Dynamics of non-spherically symmetric systems and N-body simulations in MOND

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OUTLINE

- The non-linear MOND field equation
- Analytical solutions:
  - The Kuzmin disk (Brada & Milgrom 1995)
  - New axisymmetric and triaxial models (Ciotti, Londrillo & Nipoti 2006)
- Numerical solutions:
  - A new numerical MOND potential solver (Ciotti, Londrillo & Nipoti 2006)
  - Testing the code
- Applications:
  - Estimating the solenoidal field in galaxy models
  - Vertical force in disks: MOND vs DM (Zhao, Nipoti, Londrillo & Ciotti in prep)
  - N-body simulations in MOND
- Conclusions

The non-linear MOND field equation

(Bekenstein & Milgrom 1984)

$$\nabla \cdot \left[\mu \left(\frac{\|\nabla \phi\|}{a_0} \right) \nabla \phi \right] = 4 \pi G \rho$$

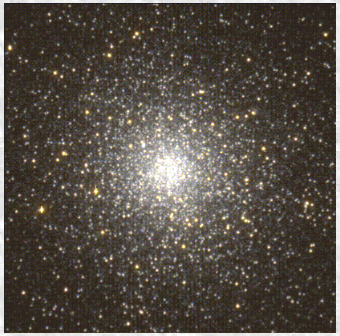
replaces the Poisson equation

$a_0 = 1.2 \times 10^{-10} \, m \, s^{-2}$ characteristic acceleration

$\vec{g} = -\nabla \phi$ gravitational field

$\mu(x) = \frac{x}{\sqrt{1+x^2}}$ the μ function (Bekenstein & Milgrom 1984)

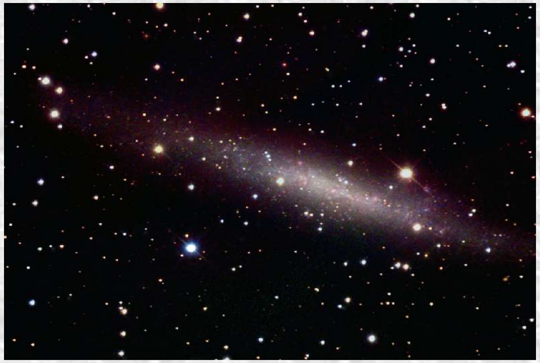
Newtonian regime
(high surface density systems)



Globular cluster 47tuc

$g \ll a_0 \Rightarrow \mu \simeq 1 \Rightarrow \nabla^2 \phi_N = 4 \pi G \rho$

Deep MOND regime
(low surface density systems)



LSB NGC1560

$g \gg a_0 \Rightarrow \mu \simeq x \Rightarrow \nabla \cdot (\|\nabla \phi\| \nabla \phi) = 4 \pi G \rho a_0$

The solenoidal field S=curl h

The MOND potential ϕ is related to the Newtonian potential ϕ_N by

$$\mu\left(\frac{|\nabla\phi|}{a_0}\right)\nabla\phi=\nabla\phi_N+\vec{S}$$

$\vec{S}=\nabla\times\vec{h}$ is an unknown solenoidal field

Only in case of for spherical, cylindrical, planar symmetry $\vec{S}=0$
=> $\mu\nabla\phi=\nabla\phi_N$ easy algebraic solution (Milgrom 1983 empirical formula)

In general $\vec{S}\neq 0$ and one has to solve the non-linear field equation

$$\nabla\cdot\left[\mu\left(\frac{\|\nabla\phi\|}{a_0}\right)\nabla\phi\right]=4\pi G\rho$$

Also for axisymmetric systems!!!

