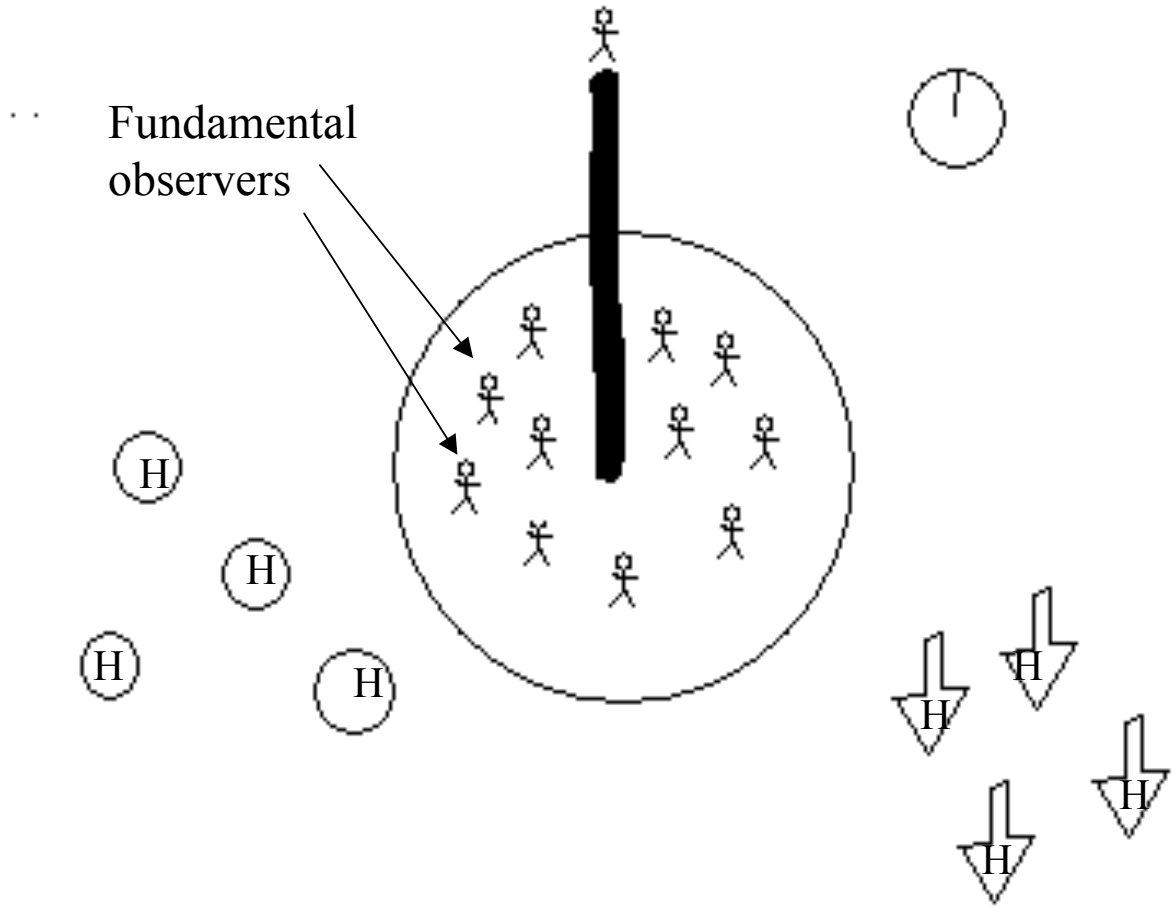
- **Past → Now**



Trafalgar Square

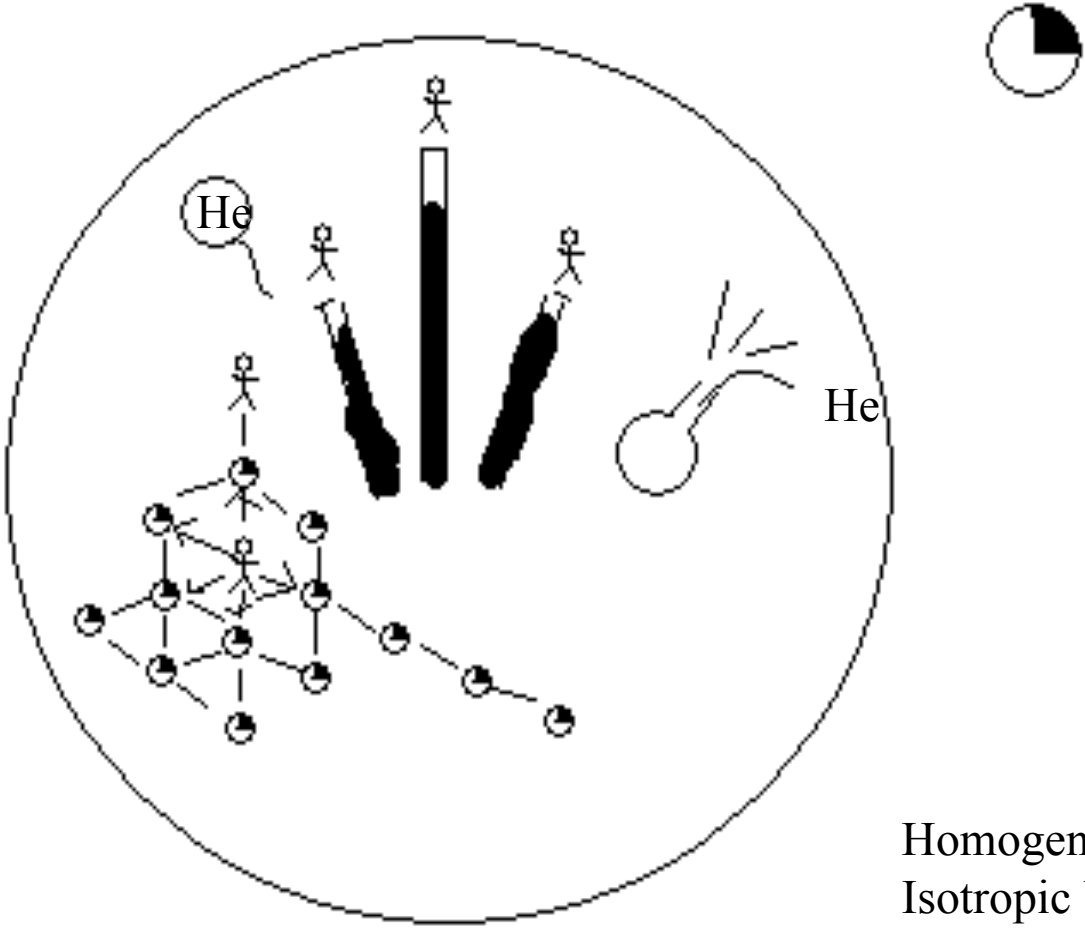
London Jan 1

Set your watches 0h:0m:0s



A comic explanation for cosmic expansion …

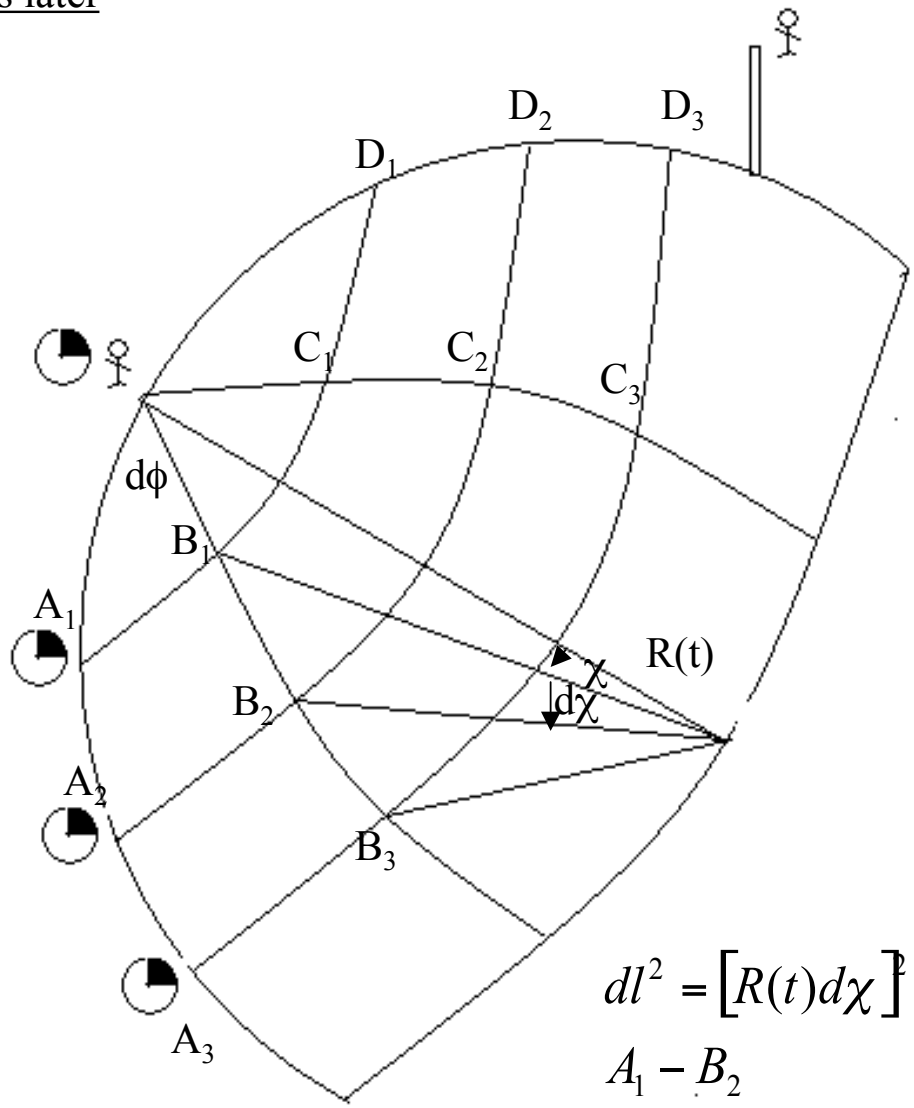
3 mins later



Homogeneous
Isotropic Universe

