AS 4022: Cosmology

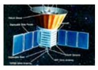
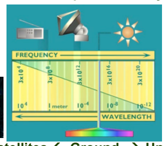
HS Zhao and K Horne

Online notes:
star-www.st-and.ac.uk/~hz4/cos/cos.html

Handouts in Library
Summary sheet of key results (from John Peacock)
take your own notes (including blackboard lectures)

Observable Space-Time and Bands

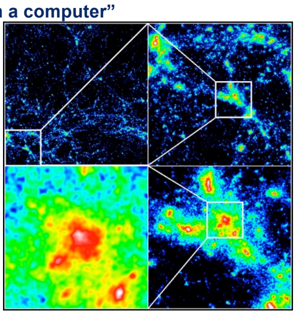
- **See What is out there? In all Energy bands**
  - Pupil → Galileo's Lens → 8m telescopes → square km arrays
  - Radio, Infrared ← optical → X-ray, Gamma-Ray (spectrum)

  - COBE satellites ← Ground → Underground DM detector
- **Know How were we created? XYZ & T ?**
  - Us, CNO in Life, Sun, Milky Way, … further and further
  - → first galaxy → first star → first Helium → first quark
  - Now → Billion years ago → first second → quantum origin

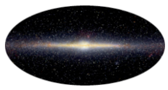
The Visible Cosmos: a hierarchy of structure and motion

- **"Cosmos in a computer"**



Observe A Hierarchical Universe

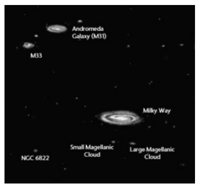
- **Planets**
  - moving around stars;
- **Stars grouped together,**
  - moving in a slow dance around the center of galaxies.



- **Galaxies themselves**
  - some 100 billion of them in the observable universe—
  - form galaxy clusters bound by gravity as they journey through the void.
- **But the largest structures of all are superclusters,**
  - each containing thousands of galaxies
  - and stretching many hundreds of millions of light years.
  - are arranged in filament or sheet-like structures,
  - between which are gigantic voids of seemingly empty space.

Cosmic Village

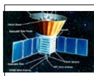
- **The Milky Way and Andromeda galaxies,**
  - along with about fifteen or sixteen smaller galaxies,
  - form what's known as the Local Group of galaxies.
- **The Local Group**
  - sits near the outer edge of a supercluster, the Virgo cluster.
  - the Milky Way and Andromeda are moving toward each other,
  - the Local Group is falling into the middle of the Virgo cluster, and



- **the entire Virgo cluster itself,**
  - is speeding toward a mass
  - known only as "The Great Attractor."

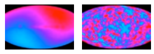
Introducing Gravity and DM (Key players)

- These structures and their movements
  - can't be explained purely by the expansion of the universe
- must be guided by the gravitational pull of matter.
- Visible matter is not enough
- one more player into our hierarchical scenario:
- dark matter.



Cosmologists hope answer these questions:

- How old is the universe? H0
- Why was it so smooth? P(k), inflation



- How did structures emerge from smooth? N-body
- How did galaxies form? Hydro

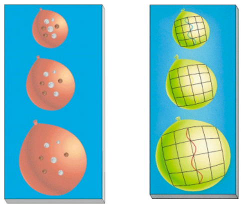
- Will the universe expand forever? Omega, Lamda
- Or will it collapse upon itself like a bubble?

1st main concept in cosmology

- Cosmological Redshift

Stretch of photon wavelength in expanding space

- Emitted with intrinsic wavelength λ_0 from Galaxy A at time $t<t_{now}$ in smaller universe $R(t) < R_{now}$



- → Received at Galaxy B now (t_{now}) with λ
- $\lambda / \lambda_0 = R_{now} / R(t) = 1+z(t) > 1$

1st main concept: Cosmological Redshift

- The space/universe is expanding,
  - Galaxies (pegs on grid points) are receding from each other
- As a photon travels through space, its wavelength becomes stretched gradually with time.
  - Photons wave-packets are like links between grid points
- This redshift is defined by:

$$z \equiv \frac{\lambda - \lambda_o}{\lambda_o}$$

$$\frac{\lambda}{\lambda_o} = 1 + z$$

- e.g. consider a quasar with redshift z=2. Since the time the light left the quasar the universe has expanded by a factor of 1+z=3. At the epoch when the light left the quasar,
  - What was the distance between us and Virgo (presently 15Mpc)?
  - What was the CMB temperature then (presently 3K)?

$$1 + z = \frac{\lambda_{now}}{\lambda(t)} \quad \text{(wavelength)}$$

$$= \frac{R_{now}}{R(t)} \quad \text{(expansion factor)}$$

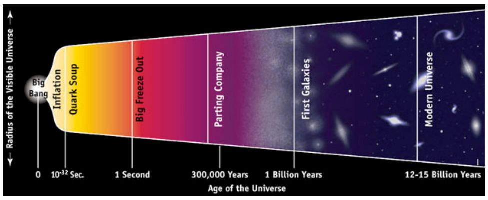
$$= \frac{T(t)}{T_{now}} \quad (\text{Photon Blackbody T } \propto 1/\lambda, why?)$$

CANDLEMAS SEMESTER 2005-06

	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
9 - 10	Classical Mechanics Prof Leonhardt ThC Physics of Electronic Devices Dr Turnbull 301	Nuclear and Particle Physics Dr McGill ThC Galaxies Dr Driver 301	Classical Mechanics Prof Leonhardt ThC Physics of Electronic Devices Dr Turnbull 301	Nuclear and Particle Physics Dr McGill ThC Physics of Electronic Devices Dr Turnbull 301	Nuclear and Particle Physics Dr McGill ThC Galaxies Dr Driver 301
10 -11	Quantum Mechanics 2 Dr Wan ThC Star Formation & Plasma Astrophysics Dr Jardine 301	Principles of Optics Dr Koenig ThC Star Formation & Plasma Astrophysics Dr Jardine 301	Quantum Mechanics 2 Dr Wan ThC Principles of Optics Dr Koenig 301	Principles of Optics Dr Koenig ThC Star Formation & Plasma Astrophysics Dr Jardine 301 Optoelectronics and Nonlinear Optics 2 Prof Krauss 233	
11 -12	Electromagnetism Dr Korolkova ThC Cosmology Prof Horne & Dr Zhao 301 Optoelectronics and Nonlinear Optics 2 Prof Krauss 233	Solid State Physics Prof Mackenzie ThC General Relativity Dr McGill 233	Electromagnetism Dr Korolkova ThC Cosmology Prof Horne & Dr Zhao 301 Optoelectronics and Nonlinear Optics 2 Prof Krauss 233	Solid State Physics Prof Mackenzie ThC General Relativity Dr McGill 233	Electromagnetism Dr Korolkova ThC General Relativity Dr McGill 233
12 - 1	Stars Dr Wood ThC Radio and Coherent Techniques Dr Lesurf 301	Stars Dr Wood ThC Quantum Mechanics 3 Dr Korolkova 301	Transferable Skills for Physicists Dr Sinclair et al. ThC Radio and Coherent Techniques Dr Lesurf 301	Stars Dr Wood ThC Quantum Mechanics 3 Dr Korolkova 301	Solid State Physics Prof Mackenzie ThC Radio and Coherent Techniques Dr Lesurf 301
2-3 2-5		Gravitational Dynamics Dr Zhao ThC Computational Physics Drs Smith, Gillies & Mazilu Comp Classroom			Gravitational Dynamics Dr Zhao ThC Computational Physics Drs Smith, Gillies & Mazilu Comp Classroom