(Bekenstein & Milgrom 1984)

An exception: the Kuzmin disk

(Brada & Milgrom 1995)

The razor-thin Kuzmin disk is the ONLY known axisymmetric model for which $\vec{S}=0$ because $g_N \equiv \|\nabla \phi_N\| = g_N(\phi_N)$

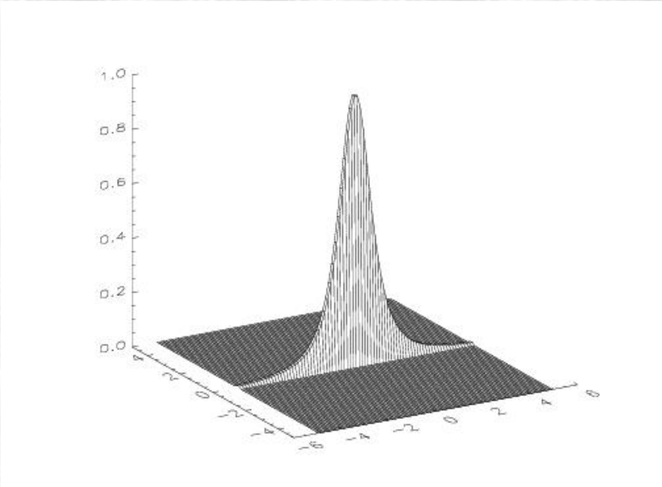
$$\rho_{KUZMIN}(R) = \frac{aM}{2\pi (R^2+a^2)^{3/2}}$$

Kuzmin disk surface density

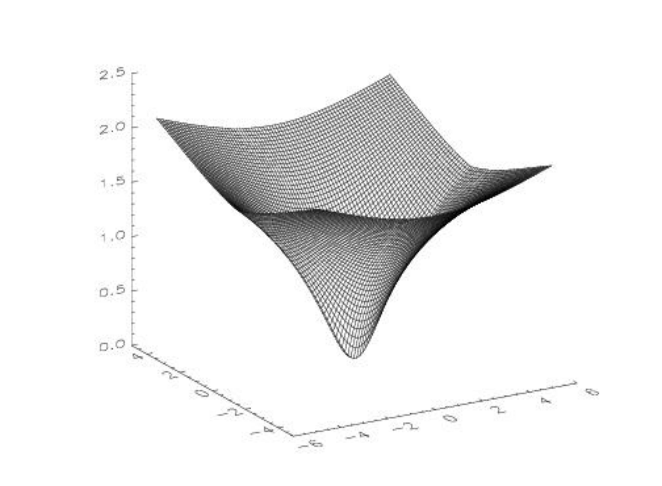
- The MOND potential of the Kuzmin disk is known **analytically**
- Useful to test MOND out of spherical symmetry
- BUT quite unrealistic as a galaxy model!

For instance, in deep MOND:

$$\phi_{KUZMIN}(R,z) = \sqrt{GMa_0} \ln (R^2 + (|z|+a)^2)^{1/2}$$



$\rho_{KUZMIN}(R,z)$



$\phi_{KUZMIN}(R,z)$

Analytical axisymmetric and triaxial MOND density-potential pairs

(Ciotti, Londrillo & Nipoti 2006, Apj)

- We propose a **general** method to build analytical axisymmetric and triaxial density-potential pairs
- ϕ-to-ρ approach: deformation of a spherically symmetric solution

1) Choose a spherical density and compute the MOND potential:

$$\rho_0(r) \Rightarrow \phi_0(r)$$

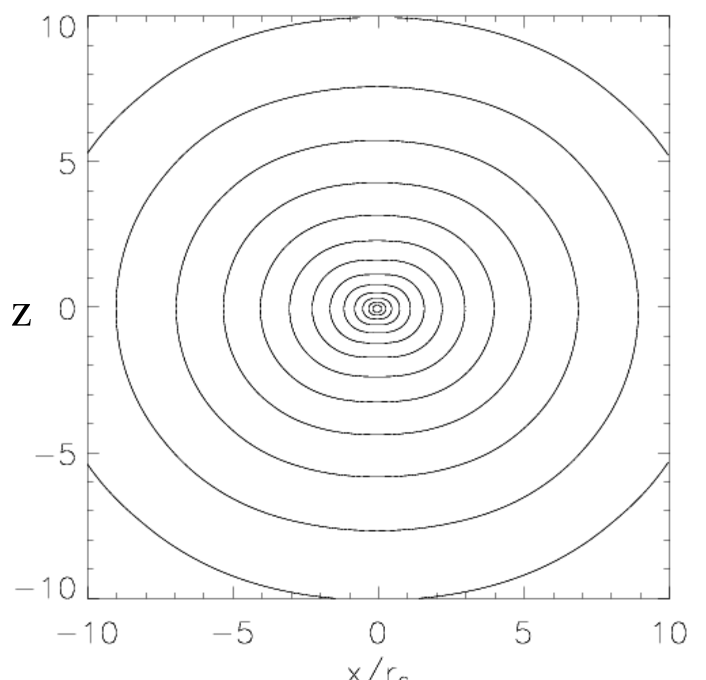
2) Add an **aspherical** function to the potential and compute the corresponding density using the MOND field equation:

$$\phi(r,\theta,\varphi) \equiv \phi_0(r) + \lambda\,\phi_1(r,\theta,\varphi) \Rightarrow \rho(r,\theta,\varphi)$$

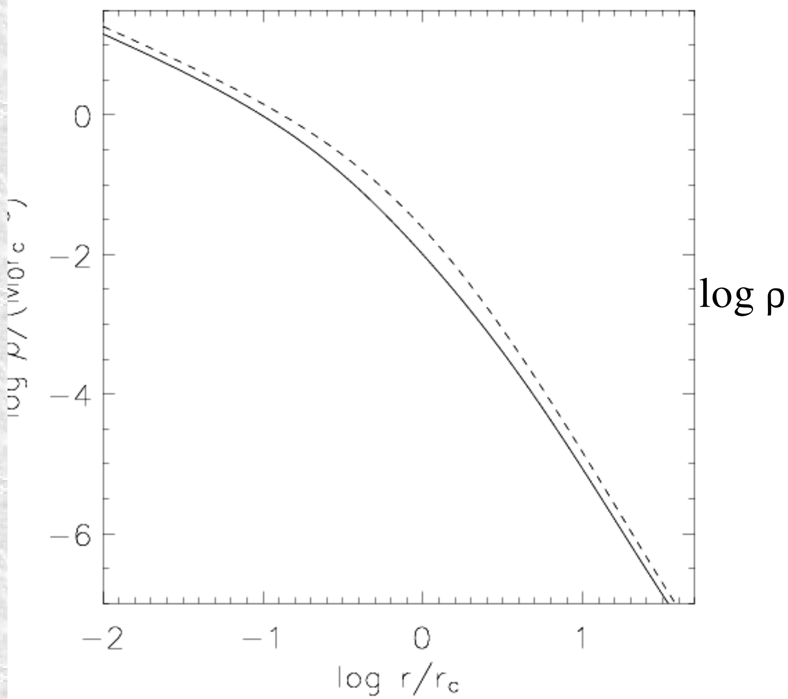
3) If the density is everywhere positive (ϕ, ρ) is an aspherical MOND density-potential pair

FOR A SUITABLE CHOICE OF ϕ_1 AND SMALL ENOUGH λ A POSITIVE DENSITY IS FOUND

An example: analytical axisymmetric and triaxial Hernquist models in MOND



Isodensity contours



Density profile

$$\rho_0(r) = \frac{M}{2\pi} \frac{1}{r(1+r)^3}$$

$$\phi_1(r,\vartheta) \propto \frac{r\cos^2\vartheta}{(r+1)^2}$$

- Analytical density & potential (+)
- General method (+)
- Realistic density profile (+)
- Significant flattening (0.6< b/a <1) (+)

- Not highly flattened systems (−)
- Density is not 100% under control (−)

A numerical potential solver is still needed

A new numerical MOND potential solver

(Ciotti, Londrillo & Nipoti 2006, Apj)

- We developed a **new numerical solver** for the non-linear MOND field equation
- Non-linear elliptic equations -> relaxation method -> **Newton iterative method**
- **Spherical coordinates** grid $(N_r, N_\vartheta, N_\varphi)$
- Spectral method in angular variables (**spherical harmonics**)
- Finite differences in radial coordinate
- The solver can be used in particle-mesh N-body codes (e.g. Londrillo & Messina 1990)
- Designed for **finite-mass, single-peaked** density distributions