Walking ↔ *Elevating* ↔ *Earth Radius Stretching* $R(t)$

Feb 14 t=45 days later



$$dl^2 = \left[R(t)d\chi\right]^2 + \left[R(t)\sin\chi d\phi\right]^2$$

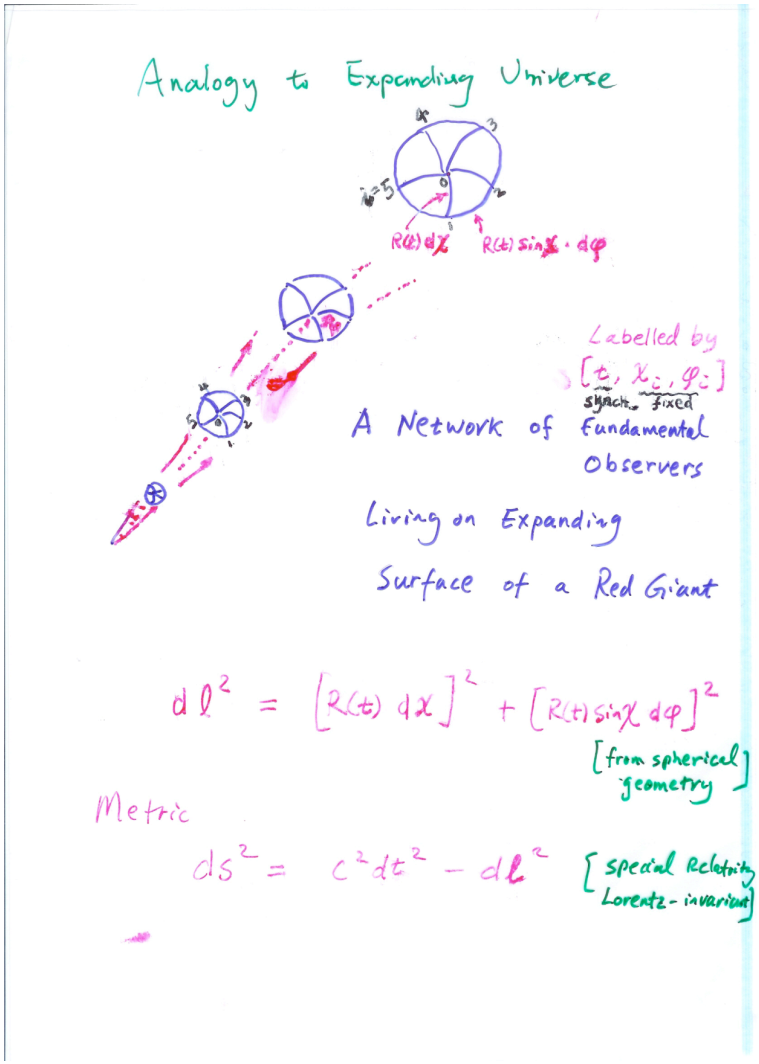
$A_1 - B_2$

Four Pillars of Hot Big Bang

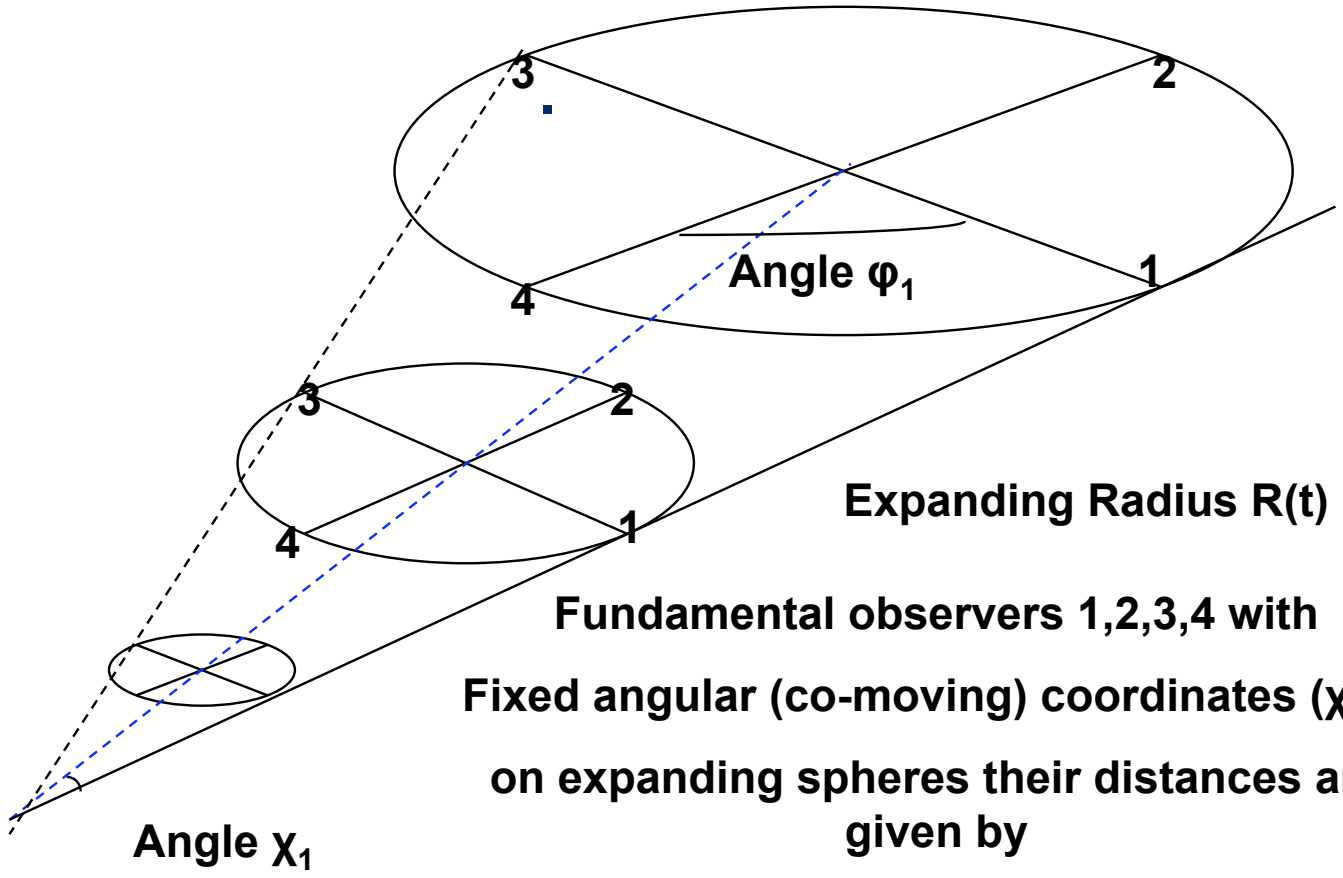
- **Galaxies moving apart from each other**
  - **Redshift or receding from each other**
  - **Universe was smaller**
- **Helium production outside stars**
  - **Universe was hot, at least 10^9K to fuse 4H → He, to overcome a potential barrier of 1MeV.**
- **Nearly Uniform Radiation 3K Background (CMB)**
  - **Universe has cooled, hence expanded by at least a factor 10^9**
- **Missing mass in galaxies and clusters (Cold Dark Matter: CDM)**
  - **Cluster potential well is deeper than the potential due to baryons**
  - **CMB temperature fluctuations: photons climbed out of random potentials of DM**