Cosmic Timeline

- Past → Now

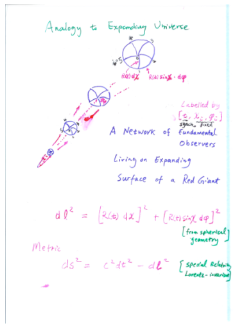


Four Pillars of Hot Big Bang

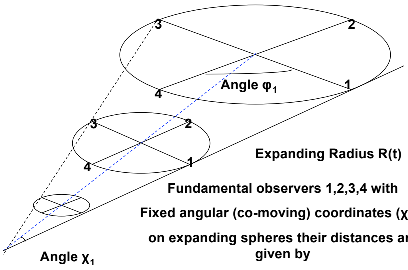
- **Galaxies moving apart from each other**
  - Redshift or receding from each other
  - Universe was smaller
- **Helium production outside stars**
  - Universe was hot, at least 10^9K to fuse 4H → He, to overcome a potential barrier of 1MeV.
- **Nearly Uniform Radiation 3K Background (CMB)**
  - Universe has cooled, hence expanded by at least a factor 10^9
- **Missing mass in galaxies and clusters (Cold Dark Matter: CDM)**
  - Cluster potential well is deeper than the potential due to baryons
  - CMB temperature fluctuations: photons climbed out of random potentials of DM

2^{nd} Concept: metric of 1+2D universe

- **Analogy of a network of civilization living on an expanding star (red giant).**
  - What is fixed (angular coordinates of the grid points)
  - what is changing (distance).
- **Figure to be scanned in.**



Analogy: a network on a expanding sphere



Expanding Radius R(t)

Fundamental observers 1,2,3,4 with Fixed angular (co-moving) coordinates (χ,φ) on expanding spheres their distances are given by

Metric at cosmic time t $ds^2 = c^2 dt^2 - dl^2$,

$dl^2 = R^2(t) (d\chi^2 + \sin^2\chi \, d\varphi^2)$

3^{rd} Concept: The Energy density of Universe

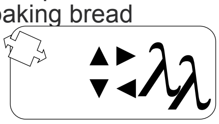
- **The Universe is made up of three things:**
  - VACUUM
  - MATTER
  - PHOTONS (radiation fields)
- **The total energy density of the universe is made up of the sum of the energy density of these three components.**

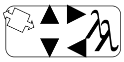
$$\varepsilon(t) = \varepsilon_{vac} + \varepsilon_{matter} + \varepsilon_{rad}$$

- **From t=0 to t=10^9 years the universe has expanded by R(t).**

Eq. of State for Expansion & analogy of baking bread

- **Vacuum~air holes in bread**
- **Matter ~nuts in bread**
- **Photons ~words painted**





- **Verify expansion doesn't change N_{hole}, N_{proton}, N_{photon}**
  - No Change with rest energy of a proton, changes energy of a photon

$$\varepsilon(t) = \rho_{eff}(t)c^2$$

$$\frac{\varepsilon(t)}{c^2} = \rho_{eff}(t)$$

- **VACUUM ENERGY:** $\rho = \text{constant} \quad \Rightarrow \mathrm{E}_{vac} \propto R^3$
- **MATTER:**

$$\rho R^3 = \text{constant}, \quad \Rightarrow m \approx \text{constant}$$

- **RADIATION:number of photons N_{ph} = constant**

$$\Rightarrow n_{ph} \approx \frac{N_{ph}}{R^3} \qquad \text{Wavelength stretchs :}\lambda \sim R$$

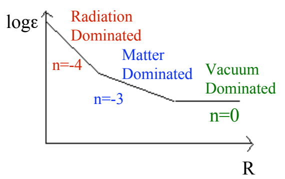
$$\text{Photons:E} = \mathrm{h}\upsilon = \frac{\mathrm{hc}}{\lambda} \sim \frac{1}{R}$$

$$\Rightarrow \varepsilon_{ph} \sim n_{ph} \times \frac{hc}{\lambda} \sim \frac{1}{R^4}$$

- **The total energy density is given by:**

$$\varepsilon \propto \varepsilon_{vac} + \varepsilon_{matter} + \varepsilon_{ph}$$

$$\propto R^0 \qquad \propto R^{-3} \qquad \propto R^{-4}$$



Key Points

- **Scaling Relation among**
  - Redshift: z,
  - expansion factor: R
    - Distance between galaxies
  - Temperature of CMB: T
    - Wavelength of CMB photons: lambda
- **Metric of an expanding 2D+time universe**
  - Fundamental observers
    - Galaxies on grid points with fixed angular coordinates
- **Energy density in**
  - vacuum, matter, photon
  - How they evolve with R or z
- **If confused, recall the analogies of**
  - balloon, bread, a network on red giant star, microwave oven

Topics
Theoretical and Observational

- **Universe of uniform density**
  - Metrics ds, Scale R(t) and Redshift
  - EoS for mix of vacuum, photon, matter
- **Thermal history**
  - Nucleosynthesis
  - He/D/H
- **Structure formation**
  - Growth of linear perturbation
  - Origin of perturbations
  - Relation to CMB

Hongsheng.Zhao (hz4)

