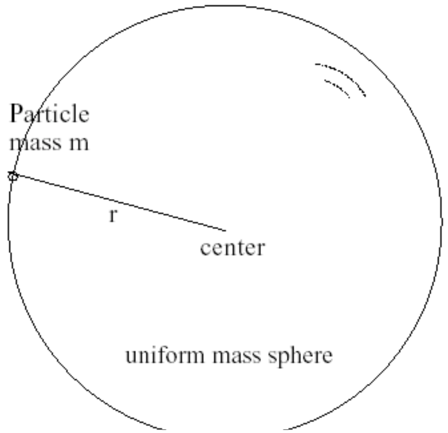
The rate of expansion of Universe

- Consider a sphere of radius $r=R(t)\ \chi$,
- If energy density inside is ρc^2

➔ Total effective mass inside is

$M = 4 \pi\rho r^3 /3$



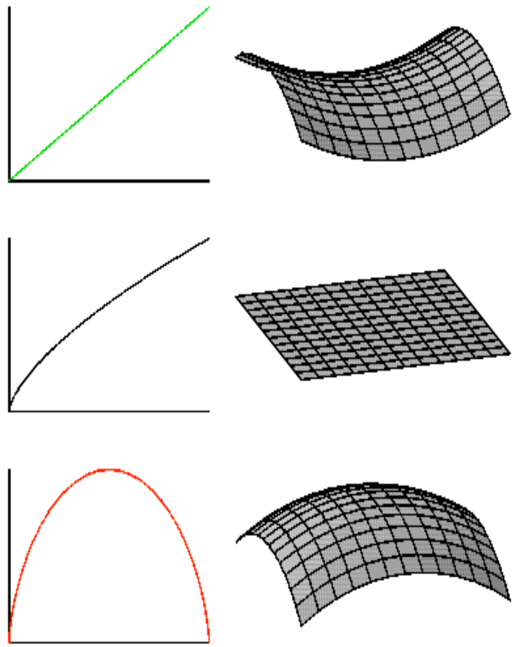
- Consider a test mass m on this expanding sphere,
- For Test mass its

Kin.Energy + Pot.E. = const E

➔ $m (dr/dt)^2/2 – G m M/r = cst$

➔ $(dR/dt)^2/2 - 4 \pi G \rho R^2/3 = cst$

cst>0, cst=0, cst<0



$$(dR/dt)^2/2 = 4 \pi G (\rho + \rho_{cur}) R^2/3$$

where cst is absorbed by $\rho_{cur} \sim R^{(-2)}$

Typical solutions of expansion rate

$H^2=(dR/dt)^2/R^2=8\pi G\ (\rho_{cur}+\rho_m+\rho_r+\rho_v)/3$

Assume domination by a component $\rho \sim R^{-n}$

Show Typical Solutions Are

$$\rho \propto R^{-n} \propto t^{-2}$$

$$n = 2(\text{curvature constant dominate})$$

$$n = 3(\textit{matter}\ \text{dominate})$$

$$n = 4(\textit{radiation}\ \text{dominate})$$

$$n \sim 0(\text{vaccum dominate}) : \ln(R) \sim t$$

- **Argue also $H = (2/n)\, t^{-1} \sim t^{-1}$. Important thing is scaling!**

Lec 4 Feb 22

Where are we heading?

