NEUTRINO DECOUPLE as Hot DM

- Neutrinos are kept in thermal equilibrium by the creating electron pairs and scattering (weak interaction):

$$\upsilon + \overline{\upsilon} \leftrightarrow e^{+} + e^{-} \qquad\qquad \upsilon + e^{-} \leftrightarrow \upsilon + e^{-}$$

- This interaction freezes out when the temperature drops to kT_{υ}~MeV~ rest mass electrons
  - Because very few electrons are around afterwards
- Argue that Neutrinos have Relativistic speeds while freezing out
  - $kT_{\upsilon} >>$ rest mass of neutrinos(~eV)
  - They are called Hot Dark Matter (HDM)
  - Move without scattering by electrons after 1 sec.

e.g., Neutrons

- Before 1 s, lots of neutrinos and electrons keep the abundance of protons about equal to that of neutrons through
  - $n + \upsilon \leftrightarrow p + e^-$
- After 1 s free-moving neutrons start to decay.
  - $n \rightarrow p + e^- + \upsilon$
  - Argue that presently fewer neutrons in nuclei than protons

thermal equilibrium number density

- The thermal equilibrium background number density of particles is given by:

$$n = \frac{g}{h^3}\int_0^\infty \frac{d\left(\frac{4\pi}{3}p^3\right)}{\exp(E/kT)\pm 1}$$

+ for Fermions

- for Bosons

$$E = \sqrt{m^2c^4 + p^2c^2}$$

- Where we have to change to momentum space and g is the degeneracy factor.

$$E = \sqrt{c^2p^2 + (mc^2)^2} \approx cp$$ relativistic cp >> mc^2

$$\approx m\,c^2 + \frac{1}{2}\frac{p^2}{m}$$ non relativistic cp < mc^2

- As kT cools, particles go from
- From Ultrarelativistic limit. ($kT >> mc^2$)

  particles behave as if they were massless→

$$n = \left(\frac{kT}{c}\right)^3 \frac{4\pi g}{(2\pi\hbar)^3} \int_0^\infty \frac{y^2 dy}{e^y \pm 1}$$

- To Non relativistic limit ($kT << 0.1mc^2$.) Here we can neglect the ±1 in the occupancy number→

$$n = e^{-\frac{mc^2}{kT}} (2mkT)^{\frac{3}{2}} \frac{4\pi g}{(2\pi\hbar)^3} \int_0^\infty e^{-y^2} y^2 dy$$

Number density of particles
(annihilating/creating in a photon bath)

$$\text{Number Density N} = \frac{g}{h^3} \cdot \int_o^\infty \frac{d\left(\frac{4\pi}{3}p^3\right)}{e^{E/kT} \pm 1}$$

$$E = \sqrt{c^2 p^2 + m^2 c^4}$$

$$\approx g.\left(\frac{KT}{hc}\right)^3 \xi \qquad \text{(Relativistic)}$$

$$\begin{cases} \xi = 0.122 \ \text{boson } \gamma \\ \ \ = 0.091 \ \text{fermion e} \end{cases}$$

$$g = 2 \quad \gamma, e, p$$
$$g = 1 \quad \nu$$

$$\approx g.\frac{\left(2\pi KT.mc^2\right)^{\frac{3}{2}}}{h^3.c^3} e^{-\frac{mc^2}{KT}} \qquad \text{Non-Relativistic}$$

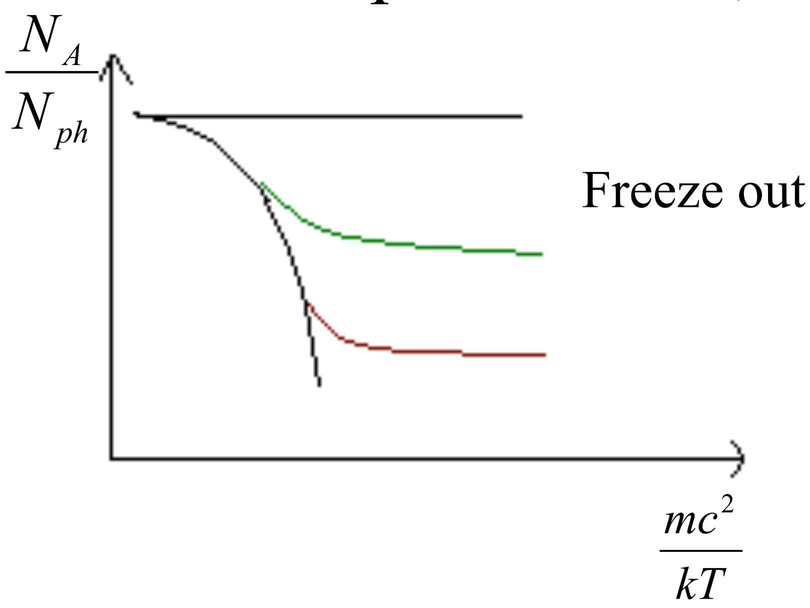
Particles Freeze Out

- Freeze-out of equilibrium (relativistic or non-relativistic) at certain temperature depending on number density, and cross-section.