- **Quest of H0 (obs.)**
  - Applications of expansion models
  - Distances Ladders
    - (GL, SZ)
- **Quest for Omega (obs.)**
  - Galaxy/SNe surveys
  - Luminosity/Correlation Functions
- **Cosmic Background**
  - COBE/MAP/PLANCK etc.
  - Parameters of cosmos

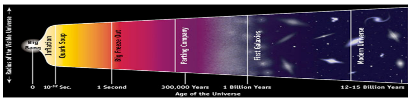
Keith D. Horne (kdh1)

Acronyms in Cosmology

- **Cosmic Background Radiation (CBR)**
  - Or CMB (microwave because of present temperature 3K)
    - Argue about 10^5 photons fit in a 10cmx10cmx10cm microwave oven. [Hint: 3kT = h c / λ]
- **CDM/WIMPs: Cold Dark Matter, weakly-interact massive particles**
  - At time DM decoupled from photons, $T \sim 10^{14}$K, kT ~ 0.1 mc^2
    - Argue that dark particles were
      - non-relativistic (v/c << 1), hence "cold".
      - Massive (m >> m_{proton} =1 GeV)

Brief History of Universe

- **Inflation**
  - Quantum fluctuations of a tiny region
  - Expanded exponentially
- **Radiation cools with expansion $T \sim 1/R \sim t^{-2/n}$**
  - He and D are produced (lower energy than H)
  - Ionized H turns neutral (recombination)
  - Photon decouple (path no longer scattered by electrons)
- **Dark Matter Era**
  - Slight overdensity in Matter can collapse/cool.
  - Neutral transparent gas
- **Lighthouses (Galaxies and Quasars) form**
  - UV photons re-ionize H
  - Larger Scale (Clusters of galaxies) form



Acronyms and Physics Behind

- **DL: Distance Ladder**
  - Estimate the distance of a galaxy of size 1 kpc and angular size 1 arcsec? [About 0.6 10^9 light years]
- **GL: Gravitational Lensing**
  - Show that a light ray grazing a spherical galaxy of 10^{10} Msun at typical b=1 kpc scale will be bend ~4GM/b/c^2 radian ~1 arcsec
  - It is a distance ladder
- **SZ: Sunyaev-Zeldovich effect**
  - A cloud of 1kev thermal electrons scattering a 3K microwave photon generally boost the latter's energy by 1kev/500kev=0.2%
  - Argue this skews the blackbody CMB, moving low-energy photons to high-energy; effect is proportional to electron column density.

- **the energy density of universe now consists roughly**
  - Equal amount of vaccum and matter,
  - 1/10 of the matter is ordinary protons, rest in dark matter particles of 10Gev
    - Argue dark-particle-to-proton ratio ~ 1
  - Photons (3K ~10^{-4}ev) make up only 10^{-4} part of total energy density of universe (which is ~ proton rest mass energy density)
    - Argue photon-to-proton ratio ~ 10^{-4} GeV/(10^{-4}ev) ~ 10^9

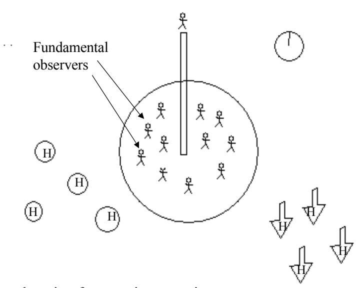
What have we learned?

- **Concepts of Thermal history of universe**
  - Decoupling
  - Last scattering
  - Dark Matter era
  - Compton scattering
  - Gravitational lensing
  - Distance Ladder
- **Photon-to-baryon ratio >>1**
- **If confused, recall the analogy of**
  - Crystalization from comic soup,
  - Last scattering photons escape from the photosphere of the sun

Trafalgar Square

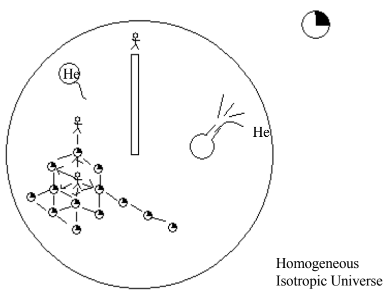
London Jan 1 2006

Set your watches 0h:0m:0s



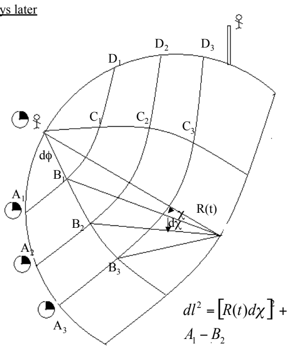
A comic explanation for cosmic expansion …

3 mins later



Walking ↔ Earth Radius Stretching R(t)

Feb 8 2006 40 days later



$$dl^2 = [R(t)d\chi]^2 + [R(t)\sin\chi d\phi]^2$$

$$A_1 - B_2$$

Lec 4 Feb 15

The rate of expansion of Universe

- **Consider a sphere of radius r=R(t) χ,**
- **If energy density inside is ρ c²**

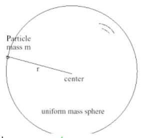
➔ Total effective mass inside is M = 4 πρ r³ /3

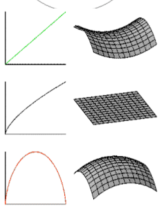
- **Consider a test mass m on this expanding sphere,**
- **For Test mass its Kin.Energy + Pot.E. = const E**

➔ m (dr/dt)²/2 – G m M/r = cst

➔(dR/dt)²/2 - 4 πG ρ R²/3 = cst

cst>0, cst=0, cst<0





Typical solutions of expansion rate

$$\frac{\dot{R}^2}{2} - \frac{4\pi G\rho R^2}{3} = cst$$

Show Typical Solutions Are

$\rho \propto R^{-n} \propto t^{-2}$

$n = 2$(curvature constant dominate)

$n = 3$(*matter* dominate)

$n = 4$(*radiation* dominate)

$n \sim 0$(vaccum dominate) : $\ln(R) \sim t$

Where are we heading?