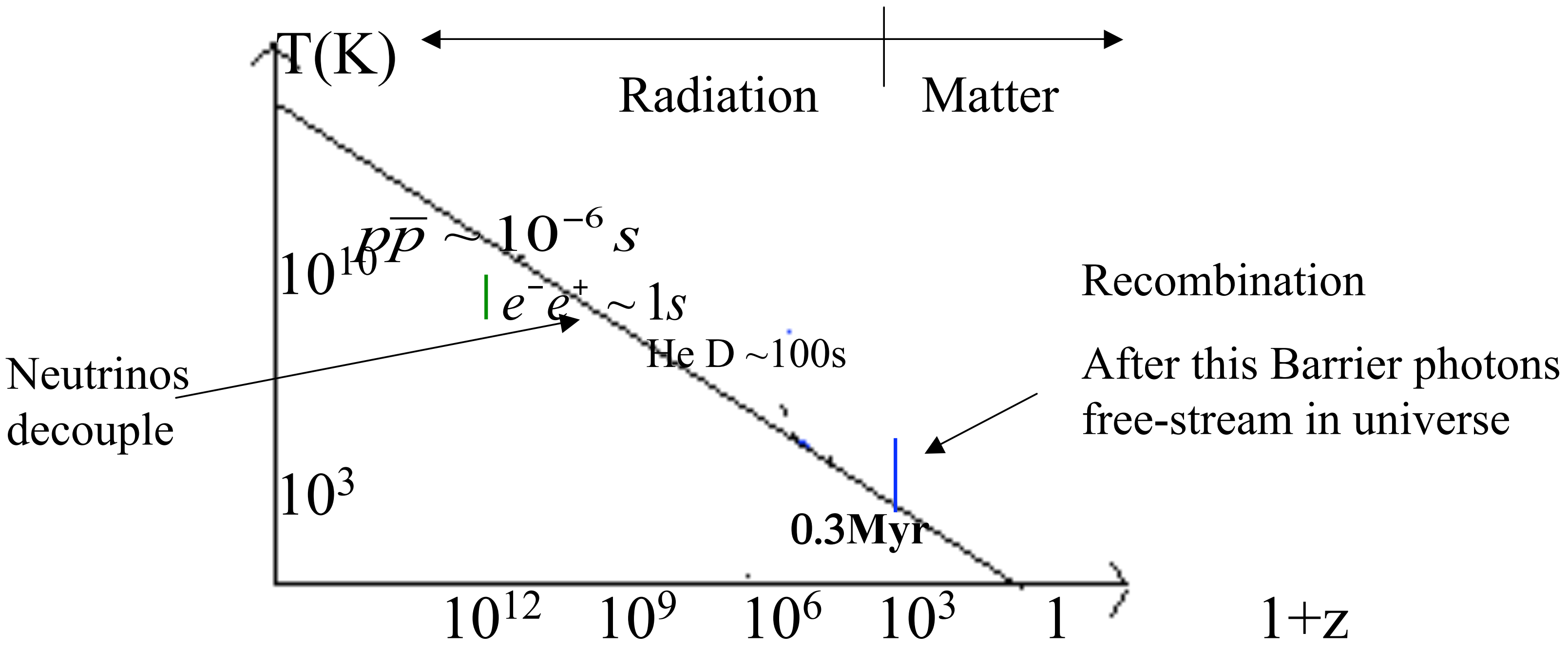
Next few lectures will cover a few chapters of

- Malcolm S. Longair's "Galaxy Formation" [Library Short Loan]