- Generally a particle A undergoes the reaction:

$$A + \overline{A} \rightarrow \gamma + \gamma$$

- When the reverse reaction rate is slower than Hubble expansion rate, it undergoes freezeout.



$\sigma_A \upsilon$ LOW→weak interaction
early freeze out while relativistic

$\sigma_A \upsilon$ HIGH→strong interaction
later freeze out at lower T

A general history of a massive particle

- Initially relativistic, dense (comparable to photon number density),
  - has frequent collisions with other species to be in thermal equilibrium and cools with CBR photon bath.

Freeze-Out

- Later, Relics Freeze-out of the cooling heat bath because
  - interactions too slow due to lower and lower density in expanding universe.
  - This defines a “last scattering surface” where optical depth drops below unity.
  - The number density falls with expanding volume of universe, but Ratio to photons kept constant.

Number density of non-relativistic particles to relativistic photons

- Reduction factor ~ $\exp(-mc^2/kT)$, which drop sharply with cooler temperature.
- Non-relativistic particles (relic) become rarer as universe cools (if maintain coupled-equilibrium).

smallest Collision cross-section

- neutrinos (Hot DM) decouple from electrons (via weak interaction) while still relativistic $kT > \Delta mc^2$.
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Small Collision cross-section

- Decouple at non-relativistic once $kT<\Delta mc^2$. Number density ratio to photon drops steeply with cooling $\exp(-\Delta mc^2/kT)$.
  - anti-protons and wimps (Cold DM) etc. decouple (stop creating/annihilating) while non-relativistic. Abundant (CDM).
  - non-relativistic and combine into lower energy state. n → H → D → He, e → Neutral H. Neutrons/electrons Rarer than Hydrogen.

- $T_c \sim 10^9$K NUCLEOSYNTHESIS (100s)
- $T_c \sim 5000$K RECOMBINATION (10^6 years) (Redshift=1000)

A worked-out exercise

$A + \bar{A} \rightarrow \gamma + \gamma$

Show at last scattering surface Optical depth $\tau = \int_0^z \sigma v \eta \mathrm{n_{ph}}(\mathrm{z}) \frac{dt}{dz} dz$

$\sim \int_0^z \sigma v \eta (1+\mathrm{z})^3 \frac{d(1+\mathrm{z})^{\text{-n/2}}}{dz} dz$

$\sim \sigma v \eta (1+\mathrm{z})^{\text{3-n/2}} \sim \sigma v \eta T^{\text{3-n/2}} \sim 1$.

where n=4 for radiation era.

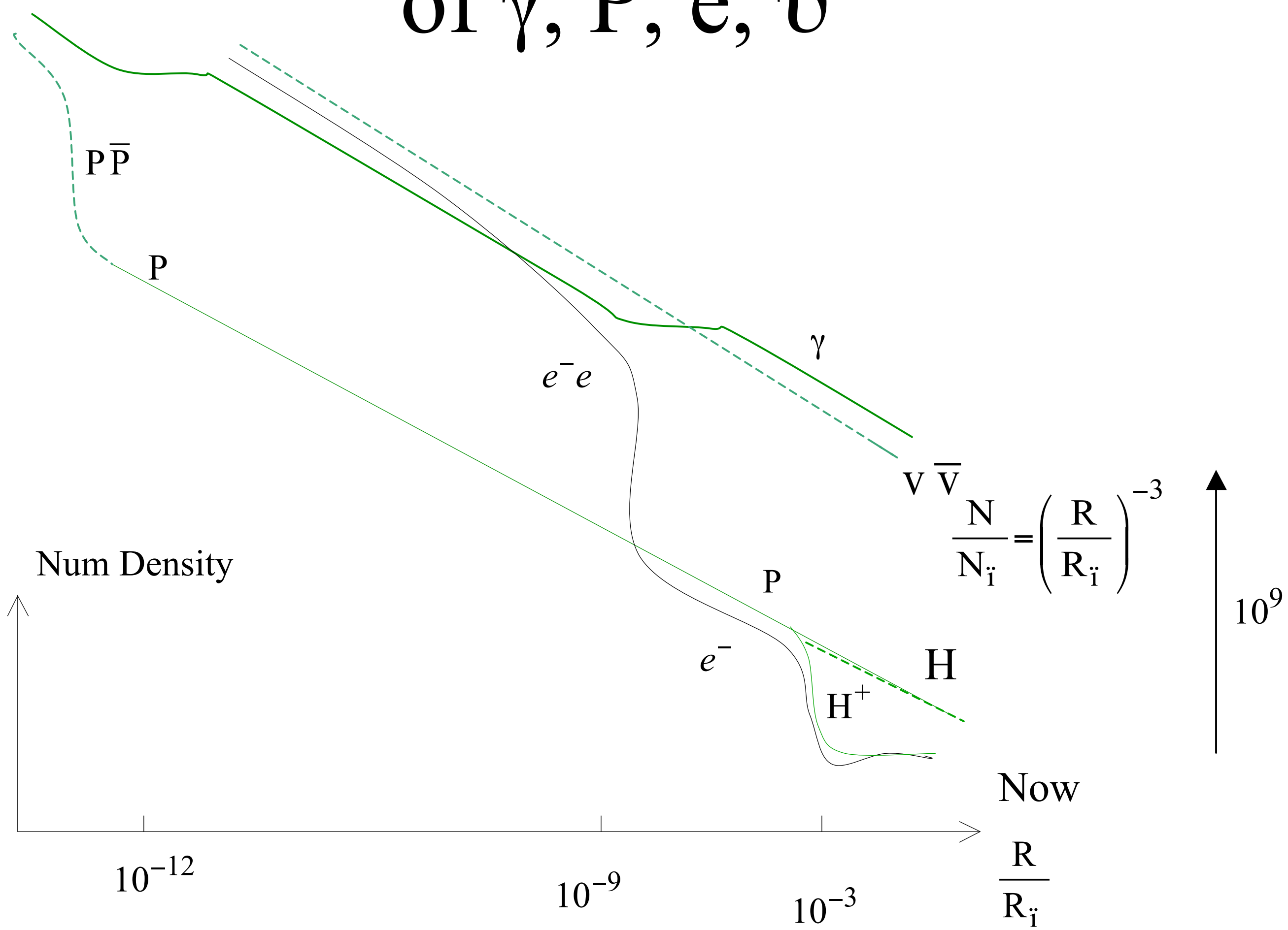
Given that Freeze-out fraction $\eta \sim \exp(-\frac{\Delta mc^2}{kT})$