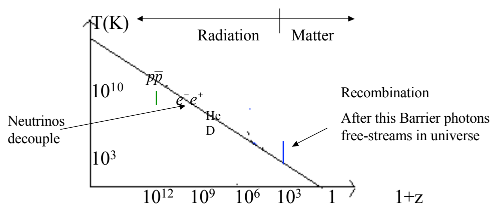
Next few lectures will cover a few chapters of

– Malcolm S. Longair's "Galaxy Formation" [Library Short Loan]

- **Chpt 1: Introduction**
- **Chpt 2: Metrics, Energy density and Expansion**
- **Chpt 9-10: Thermal History**

Thermal Schedule of Universe [chpt 9-10]

- **At early times, photons are typically energetic enough that they interact strongly with matter so the whole universe sits at a temperature dictated by the radiation.**
- **The energy state of matter changes as a function of its temperature and so a number of key events in the history of the universe happen according to a schedule dictated by the temperature-time relation.**



- At early times energy density of photons are high enough to produce particle pairs
  - the number density of photons was so high, and typical photons were so energetic
  - PHOTON+PHOTON←→PARTICLE +ANTI-PARTICLE
- The kinds of particles and anti-particles that are created depends on photon energy spectrum
  - Particularly, depends on the average energy per photon, which depends on the temperature.
  - If the photon energy is less than $m_p c^2$ then m_p can't be created;
  - as universe cools, more massive particles ceased to be created, while less massive particles were still allowed to be created.

For example,

- the window of opportunity for creating electrons is longer than creating protons.
  - Argue it is because a proton is 2000 times heavier than an electron.
  - In fact 10^{-6}s to 0.1s is the window of opportunity for creating protons and neutrons and other "normal" particles.
  - Electrons were around till 1 sec.

NEUTRINO DECOUPLE as Hot DM

- Neutrinos are kept in thermal equilibrium by creating electron pairs and scattering (weak interaction):

$$\upsilon + e^- \leftrightarrow \upsilon + e^-$$

$$\upsilon + \overline{\upsilon} \leftrightarrow e^+ + e^-$$

- This interaction freezes out when the temperature drops to kT_υ ~MeV~ rest mass electrons
  - Because very few electrons are around afterwards
  - Move without scattering by electrons after 1 sec.
- Argue that Neutrinos have Relativistic speeds while freezing out
  - kT_υ >> rest mass of neutrinos(~eV)
  - They are called Hot Dark Matter (HDM)

e.g., Argue why fewer neutrons in universe than protons

- Before 1 s, lots of neutrinos and electrons keep the abundance of protons and neutrons about equal through
  - n + υ ←→ p + e⁻
- After 1 s free-moving neutrons (which is slightly more massive than protons) start to decay.
  - n → p + e⁻ + υ

thermal equilibrium number density

- The thermal equilibrium background number density of particles is given by:

$$n = \frac{g}{h^3}\int_0^\infty \frac{d\left(\frac{4\pi}{3}p^3\right)}{\exp(E/kT)\pm 1} \qquad \begin{array}{l} + \text{ for Fermions} \\ - \text{ for Bosons} \end{array}$$

- Where we have to change to momentum space and g is the degeneracy factor.

$$E = \sqrt{c^2p^2 + (mc^2)^2} \approx cp \quad \text{relativistic cp >> mc}^2$$

$$\approx m\,c^2 + \frac{1}{2}\frac{p^2}{m} \quad \text{non relativistic cp < mc}^2$$

- As kT cools, particles go from
- From Ultrarelativistic limit. ($kT >> mc^2$) particles behave as if they were massless→

$$n = \left(\frac{kT}{c}\right)^3 \frac{4\pi g}{(2\pi\hbar)^3}\int_0^\infty \frac{y^2dy}{e^y \pm 1} \Rightarrow n \sim T^3$$

- To Non relativistic limit ($kT << 0.1mc^2$.) Here we can neglect the ±1 in the occupancy number→

$$n = e^{-\frac{mc^2}{kT}}(2mkT)^{\frac{3}{2}}\frac{4\pi g}{(2\pi\hbar)^3}\int_0^\infty e^{-y^2}y^2dy \Rightarrow n \sim T^{\frac{3}{2}}e^{-\frac{mc^2}{kT}}$$

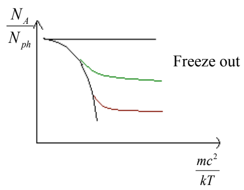
Particles Freeze Out

- Freeze-out of equilibrium (relativistic or non-relativistic) at certain temperature depending on number density, and cross-section.

- Generally a particle A undergoes the reaction:

$$A + \bar{A} \rightarrow \gamma + \gamma$$

- When the reverse reaction rate is slower than Hubble expansion rate, it undergoes freezeout.



$\sigma_A \upsilon$ LOW→weak interaction early freeze out while relativistic

$\sigma_A \upsilon$ HIGH→strong interaction later freeze out at lower T

A general history of a massive particle

- Initially relativistic, dense (comparable to photon number density),
  - has frequent collisions with other species to be in thermal equilibrium and cools with CBR photon bath.

Freeze-Out

- Later, Relics Freeze-out of the cooling heat bath because
  - interactions too slow due to lower and lower density in expanding universe.
  - This defines a “last scattering surface” where optical depth drops below unity.
  - The number density falls with expanding volume of universe, but Ratio to photons kept constant.

Number density of non-relativistic particles to relativistic photons

- Reduction factor ~ exp(-mc^2/kT), which drop sharply with cooler temperature.
- Non-relativistic particles (relic) become *much rarer* by exp(-10) as universe cools below 0.1 mc^2
  - So rarer that infrequent collisions can no longer maintain coupled-equilibrium.
  - So Decouple and Freeze-out

smallest Collision cross-section

- neutrinos (Hot DM) decouple from electrons (due to very weak interaction) while still hot (relativistic $kT>mc^2$).
-

Small Collision cross-section

- **Decouple non-relativisticly once $kT<mc^2$. Number density ratio to photon drops steeply with cooling $\exp(- mc^2/kT)$.**
  - **anti-protons and wimps (Cold DM) etc. decouple (stop creating/annihilating) while non-relativistic. Abundant (CDM).**
- **$T_c \sim 10^9$K NUCLEOSYNTHESIS (100s)**
- **$T_c \sim 5000$K RECOMBINATION (10^6 years) (Redshift=1000)**

A worked-out exercise

$A+\bar{A} \to \gamma + \gamma$

Show at last scattering surface Optical depth $\tau = \int_0^z \sigma v \eta n_{ph}(z) \frac{dt}{dz} dz$

$\sim \int_0^z \sigma v \eta (1+z)^3 \frac{d(1+z)^{-n/2}}{dz} dz$

$\sim \sigma v \eta (1+z)^{3-n/2} \sim \sigma v \eta T^{3-n/2} \sim 1$.

where n=4 for radiation era.