→ Literature: very little work on numerical solution of the MOND field equation
(Brada & Milgrom 1995, 1999: Cartesian coordinates + multigrid method)

NUMERICAL SOLUTIONS

The numerical method

$$\hat{M}[\phi(\mathbf{x})] \equiv \nabla\cdot\left[\mu\left(\frac{g}{a_0}\right)\nabla\phi(\mathbf{x})\right] - 4\pi G\rho(\mathbf{x}) = 0, \quad g = O(r^{-1}) \text{ for } r\to\infty$$

$$\phi^{(n+1)} = \phi^{(n)} + \delta\phi^{(n)}$$

NEWTON ITERATION

$$\delta\hat{M}^{(n)}\left[\delta\phi^{(n)}\right] = -\hat{M}\left[\phi^{(n)}\right]$$

$$\delta\hat{M}^{(n)} \equiv \nabla\cdot\left[\mu^{(n)}\nabla + \mu'^{(n)}\mathbf{g}^{(n)}\left(\mathbf{g}^{(n)}\cdot\nabla\right)\right]$$

$$\hat{M}\left[\phi^{(n+1)}\right] - \hat{M}\left[\phi^{(n)}\right] = \delta\hat{M}^{(n)}\left[\delta\phi^{(n)}\right] + O\left[\left(\delta\phi^{(n)}\right)^2\right]$$

Exact operator:
- **Quadratic convergence**
- **Inversion of a 3-D matrix required**
- **Numerical difficulties**

NO!

INEFFICIENT!

$$\delta\hat{\mathcal{M}}^{(n)} \equiv \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\bar{\mu}^{(n)}(r)\frac{\partial}{\partial r}\right) + \bar{\mu}^{(n)}(r)\left(\hat{L}_\vartheta + \hat{L}_\varphi\right)\right]$$

$$\hat{L}_\vartheta \equiv \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial}{\partial\vartheta}\right), \quad \hat{L}_\varphi \equiv \frac{1}{\sin\vartheta}\frac{\partial^2}{\partial\varphi^2}$$

$$\bar{\mu}^{(n)}(r) = (1/4\pi)\int \mu^{(n)}(r,\vartheta,\varphi)\sin\vartheta\, d\vartheta\, d\varphi$$

$$\hat{M}\left[\phi^{(n+1)}\right] - \hat{M}\left[\phi^{(n)}\right] = \delta\hat{\mathcal{M}}^{(n)}\left[\delta\phi^{(n)}\right] + O\left[\delta\phi^{(n)}\right]$$

Approximate operator:
- **Only Linear convergence**
- **Exploits spherical harmonics**
- **Inversion of a 1-D matrix**

YES!

ACCURATE AND EFFICIENT!

$$\delta\phi^{(n)}(r,\vartheta,\varphi) = \sum_{l,m}\delta\phi^{(n)}_{l,m}(r)Y^m_l(\vartheta,\varphi)$$