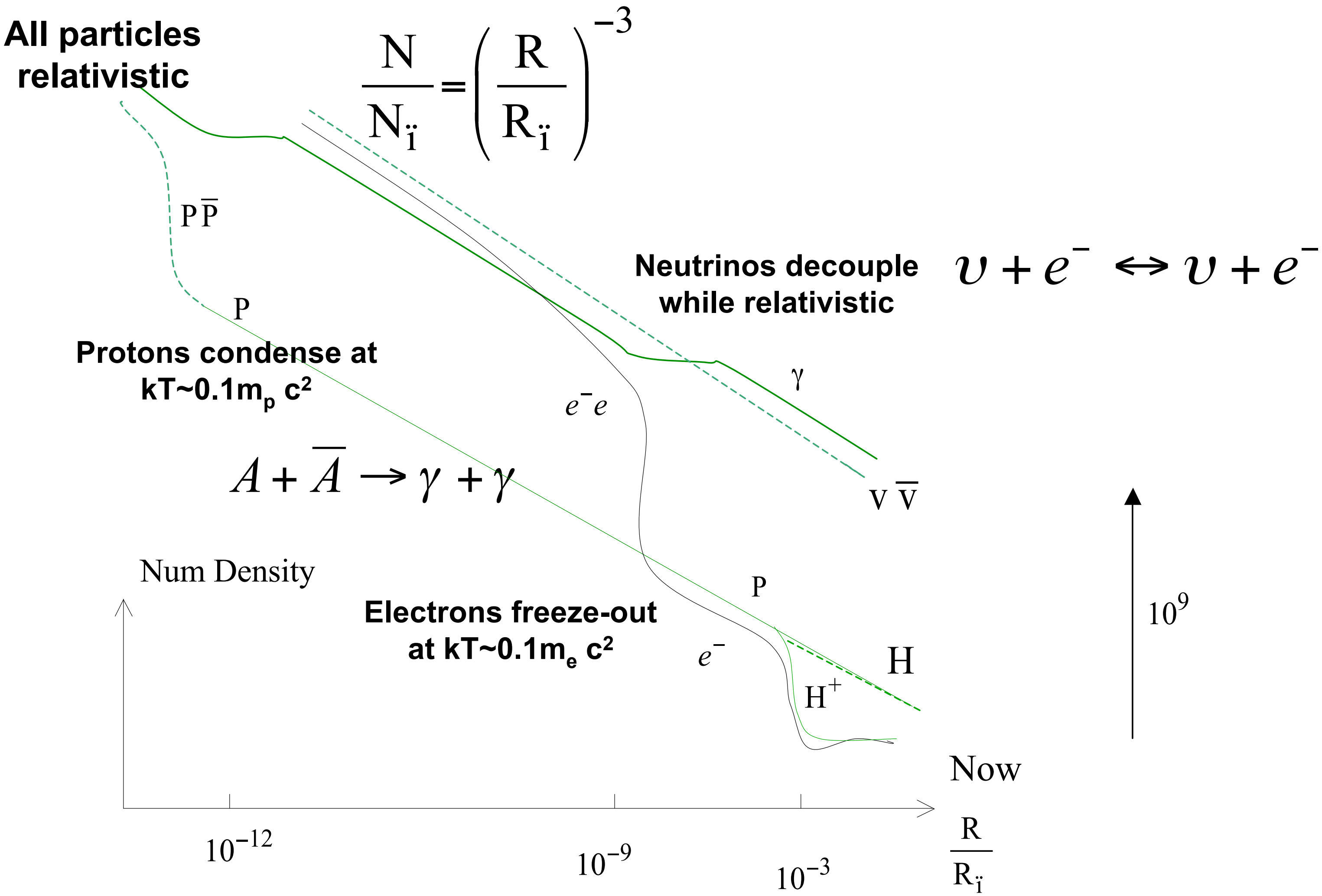
- **Chpt 1: Introduction**
- **Chpt 2: Metrics, Energy density and Expansion**
- **Chpt 9-10: Thermal History**

Thermal Schedule of Universe [chpt 9-10]

- At very early times, photons are typically energetic enough that they interact strongly with matter so the whole universe sits at a temperature dictated by the radiation.
- The energy state of matter changes as a function of its temperature and so a number of key events in the history of the universe happen according to a schedule dictated by the temperature-time relation.
- Crudely $(1+z) \sim 1/R \sim (T/3) \sim 10^9 \, (t/100s)^{(-2/n)} \sim 1000 \, (t/0.3Myr)^{-2/n}$, $H \sim 1/t$
- $n \sim 4$ during radiation domination