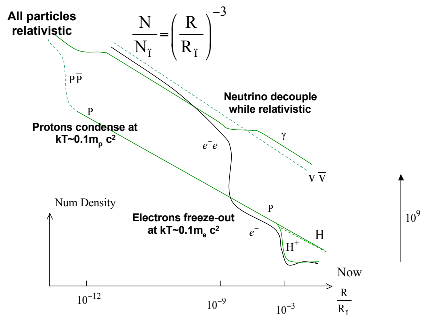
Given that Freeze-out fraction $\eta \sim \exp(-\frac{\Delta mc^2}{kT})$

and assume decouple at $kT \sim mc^2/\ln(1/\eta)$,

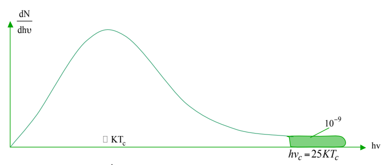
Argue cosmic abundance

$\Omega \sim \eta m \sim T^{-1} m/(\sigma v) \sim (\sigma v)^{-1}$

A summary: Evolution of Number Densities of γ, P, e, υ



particles of energy $E_c = h\nu_c$ unbound by high energy tail of photon bath



$$\frac{N(>h\nu_c)}{N_{ph}} \approx e^{-\frac{h\nu_c}{KT_c}} \left(\frac{h\nu_c}{KT_c}\right)^2 .O(1)$$

$$\sim e^{-25} \times 25^2$$

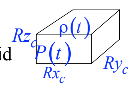
$$\sim 10^{-9} \sim \frac{N_B}{N_{ph}}$$

\# hardest photons ~ # baryons

If run short of hard photon to unbind => "Freeze-out" => $KT_c \sim \frac{h\nu_c}{25}$

Evolution of Sound Speed

Expand a box of fluid



Sound Speed $C_s^2 = \frac{\partial P / \partial(vol)}{\partial \rho / \partial(vol)}$

$= \frac{\partial P / \partial R}{\partial \rho / \partial R}$

$Vol = R^3(t) x_c y_c z_c$

$\propto R^3(t)$

Coupled radiation-baryon relativistic fluid

	Radiation	Matter
Where fluid density $\rho(t) =$	ρ_r	ρ_m
Fluid pressure $P(t) =$	$\frac{c^2}{3}\rho_r$	$\frac{\rho_m}{\mu} . KT_m$

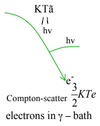
Matter number density — Random motion energy Non-Relativistic IDEAL GAS

Note $\rho_r \propto R^{-4}$

$\rho_m \propto R^{-3}$ Neglect $\frac{1}{\mu} KT_m << c^2$

Show $C_s^2 = c^2/3 /(1+Q)$, $Q = (3 \rho_m) /(4 \rho_r)$, ➔ Cs drops
- **from c/sqrt(3) at radiation-dominated era**
- **to c/sqrt(5.25) at matter-radiation equality**

Coupled Photon-Baryon Fluid



Keep electrons hot Te ~ Tr until redshift z

$$Tr \approx 1500 \times \left(\frac{1+z}{500}\right)$$

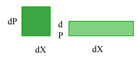
Temperature and Sound Speed of Decoupled Baryonic Gas

After decoupling (z<500),
Cs ~ 6 (1+z) m/s because

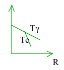
$d^3P\, d_x^3$ invarient phase space volume

So: $P \propto x^{-1} \propto R^{-1}$

$$\frac{3}{2}\times T_e = mv^2/2 \propto R^{-2} \quad Te \sim 1500K?\left(\frac{1+z}{500}\right)^2$$



$$T_e \propto C_s^2 \propto R^{-2}$$



Until reionization z ~ 10 by stars quasars

What have we learned? Where are we heading?

- **Sound speed of gas before/after decoupling**

Topics Next:

- **Growth of [chpt 11 bankruptcy of uniform universe]**
  - **Density Perturbations (how galaxies form)**
  - **peculiar velocity (how galaxies move and merge)**
- **CMB fluctuations (temperature variation in CMB)**
- **Inflation (origin of perturbations)**

Peculiar Motion

- **The motion of a galaxy has two parts:**

$$\vec{v} = \frac{d}{dt}\left[R(t)\theta(t)\right]$$

$$= \dot{R}(t).\theta + R(t)\dot{\theta}(t)$$

$\underset{\sim}{x}^c \leftrightarrow \underset{\to}{\theta}$

Proper length vector

Uniform expansion v_o

Peculiar motion δv

Damping of peculiar motion (in the absence of overdensity)

-
- **Generally peculiar velocity drops with expansion.**

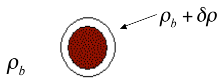
$$R^2\dot{\theta} = R*(R\dot{\theta}) = \text{constant}\sim\text{"Angular Momentum"}$$

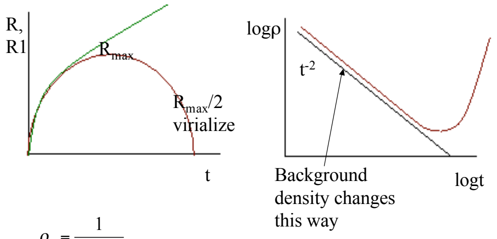
- **Similar to the drop of (non-relativistic) sound speed with expansion**

$$\delta v = R(t)\dot{x}_c = \frac{\text{constant}}{\text{R(t)}}$$

Non-linear Collapse of an Overdense Sphere

- **An overdense sphere is a very useful non linear model as it behaves in exactly the same way as a closed sub-universe.**
- **The density perturbations need not be a uniform sphere: any spherically symmetric perturbation will clearly evolve at a given radius in the same way as a uniform sphere containing the same amount of mass.**





$$\rho_b = \frac{1}{6\pi G t^2}$$

Gradual Growth of perturbation

$$\rightarrow \frac{\delta\rho}{\rho} = \frac{3c^2}{8\pi G}\frac{1}{\rho R^2} \propto \begin{cases} R^2 \text{ (mainly radiation } \rho \propto R^{-4}) \\ R \text{ (mainly matter } \rho \propto R^{-3}) \end{cases}$$

Perturbations Grow!

Verify δ change by a factor 10 between z=10 to z=100? And a factor 100 between z=10^5 to z=10^6?