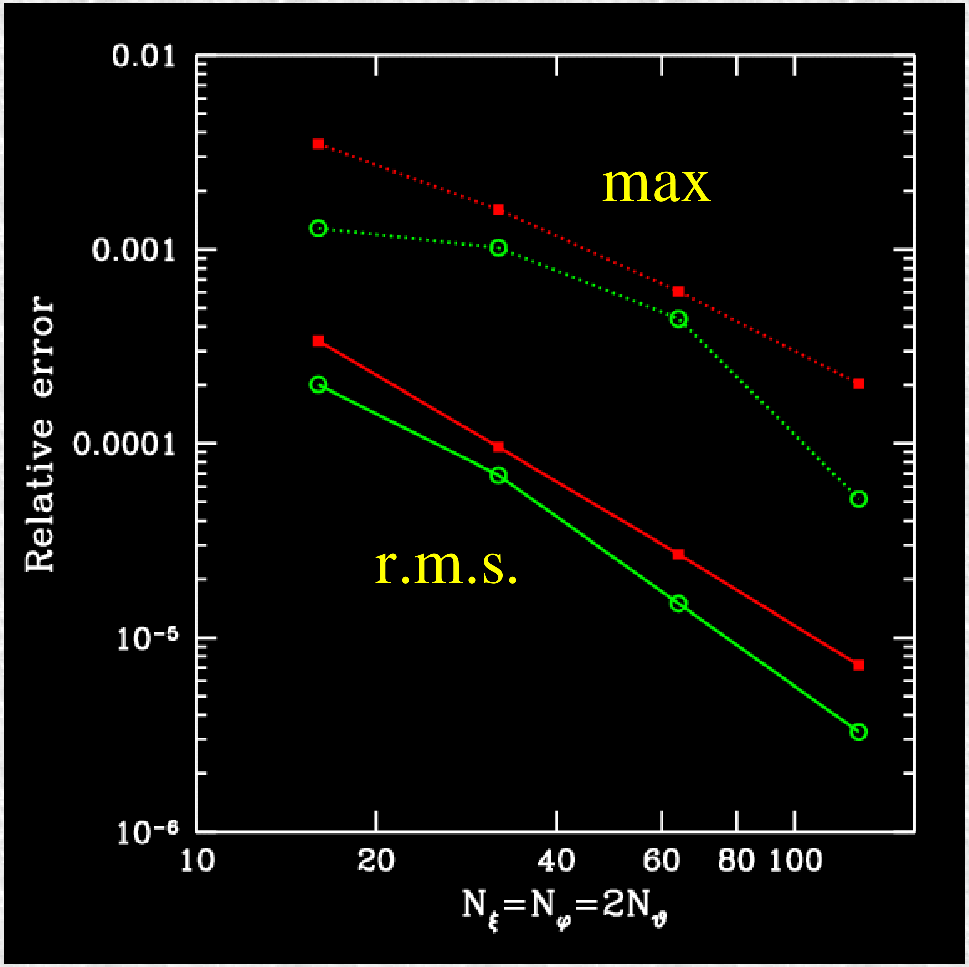
$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\bar{\mu}^{(n)}\frac{\partial}{\partial r}\right) - \bar{\mu}^{(n)}(r)l(l+1)\right]\delta\phi^{(n)}_{l,m}(r) = -\hat{M}\left[\phi^{(n)}\right]_{l,m}$$

AT EACH ITERATION STEP, ONE RADIAL EQUATION FOR EACH *(l,m)* COMPONENT

TESTING THE NEW NUMERICAL SOLVER

Comparison with non-spherical analytical solutions:

- Kuzmin disk
- Triaxial Hernquist models

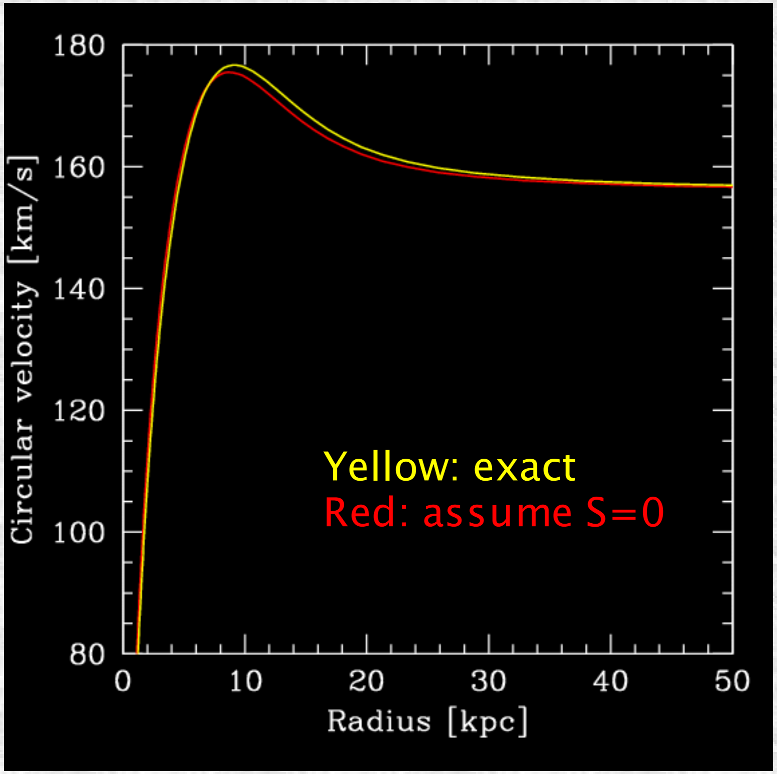


HOW IMPORTANT IS THE SOLENOIDAL FIELD S?