and assume decouple at $kT \sim mc^2/\ln(1/\eta)$,

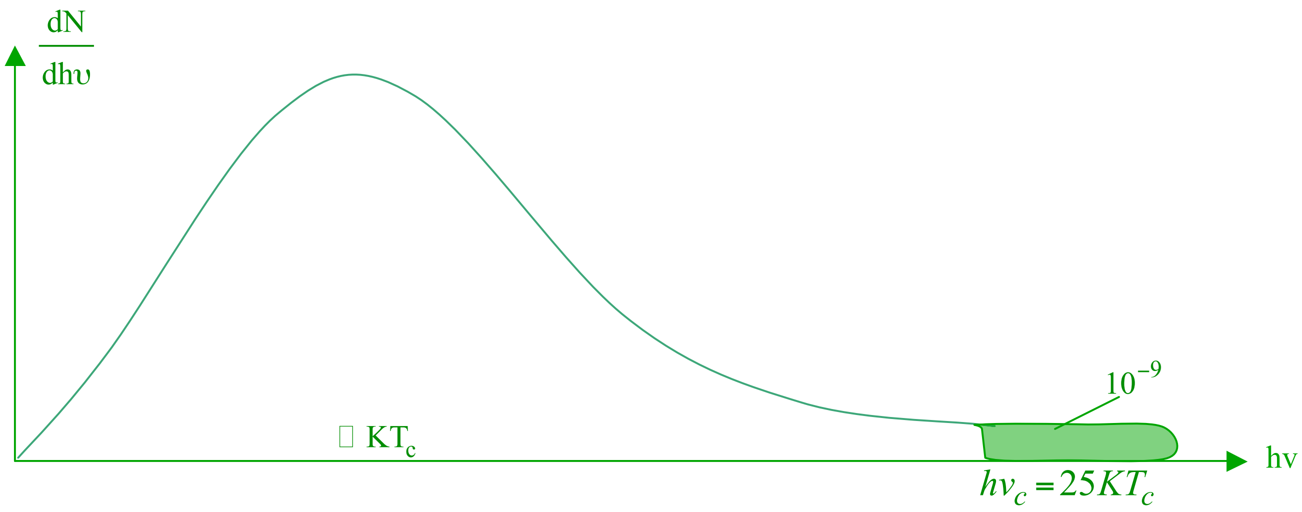
Argue cosmic abundance

$\Omega \sim \eta m \sim T^{-1} m/(\sigma v) \sim (\sigma v)^{-1}$

A summary: Evolution of Number Densities of γ, P, e, υ



Energetic Tail of Photon Bath



$$\frac{N(>h\nu_c)}{N_{ph}} \approx e^{-\frac{h\nu_c}{KT_c}} \left(\frac{h\nu_c}{KT_c}\right)^2 .O(1)$$

$$\sim e^{-25} \times 25^2$$

$$\sim 10^{-9} \sim \frac{N_B}{N_{ph}}$$

$$\text{"Freeze-out" } KT_c \sim \frac{h\nu_c}{25}$$

hardest photons

~ # baryons "Freeze-out"