2nd Concept: metric of 1+2D universe

- **Analogy of a network of civilization living on an expanding star (red giant).**
  - **What is fixed (angular coordinates of the grid points)**
  - **what is changing (distance).**



Analogy: a network on a expanding sphere



Expanding Radius R(t)

Fundamental observers 1,2,3,4 with

Fixed angular (co-moving) coordinates (χ,φ)

on expanding spheres their distances are given by

Metric at cosmic time t $ds^2 = c^2\,dt^2 - dl^2$,

$$dl^2 = R^2(t)\,(d\chi^2 + \sin^2\chi\, d\varphi^2)$$

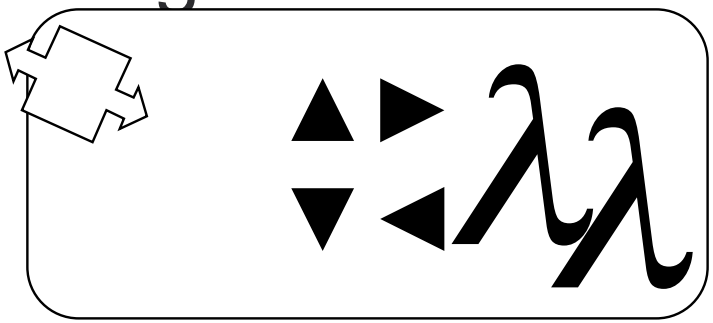
3rd Concept: The Energy density of Universe

- The Universe is made up of three things:
  - VACUUM
  - MATTER
  - PHOTONS (radiation fields)
- The total energy density of the universe is made up of the sum of the energy density of these three components.

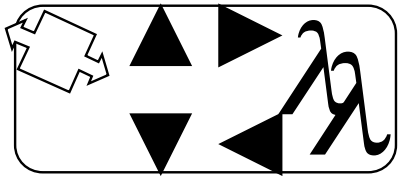
$$\varepsilon(t) = \varepsilon_{vac} + \varepsilon_{matter} + \varepsilon_{rad}$$

- From t=0 to $t=10^9$ years the universe has expanded by R(t).