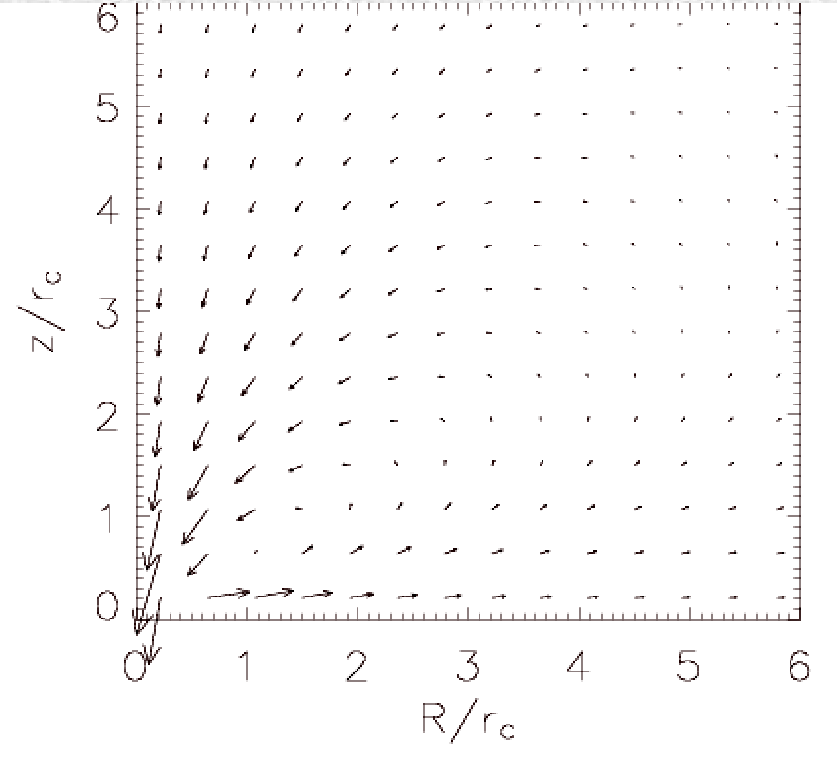
$$\vec{S} = \mu \nabla \phi - \nabla \phi_N$$

We used the numerical solver to estimate the solenoidal field S in astrophysically relevant systems

- S is typically small compared to the MOND acceleration g ($S/g<0.1$) (in agreement with Brada & Milgrom 1995)
- This is not always true: in deep MOND systems (e.g. low-surface density axisymmetric Hernquist models) s/g is as high as 0.6 at the centre!