A summary: Evolution of Number Densities of γ, P, e, υ



A busy schedule for the universe

- **Universe crystalizes with a sophisticated schedule, much more confusing than simple expansion!**
  - **Because of many bosonic/fermionic players changing balance**
  - **Various phase transitions, numbers NOT conserved unless the chain of reaction is broken!**
  - **$p + p^- <-> \gamma + \gamma$ (baryongenesis)**
  - **$e + e^+ <-> \gamma + \gamma$, $v + e <-> v + e$ (neutrino decouple)**
  - **$n <\to p + e^- + v$, $p + n <\to D + \gamma$ (BBN)**
  - **$H^+ + e^- <\to H + \gamma$, $\gamma + e <-> \gamma + e$ (recombination)**

- **Here we will try to single out some rules of thumb.**
  - **We will caution where the formulae are not valid, exceptions.**
  - **You are not required to reproduce many details, but might be asked for general ideas.**

What is meant Particle-Freeze-Out?

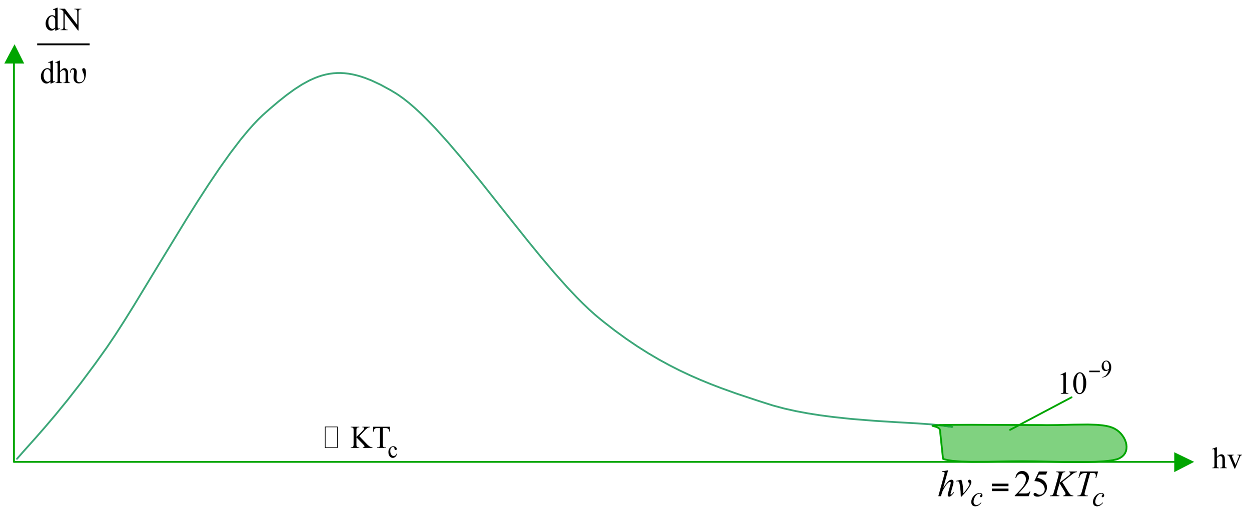
- **Freeze-out of equilibrium means NO LONGER in thermal equilibrium, means insulation.**
- **Freeze-out temperature means a species of particles have the SAME TEMPERATURE as radiation up to this point, then they bifurcate.**

- **Decouple = switch off = the chain is broken = Freeze-out**

A general history of a massive particle

- **Initially mass doesn't matter in hot universe**
- **relativistic, dense (comparable to photon number density $\sim T^3 \sim R^{-3}$),**
  - **frequent collisions with other species to be in thermal equilibrium and cools with photon bath.**
  - **Photon numbers (approximately) conserved, so is the number of relativistic massive particles**

energy distribution in the photon bath



hardest photons

Initially zero chemical potential (~ Chain is on, equilibrium with photon)

- **The number density of photon or massive particles is :**

$$n = \frac{g}{h^3}\int_0^\infty \frac{d\left(\frac{4\pi}{3}p^3\right)}{\exp\left(E/kT\right) \pm 1}$$