Equations governing Fluid Motion

$$\nabla^2\phi = 4\pi G\rho \qquad \text{(Poissons Equation)}$$

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{dt}} = \frac{d\ln\rho}{dt} = -\vec{\nabla}.\vec{v} \qquad \text{(Mass Conservation)}$$

$$\frac{\mathrm{dv}}{\mathrm{dt}} = -\vec{\nabla}\phi - c_s^2\vec{\nabla}\ln\rho \qquad \text{(Equation of motion)}$$

$$\frac{\nabla P}{\rho} \quad \text{since } \partial P = c_s^2 \partial\rho$$

Decompose into unperturbed + perturbed

- **Let**

$\rho = \rho_o + \delta\rho$

$v = v_o + \delta v = \dot{R}\chi_c + R\dot{\chi}_c$

$\phi = \phi_o + \delta\phi$

- **We define the Fractional Density Perturbation:**

$$\delta = \frac{\delta\rho}{\rho_o} = \delta(t)\exp(-i\vec{k}\bullet\vec{x}),$$

$$|\vec{k}| = 2\pi/\lambda, \quad \text{where } \lambda = R(t)\lambda_c$$

$$\vec{k}\bullet\vec{x} = \vec{k}_c\bullet\vec{x}_c \qquad x(t) = R(t)\chi_c$$

- **Motion driven by gravity:** $\vec{g}_o(t) + \vec{g}_1(\theta, t)$
  due to an overdensity: $\rho(t) = \rho_o(1 + \delta(\theta, t))$
- **Gravity and overdensity by Poissons equation:**
- **Continuity equation:**

$$-\vec{\nabla}\bullet g_1 = 4\pi G\rho_o\delta$$

Peculiar motion δv and peculiar gravity g_1 both scale with δ and are in the same direction.

$$-\vec{\nabla}\bullet\delta\vec{v} = \frac{d}{dt}(\delta(\theta, t))$$

The over density will rise if there is an inflow of matter

THE equation for structure formation

- **In matter domination**
- **Equation becomes**

$$\frac{\partial^2\delta}{\partial t^2} + 2\frac{\dot{R}}{R}\frac{\partial\delta}{\partial t} = (4\pi G\rho_o + c_s^2\nabla^2)\delta$$

$-c_s^2 k^2$

Gravity has the tendency to make the density perturbation grow exponentially.

Pressure makes it oscillate

e.g., Nearly Empty Pressure-less Universe

$$\rho \sim 0$$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2}{t}\frac{\partial \delta}{\partial t} = 0, \quad H = \frac{\dot{R}}{R} = \frac{1}{t} \quad (R \propto t)$$

$$\delta \propto t^0 = \text{constant}$$

$$\rightarrow \text{no growth}$$

What have we learned? Where are we heading?

- **OverDensity grows as**
  - R (matter) or R^2 (radiation)
- **Peculiar velocity points towards overdensities**
- **Topics Next: Jeans instability**

The Jeans Instability

- **Case 1- no expansion** $\dot{R} = 0$
  - Verify the density contrast δ has a wave-like form

for the harmonic oscillator equation

$$\delta = \delta_o \exp(i\bar{k}.\bar{r} - i\omega t)$$

where we have the dispersion relation

$$\frac{\partial^2 \delta}{\partial t^2} + 2 * 0 * \frac{\partial \delta}{\partial t} = -\omega^2 \delta$$

$$\omega^2 \equiv c_s^2 k^2 - 4\pi G \rho$$

Pressure support ($c_s^2 k^2$) — gravity ($4\pi G\rho$)

- **At the (proper) JEANS LENGTH scale we switch from**
  - standing sound waves for shorter wavelengths to
  - the exponential growth of perturbations for long wavelength modes

- **$\lambda < \lambda_J$, $\omega^2 > 0$ → oscillation of the perturbation.**
- **$\lambda \geq \lambda_J$, $\omega^2 \leq 0$ → exponential growth/decay**

$$\lambda_J = c_s \tau, \quad \text{where timescale } \tau = \sqrt{\frac{\pi}{G\rho}}$$

$$\delta \propto \exp(\pm \Gamma t) \text{ where } \Gamma = \sqrt{-\omega^2}$$

Jeans Length in background of constant or falling density

- **Background of Constant density :**
  - Application: Collapse of clouds, star formation.
  - Timescale:

$$\tau = (G\rho/\pi)^{-\frac{1}{2}}$$

$\sim$ dynamical collapse time

for region of uniform density ρ.

- **Background of Falling density**
  - Expanding universe $G\rho \sim t^{-2}$,
  - Instantaneous Jeans length $\sim c_s t$

Jeans Instability

- **Case 2: on very large scale $\lambda >> \lambda_J$ of Expanding universe**
  - Neglect Pressure (restoring force) term

$$c_s^2 k^2 << 4\pi G \rho = c_s^2 k_J^2$$

$$\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} = 4\pi G \rho_m \delta$$

$2H \rightarrow 2/(3t)$; $4\pi G\rho_m \rightarrow 2/(3t^2)$

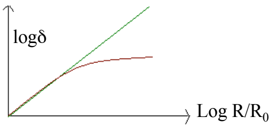
E.g.,

- Einstein de Sitter Universe

$$\Omega_M = 1, H = \frac{\dot{R}}{R} = \frac{2}{3t}$$

$$\text{Verify Growth Solution } \delta \propto R \propto t^{\frac{2}{3}} \propto \frac{1}{1+z}$$

- Generally



Case III: Relativistic (photon) Fluid

- equation governing the growth of perturbations being:

$$\Rightarrow \frac{d^2\delta}{dt^2} + 2H\frac{d\delta}{dt} = \delta.\left(\frac{32\pi G\rho}{3} - k^2 c_s^2\right)$$

- Verify

1/t $1/t^2$

- Oscillation solution on small scale $2\pi/k = \lambda<\lambda_J$

$$\Rightarrow \delta \propto t \propto R^2 \quad \text{for length scale } \lambda >> \lambda_J \sim c_s t$$

Jeans Mass Depends on the Species of the Fluid that dominates

- If Photon dominates:

$$M_J^\gamma = \rho_\gamma(t)\frac{4\pi}{3}\left(\frac{\lambda_J}{2}\right)^3 \propto \frac{1}{6t^2}\left[\frac{c}{\sqrt{3}}t\right]^3 \propto t^1 \propto (1+z)^{-2}$$

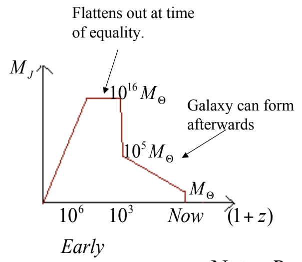
$c_s t$=distance travelled since big bang