MOND rotation curve for an exponential disk



$\frac{\vec{S}}{\|\nabla \phi\|}$ for an axysymmetric Hernquist model

VERTICAL FORCE IN DISK GALAXIES IN MOND AND DM

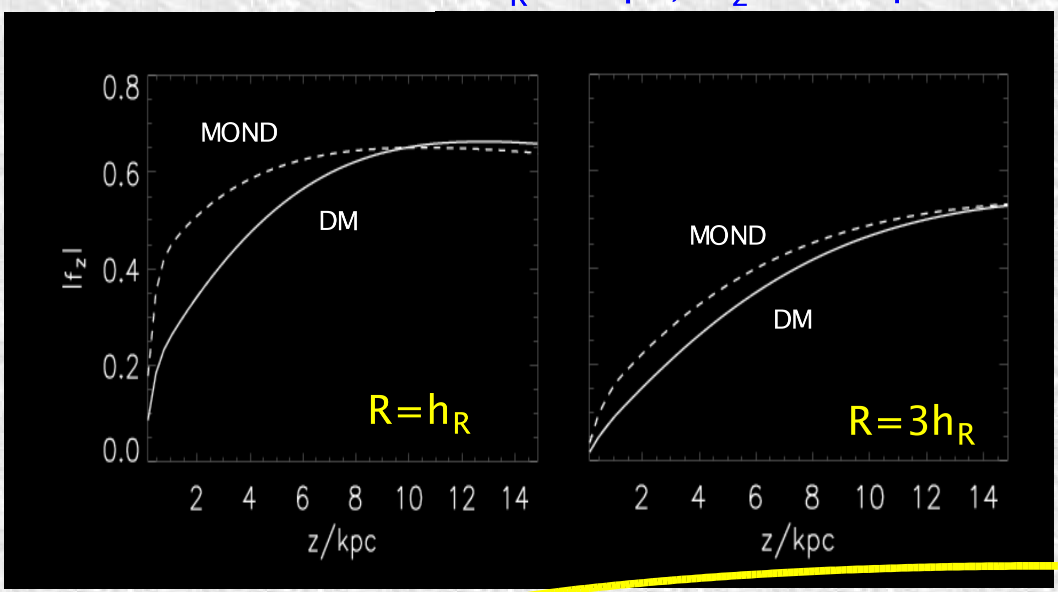
Preliminary results (Zhao, Nipoti, Londrillo & Ciotti in prep)

- Given a the surface density of baryons in a disk galaxy MOND predicts the vertical force field
- For the same galaxy, in a DM scenario the disk+(spherical)halo model reproducing the rotation curve predicts a different vertical force field
- Good measures of the vertical velocity dispersion of observed disk galaxis can discriminate between the two scenarios

NUMERICAL SOLUTION

Exponential disk: $M=10^{10}$ Msun

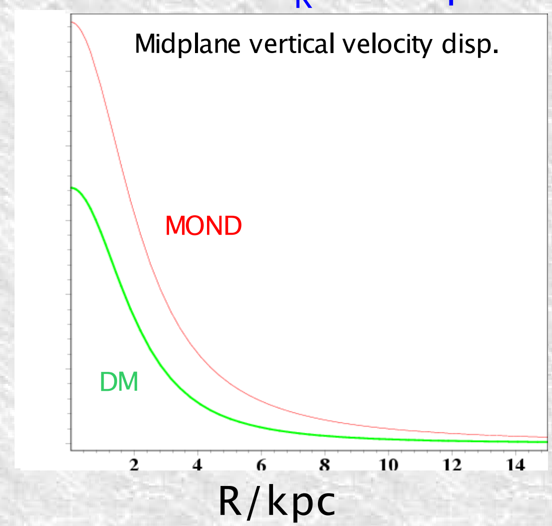
$h_R= 3$kpc, $h_z= 0.3$ kpc



ANALYTICAL SOLUTION

Kuzmin disk: $M=10^{10}$ Msun

$h_R= 2.5$kpc



Both MOND and disk+DM halo reproduce the **same rotation curve**
BUT **MOND** PREDICTS **HIGHER VERTICAL VELOCITY DISPERSION**
THAN DISK+DARK MATTER HALO NEWTONIAN GRAVITY

Application to observational data:
MILKY WAY (vertical vel disp. in the solar neighborhood) / other galaxies

N-body simulations in MOND

- No Green function --> NO TREECODE, NO DIRECT N-BODY CODE
- Instead use --> PARTICLE-MESH CODES
- WE CANNOT NEGLECT THE SOLENOIDAL FIELD S
  (even if S is typically small in stationary systems!)
- WE MUST SOLVE EXACTLY THE FIELD EQUATION