Eq. of State for Expansion & analogy of baking bread



- Vacuum~air holes in bread
- Matter ~nuts in bread
- Photons ~words painted



- Verify expansion doesn't change N_{hole}, N_{proton}, N_{photon}
  - No Change with rest energy of a proton, changes energy of a photon

$$\varepsilon(t) = \rho_{eff}(t)c^2$$

$$\frac{\varepsilon(t)}{c^2} = \rho_{eff}(t)$$

- **VACUUM ENERGY:** $\rho = \text{constant} \quad \Rightarrow \mathrm{E}_{vac} \propto R^3$

- **MATTER:**

$$\rho R^3 = \text{constant}, \quad \Rightarrow m \approx \text{constant}$$

- **RADIATION: number of photons N_{ph} = constant**

$$\Rightarrow n_{ph} \approx \frac{N_{ph}}{R^3}$$

Wavelength stretches : $\lambda \sim R$

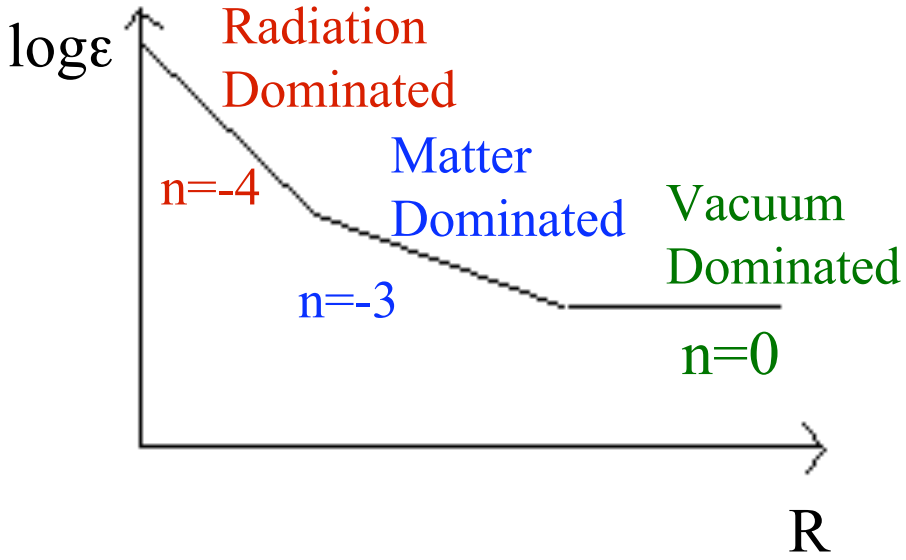
$$\text{Photons: E} = \text{h}\upsilon = \frac{\text{hc}}{\lambda} \sim \frac{1}{R}$$

$$\Rightarrow \varepsilon_{ph} \sim n_{ph} \times \frac{hc}{\lambda} \sim \frac{1}{R^4}$$

- **The total energy density is given by:**

$$\varepsilon \propto \varepsilon_{vac} + \varepsilon_{matter} + \varepsilon_{ph}$$

$$\propto R^{0} \qquad \propto R^{-3} \qquad \propto R^{-4}$$



Key Points

- **Scaling Relation among**
  - Redshift: z,
  - expansion factor: R
    - Distance between galaxies
  - Temperature of CMB: T
    - Wavelength of CMB photons: lambda
- **Metric of an expanding 2D+time universe**
  - Fundamental observers
    - Galaxies on grid points with fixed angular coordinates
- **Energy density in**
  - vacuum, matter, photon
  - How they evolve with R or z

- **If confused, recall the analogies of**
  - balloon, bread, a network on red giant star, microwave oven

Topics
Theoretical and Observational

- **Universe of uniform density**
  - Metrics ds, Scale R(t) and Redshift
  - EoS for mix of vacuum, photon, matter
- **Thermal history**
  - Nucleosynthesis
  - He/D/H
- **Structure formation**
  - Growth of linear perturbation
  - Origin of perturbations
  - Relation to CMB