- If DarkMatt dominates & decoupled from photon:

$$M_J^D = \rho_D(t)\frac{4\pi}{3}\left(\frac{\lambda_J}{2}\right)^3 \propto t^{-2}\left[c_s t\right]^3 \propto t^{-1}$$

$$\text{non-relativistic cooling of random motion } c_s \propto 1/R \propto t^{-2/3}$$

- Jeans Mass past and now



$$\text{Note: } R \propto (1+z)^{-1}$$

Summary: Dark Matter Overdensity Growth Condition

- GROW [~Collapse] only if
  - During matter-domination ($t > t_{eq}$) [chpt 11.4] or
  - during radiation domination, but on proper length scales larger than
    - sound horizon ($\lambda > c_s$ t) [chtp12.1] &
    - free-streaming length of relativistic dark matter ($\lambda > c\ t_{fs}$) [chpt 13.3]

Lec 8

- What have we learned: [chpt 11.4]
  - Conditions of gravitational collapse (=growth)
  - Stable oscillation (no collapse) within sound horizon if pressure-dominated
- Where are we heading:
  - Cosmic Microwave Background [chpt 15.4]
    - As an application of Jeans instability
  - Inflation in the Early Universe [chpt 20.3]

Theory of CMB Fluctuations

- **Linear theory of structure growth predicts that the perturbations:**

δ_D in dark matter $\dfrac{\delta\rho_{\rm D}}{\rho_{\rm D}}$

δ_B in baryons $\dfrac{\delta\rho_{\rm B}}{\rho_{\rm B}}$

δ_r in radiation $\dfrac{\delta\rho_{\rm r}}{\rho_r}$ Or $\tilde{\delta}_r = \dfrac{3}{4}\delta_r = \dfrac{\delta n_\gamma}{n_\gamma}$

will follow the following coupled equations.

$$\frac{d^2}{dt^2}\begin{pmatrix}\delta_D\\ \delta_B\\ \tilde{\delta}_r\end{pmatrix} + 2H(t)\frac{d}{dt}\begin{pmatrix}\delta_D\\ \delta_B\\ \tilde{\delta}_r\end{pmatrix} + k^2\begin{pmatrix}c_{s,D}^2\delta_D\\ c_{s,B}^2\delta_B\\ c_{s,r}^2\tilde{\delta}_r\end{pmatrix} = \nabla^2\Psi = -k^2\Psi$$

- **Where ψ is the perturbation in the gravitational potential, with**

$$\Psi_{x,t} \propto \Psi(t)\exp(i\vec{k}.\vec{x})$$

Gravitational Coupling

$$\Psi = 4\pi G\delta\rho_D + 4\pi G\delta\rho_B + 8\pi G\delta\rho_r$$
$$= 4\pi G\rho_{crit}\times\left[\Omega_D\delta_D + \Omega_B\delta_B + 2\Omega_r\delta_r\right]$$

- **Each eq. is similar to a forced spring**

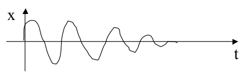
Restoring

$$\frac{d^2x}{dt^2} = \frac{F}{m} - \omega^2 x - \mu\frac{dx}{dt}$$

Term due to friction

$$\frac{d^2x}{dt^2} + \mu\frac{dx}{dt} + \omega^2 x = \frac{F(t)}{m}$$

(Displacement for Harmonic Oscillator)



- **The solution of the Harmonic Oscillator [within sound horizon] is:**

$$\delta(t) = A_1\cos kc_s t + A_2\sin kc_s t + A_3$$

For B or R

$$c_s^2 = \frac{c^2}{3(1+Q)} \qquad Q = \frac{3\rho_B}{4\rho_R} \quad \text{(Q varies with time)}$$

- **Amplitude is sinusoidal function of k c_s t**
  - if k=constant and oscillate with t
  - or t=constant and oscillate with k.

- **We don't observe the baryon overdensity δ_B directly**
- **-- what we actually observe is temperature fluctuations.**

$$\frac{\Delta T}{T} = \frac{\Delta n_\gamma}{3n_\gamma} = \frac{\delta_B}{3} = \frac{\tilde{\delta}_R}{3}$$

$$n_\gamma \sim R^{-3} \propto T^3$$
$$\varepsilon_\gamma \sim n_\gamma kT \propto T^4$$

- **The driving force is due to dark matter over densities.**
- **The observed temperature is:**

$$\left(\frac{\Delta T}{T}\right)_{obs} = \frac{\delta_B}{3} + \frac{\psi}{c^2}$$

Effect due to having to climb out of gravitational well

- **The observed temperature also depends on how fast the Baryon Fluid is moving.**

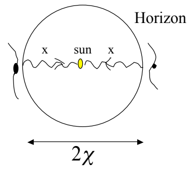
Velocity Field $\nabla v = -\dfrac{d\delta_B}{dt}$

$$\left(\frac{\Delta T}{T}\right)_{obs} = \frac{\delta_B}{3} + \frac{\psi}{c^2} \pm \frac{v}{c}$$

Doppler Term

Inflation in Early Universe [chtp 20.3]

- **Problems with normal expansion theory:**
  - What is the state of the universe at t→0? (radiation) or exotic scalar field? Pure E&M field
  - Why is the initial universe so precisely flat?
  - What makes the universe homogeneous/similar in opposite directions of horizon?
- **Solutions:**
  - Maybe the horizon can be pushed to infinity?
  - Maybe there is no horizon?
  - Maybe everything was in Causal contact at early times?



Why these two galaxies so similar without communicating yet?

$$\frac{\varepsilon_K(z)}{\varepsilon(z)} = \frac{\varepsilon_K(0)\times R^{-2}}{\varepsilon(0)\times R^{-n}} \sim R^{n-2} \sim 0 \text{ at } t=0$$

Why curvature term so small (universe so flat) at early universe if radiation dominates n=4 >2?

What have we learned?

- **What determines the patterns of CMB at last scattering**
  - Analogy as patterns of fine sands on a drum at last hit.
- **The need for inflation to**
  - Bring different regions in contact
  - Create a flat universe naturally.