+ for Fermions

- for Bosons

- **Where we count the number of particles occupied in momentum space and g is the degeneracy factor. Assuming zero cost to annihilate/decay/recreate.**

$$E = \sqrt{c^2 p^2 + (mc^2)^2} \approx cp \quad \text{relativistic cp >> mc}^2$$

$$\approx m\,c^2 + \frac{1}{2}\frac{p^2}{m} \quad \text{non relativistic cp < mc}^2$$

- **As kT cools, particles go from**
- **From Ultrarelativistic limit. (kT>>mc²)**

particles behave as if they were massless→

$$n = \left(\frac{kT}{c}\right)^3 \frac{4\pi g}{(2\pi\hbar)^3} \int_0^\infty \frac{y^2 dy}{e^y \pm 1} \Rightarrow n \sim T^3$$

- **To Non relativistic limit (θ=mc²/kT > 10 , i.e., kT<< 0.1mc²)**
Here we can neglect the ±1 in the occupancy number→

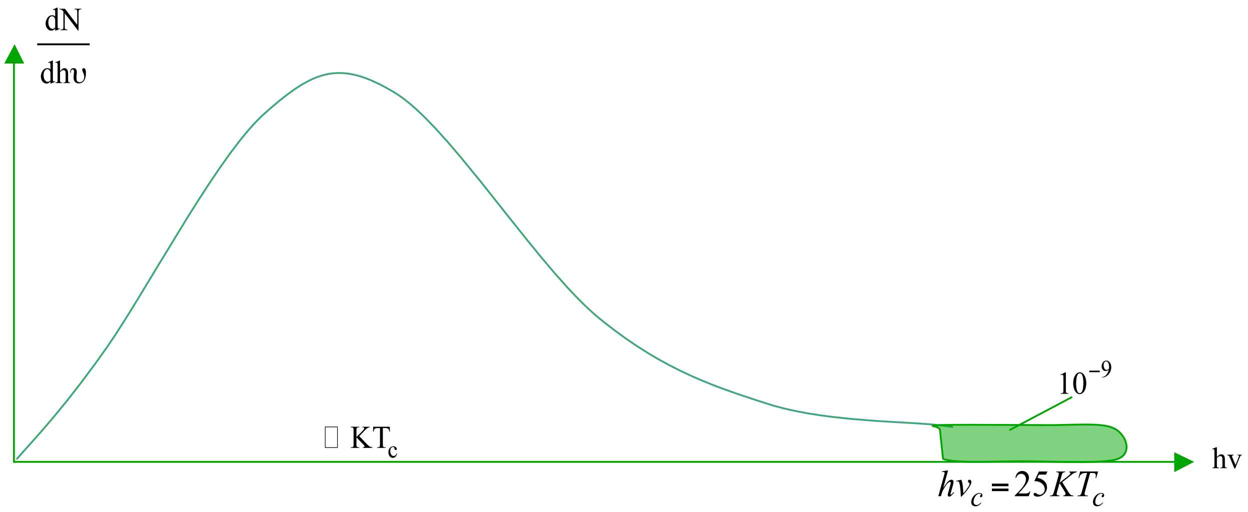
$$n = e^{-\frac{mc^2}{kT}} (2mkT)^{\frac{3}{2}} \frac{4\pi g}{(2\pi\hbar)^3} \int_0^\infty e^{-y^2} y^2 dy \Rightarrow n \sim T^{\frac{3}{2}} e^{-\frac{mc^2}{kT}}$$

When does freeze-out happen?