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- **Quest of H0 (obs.)**
  - Applications of expansion models
  - Distances Ladders
    - (GL, SZ)
- **Quest for Omega (obs.)**
  - Galaxy/SNe surveys
  - Luminosity/Correlation Functions
- **Cosmic Background**
  - COBE/MAP/PLANCK etc.
  - Parameters of cosmos

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Lec 3

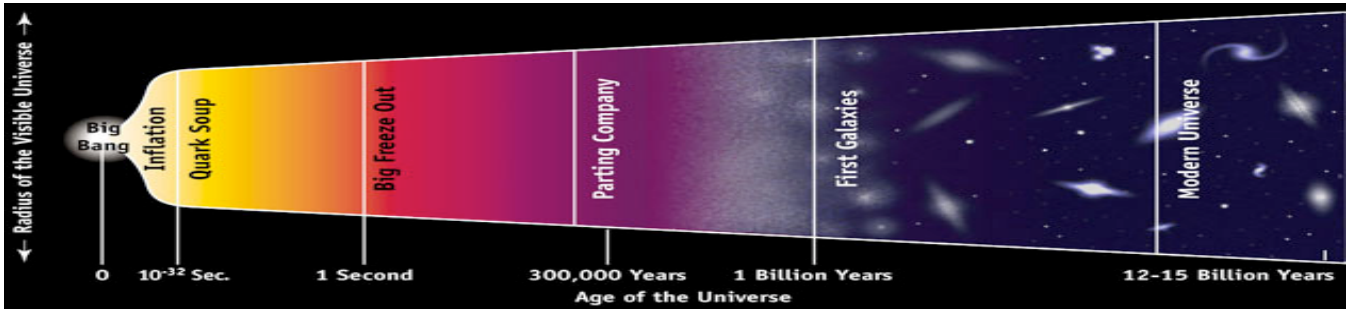
Acronyms in Cosmology

- **Cosmic Background Radiation (CBR)**
  - **Or CMB (microwave because of present temperature 3K)**
    - Argue **about 10^5 photons fit in a 10cmx10cmx10cm microwave oven. [Hint: $3kT = h c / \lambda$]**

- **CDM/WIMPs: Cold Dark Matter, weakly-interact massive particles**
  - **At time DM decoupled from photons, T ~ 10^{14}K, kT ~ 0.1 mc^2**
    - Argue **that dark particles were**
      - **non-relativistic (v/c << 1), hence "cold".**
      - **Massive (m >> m_{proton} =1 GeV)**

Brief History of Universe

- **Inflation**
  - Quantum fluctuations of a tiny region
  - Expanded exponentially
- **Radiation cools with expansion $T \sim 1/R \sim t^{-2/n}$**
  - He and D are produced (lower energy than H)
  - Ionized H turns neutral (recombination)
  - Photon decouple (path no longer scattered by electrons)
- **Dark Matter Era**
  - Slight overdensity in Matter can collapse/cool.
  - Neutral transparent gas
- **Lighthouses (Galaxies and Quasars) form**
  - UV photons re-ionize H
  - Larger Scale (Clusters of galaxies) form



Acronyms and Physics Behind

- **DL: Distance Ladder**
  - **Estimate the distance of a galaxy of size 1 kpc and angular size 1 arcsec? [About 0.6 10^9 light years]**

- **GL: Gravitational Lensing**
  - **Show that a light ray grazing a spherical galaxy of 10^{10} Msun at typical b=1 kpc scale will be bent ~$4GM/bc^2$ radian ~1 arcsec**
  - **It is a distance ladder**

- **SZ: Sunyaev-Zeldovich effect**
  - **A cloud of 1kev thermal electrons scattering a 3K microwave photon generally boost the latter's energy by 1kev/500kev=0.2%**
  - **This skews the blackbody CMB, moving low-energy photons to high-energy; effect is proportional to electron column density.**

- **the energy density of universe now consists roughly**
  - Equal amount of vacuum and matter,
  - 1/10 of the matter is ordinary protons, rest in dark matter particles of 10Gev
    - Argue dark-particle-to-proton ratio ~ 1
  - Photons (3K ~10^{-4}ev) make up only 10^{-4} part of total energy density of universe (which is ~ proton rest mass energy density)
    - Argue photon-to-proton ratio ~ 10^{-4} GeV/(10^{-4}ev) ~ 10^{9}

What have we learned?