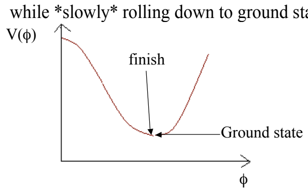
Inflationary Physics

- **Involve quantum theory to z~10^{32} and perhaps a scalar field φ(x,t) with energy density**

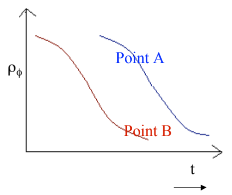
$$\rho_\phi = \tfrac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + V(\phi) \sim R(t)^{-n}, \text{ where } n \ll 1$$

fluctuate between neighbouring points [A,B]

while *slowly* rolling down to ground state



- **A slightly different finishing time (Quantum Effect) of inflation at different positions leads to slight perturbations to curvatures, which seed structure formation.**



- **Speculative at best.**

Inflation as Solution of problems of curvature, horizon

- **Consider a general expansion law**

Consider universe goes through a phase with

$\rho(t) \sim R(t)^{-n}$

$R(t) \sim t^q$ *where* q=2/n

- **Inflation is n<2**

Inflation broadens Horizon

- **Light signal travelling with speed c on an expanding sphere R(t), e.g., R(t)=1lightyr (t/1yr)^q**
  - **Emitted from time t_i**
  - **By time t=1yr will spread across (co-moving coordinate) angle x_c**

Horizon in co-moving coordinates

$$x_c = \int_{t_i}^{1} \frac{cdt}{R(t)} = \int_{t_i}^{1} \frac{cdt}{t^q} = \frac{(1^{1-q} - t_i^{1-q})}{(1-q)}$$

Normally $x_c < \frac{1}{(1-q)}$ is finite if q=2/n<1

(e.g., n=3 matter-dominate or n=4 photon-dominate)

INFLATION phase $x_c = \frac{(t_i^{1-q} - 1)}{(q-1)}$ can be very large for very small t_i if q=2/n>1

(e.g., $t_i = 0.01, q = 2, x_c = 99 >> \pi$, Inflation allows we see everywhere)

E.g.

- **As in previous universe but with n=3, Argue that the horizon of a non-relativistic moving ant at time t=1yr is also finite.**
- **Assuming the ant moves with 1cm/sec now, but was faster earlier on, estimate the age of universe when it was moving relativistically? Estimate how much it has moved from time zero to t=1 yr. What fraction of the length was in the relativistic phase?**

Inflation dilutes the effect of initial curvature of universe

$$\frac{\varepsilon_K(R)}{\varepsilon\ (R)} = \frac{\varepsilon_K(R_i)}{\varepsilon(R_i)}\left(\frac{R}{R_i}\right)^{n-2} \sim 0 \text{ (for n<2) sometime after } R >> R_i$$

even if initially the universe is curvature-dominated $\frac{\varepsilon_K(R_i)}{\varepsilon(R_i)} = 1$

E.g.

If a toy universe starts with $\frac{\varepsilon_K(R_i)}{\varepsilon(R_i)} = 0.1$ inflates from $t_i=10^{-40}$sec to t_f=1sec with n=1,

and then expand normally with n=4 to t=1 year,

SHOW at this time the universe is far from curvature-dominated.

Exotic Pressure drives Inflation

$$P = -\frac{d(\rho c^2 R^3)}{d(R^3)}$$

=>

$$\frac{\rho}{3} + \frac{P}{c^2} = -\frac{d(\rho R^2)}{3RdR} = \frac{n-2}{3}\rho \quad \text{if } \rho \sim R^{-n}$$

=>

$P/\rho c^2$=(n-3)/3

Inflation $n < 2$ requires exotic (negative) pressure,

define w=$P/\rho c^2$, then w = (n-3)/3<0,

Verify negligble pressure for cosmic dust (matter),

Verify for radiation P=$\rho c^2 / 3$

Verify for vaccum P=$-\rho c^2$

What Have we learned?

- **How to calculate Horizon.**
- **The basic concepts and merits of inflation**
- **Pressure of various kinds (radiation, vaccum, matter)**

Tutorial

- **Consider an micro-cosmos of N-ants habitating on an expanding sphere of radius R=R_0 $(t/t_0)^q$, where presently we are at t=t_0 =1year, R=R_0 =1m. Let q=1/2, N=100, and the ants has a cross-length σ=1cm for collision. Let each ant keeping its random angular momentum per unit mass J=1m*1(m/yr) with respect to the centre of the sphere.**
  - **What is the present rate of expansion dR/dt/R = in units of 1/yr,**
  - **How does the ant random speed, ant surface density, change as function of cosmic time?**
  - **Light emitted by ant-B travel half circle and reach ant-A now, what redshift was the light emitted?**
  - **What is the probability that the ant-A would encounter another ant from time t_1 to time t_2. How long has it travelled? Calculate assume t_1 =1/2 yr, t_2 =2yr.**

- Show the age of the universe is t=1sec at $z\sim10^{10}$; assume crudely that at matter-radiation equality $z=10^3$ and age $t=10^6$ yr
  - Argue that a void in universe now originates from an under-dense perturbation at $z=10^{10}$ with δ about 10^{-17}.
  - The edge of the void are lined up by galaxies. What direction is their peculiar gravity and peculiar motion?
- A patch of sky is presently hotter in CMB by 3 micro Kelvin than average. How much was it hotter than average at the last scattering (z=1000)?

Expectations for my part of the Exam

- Remember basic concepts (or analogies)
  - See list
- Can apply various scaling relations to do *some* of the short questions at the lectures.
  - See list
- *Relax* you will be fine.
  - thermal history and structure formation are advanced subjects with lots of details. Don't worry about details and equations, just be able to recite the big picture.
- *If you like*, you can read reference texts to have deeper understanding of the lectured material.
  - Only material on this Final Notes is examinable.

List of keys

- Scaling relations among
  - Redshift z, wavelength, temperature, cosmic time, energy density, number density, sound speed
  - Definition formulae for pressure, sound speed, horizon
  - Metrics in simple 2D universe.
- Describe in words the concepts of
  - Fundamental observers
  - thermal decoupling
    - Common temperature before,
    - Fixed number to photon ratio after
    - Hot and Cold DM.
  - gravitational growth.
    - Over-density,
    - direction of peculiar motion driven by over-density, but damped by expansion
    - pressure support vs. grav. collapse

Why Analogies in Cosmology

- Help you memorizing
  - Cosmology calls for knowledge of many areas of physics.
  - Analogies help to you memorize how things move and change in a mind-boggling expanding 4D metric.
- *Help you reason*, avoid "more equations, more confusions".
  - During the exam, You might be unsure about equations and physics,
    - the analogies *help you reason* and *recall* the right scaling relations, and get the big picture right.
- *Months after the exam*,
  - Analogies go a long way
- Enjoy Prof. Horne's Lectures