- **Happens when KT cools 10-20 times below mc^2, run out of photons to create the particles**
  - **Non-relativisitic decoupling**

- **Except for neutrinos**

particles of energy $E_c = h\nu_c$ unbound by high energy tail of photon bath



hardest photons

~ # baryons

If run short of hard photon to unbind => "Freeze-out" => $KT_c \approx \frac{h\nu_c}{25}$

Rule 1. Competition of two processes

- **Interactions keeps equilibrium:**
  - **E.g., a particle A might undergo the annihilation reaction:**

$$A + \overline{A} \rightarrow \gamma + \gamma$$

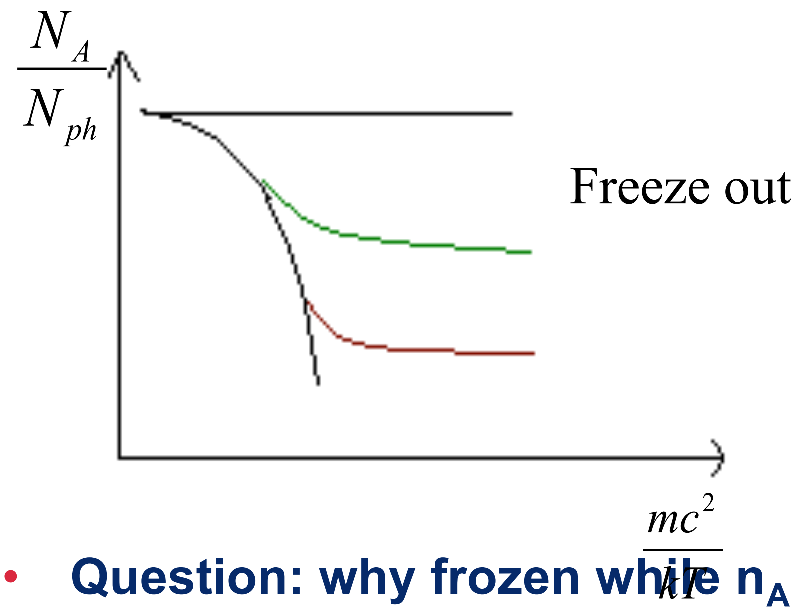
- **depends on cross-section** σ and speed v. & most importantly
  - the **number density n of photons** (falls as $t^{(-6/n)}$, Why? Hint $R \sim t^{(-2/n)}$)

- **What insulates: the increasing gap of space between particles due to Hubble expansion $H \sim t^{-1}$.**

- **Question: which process dominates at small time? Which process falls slower?**

Rule 2. Survive of the weakest

- While in equilibrium, $n_A/n_{ph} \sim \exp(-\theta)$. (Heavier is rarer)
- When the reverse reaction rate $\sigma_A \upsilon$ is slower than Hubble expansion rate H(z) , the abundance ratio is frozen $N_A/N_{ph} \sim 1/(\sigma_A \upsilon)\ /T_{freeze}$



$\sigma_A \upsilon$ LOW→ (v) smallest interaction, early freeze-out while relativistic

$\sigma_A \upsilon$ HIGH→ later freeze-out at lower T

- Question: why frozen while n_A , n_{ph} both drop as $T^3 \sim R^{-3}$.
- $\rho_A \sim n_{ph}/(\sigma_A \upsilon)$, if $m \sim T_{freeze}$

Effects of freeze-out

- **Number of particles change (reduce) in this phase transition,**
  - **(photons increase slightly)**
- **Transparent to photons or neutrinos or some other particles**
- **This defines a "last scattering surface" where optical depth to future drops below unity.**

Number density of non-relativistic particles to relativistic photons

- **Reduction factor ~ exp(- θ), $\theta = mc^2/kT$, which drop sharply with cooler temperature.**
- **Non-relativistic particles (relic) become *much rarer* by exp(-θ) as universe cools below mc^2/θ,**

  $\theta \sim 10–25.$
  - **So rare that infrequent collisions can no longer maintain coupled-equilibrium.**
  - **So Decouple = switch off = the chain is broken = Freeze-out**

After freeze-out

- **Particle numbers become conserved again.**
- **Simple expansion.**
  - **number density falls with expanding volume of universe, but Ratio to photons kept constant.**