- **Concepts of Thermal history of universe**
  - **Decoupling**
  - **Last scattering**
  - **Dark Matter era**
  - **Compton scattering**
  - **Gravitational lensing**
  - **Distance Ladder**
- **Photon-to-baryon ratio >>1**

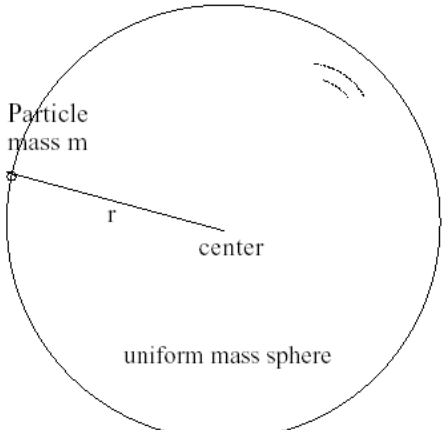
- **If confused, recall the analogy of**
  - **Crystalization from comic soup,**
  - **Last scattering photons escape from the photosphere of the sun**

The rate of expansion of Universe

- **Consider a sphere of radius r=R(t) χ,**
- **If energy density inside is ρ c^2**

➔ **Total effective mass inside is**

M = 4 πρ r^3 /3



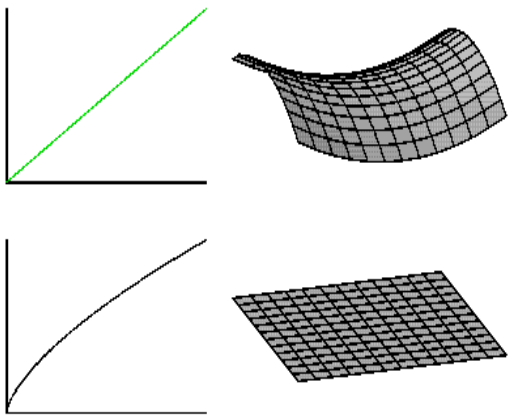
- **Consider a test mass m on this expanding sphere,**
- **For Test mass its**

Kin.Energy + Pot.E. = const E

➔ **m $(dr/dt)^2/2$ – G m M/r = cst**

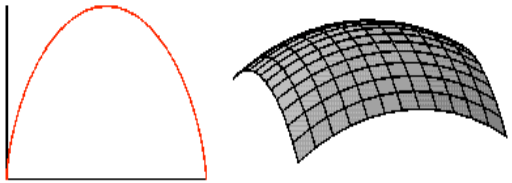
➔ **$(dR/dt)^2/2$ - 4 πG ρ $R^2/3$ = cst**

cst>0, cst=0, cst<0



$(dR/dt)^2/2$ = 4 πG $(\rho + \rho_{cur})$ $R^2/3$

where cst is absorbed by $\rho_{cur} \sim R^{(-2)}$



Typical solutions of expansion rate

$H^2=(dR/dt)^2/R^2=8\pi G\,(\rho_{cur}+\rho_m+\rho_r+\rho_v)/3$

Assume domination by a component $\rho \sim R^{-n}$

Show Typical Solutions Are

$$\rho \propto R^{-n} \propto t^{-2}$$

$$n = 2(\text{curvature constant dominate})$$

$$n = 3(\textit{matter}\ \text{dominate})$$

$$n = 4(\textit{radiation}\ \text{dominate})$$

$$n \sim 0(\text{vaccum dominate}) : \ln(R) \sim t$$

- **Argue also $H = (2/n)\, t^{-1} \sim t^{-1}$. Important thing is scaling!**