Weekly Tutorial Questions for AS4022 [2012 version]

Q1. Argue about 10^5 photons fit in a 10cm x 10cm x 10cm microwave oven. [Hint: $3kT = h c / \lambda$] Show the approximate solutions R(t) of the Friedman equation $(dR/dt/R)^2 = A R^{-n}$ where n=4 for radiation, n=3 for matter, and n=2 (negative) curvature, and n=0 for vacuum energy, and A=0 for static universe.
Consider a micro-cosmos of N-ants inhabiting an expanding spherical surface of radius $R=R_0 (t/t_0)^a$, where presently we are at $t=t_0$ =1min, $R=R_0$ =1lightsecond. Let a=1/2, N=100. What is the present rate of expansion dR/dt/R = in units of 1/min? How does the ant surface density change as function of cosmic time? [due 21Sept]

Q2. A baby universe is initially at time t_i=10^{-40} sec flat with an Omega = 1.0. If this toy universe expands first under certain energy density $\rho \sim R^{-n}$ (n=1) from time t_i to time t_f=1sec, and then expands normally under radiation with $\rho \sim R^{-n}$ (n=4) from t_f=1sec until its 1-year birthday t_b =1 year. Prove that this universe satisfies the thermo-dynamical law PdV = -dE, if $V=R^3$, $E=V \rho c^2$, the pressure $P= [(n-3)/3] \rho c^2$. [due 28Sept]

Q3. Adopt a cosmic age of 13Gyrs and a division of energy (0.7:0:2999:0.0001:0.0) =(Omega of vacuum: matter: radiation: curvature). Was the CMB temperature high enough to ionise hydrogen during the radiation era?
Estimate the fraction of the time of the universe that the radiation dominates. [Hint: -13.6eV is the energy for the ground state of hydrogen]. [due 5Oct]

Q4. Derive the time-redshift relation for a flat universe; Do a Taylor expansion of the angular diameter distance at low z. Use the Friedman equation to argue that in a universe made purely of normal matter, photons, and ordinary neutrinos, has a negative d^2R/dt^2. [due 12Oct]

Q5. For a coupled radiation-baryon fluid, the sound speed $Cs^2 = c^2/3/(1+Q)$, $Q = (3 \rho_b) / (4 \rho_r)$. This sound speed Cs drops from c/sqrt(3) at radiation-dominated era to c/sqrt(Y) at dark-matter-radiation equality. Estimate Y. Explain why the CMB has a regular pattern in the k-space, and estimate the sound horizon. [due 19Oct]

Q6. Estimate number density ratio of hot/cold/baryonic DM. [due 26 Oct, Q&A session Friday afternoon just before consolidation week]