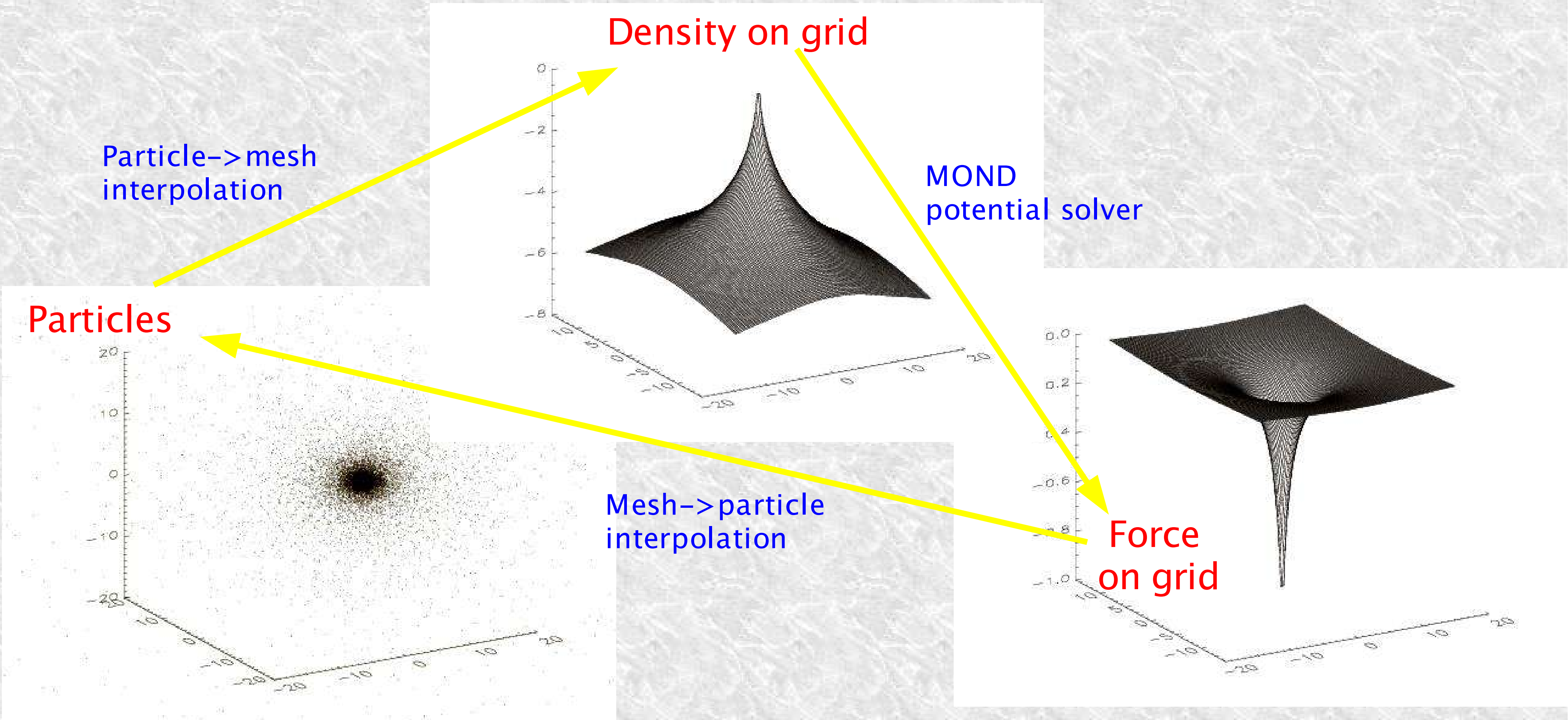
If one neglects the solenoidal field S, momentum is not conserved (Felten 1984, Bekenstein & Milgrom 1984)

The ONLY MOND N-body simulations so far were those by Brada & Milgrom (1999, 2000). Few applications: disk stability, external field effect

A new particle-mesh MOND N-body code

(Londrillo, Nipoti & Ciotti in preparation)

- We developed a new code to run N-body simulations in MOND
- Standard particle-mesh technique used in Newtonian codes
- The Poisson solver is replaced by our new MOND potential solver
- Standard leap-frog time integration
- The code is (partly) parallel



Simulations of dissipationless collapse in MOND

(Nipoti, Londrillo & Ciotti in preparation)

- We ran simulations of cold collapse of a set of N-particles in MOND
- N=1-2 x10^6 particles
- Initial conditions: clumpy, spherically symmetric Plummer distribution with particles at rest
- We check energy, linear and angular momentum conservation

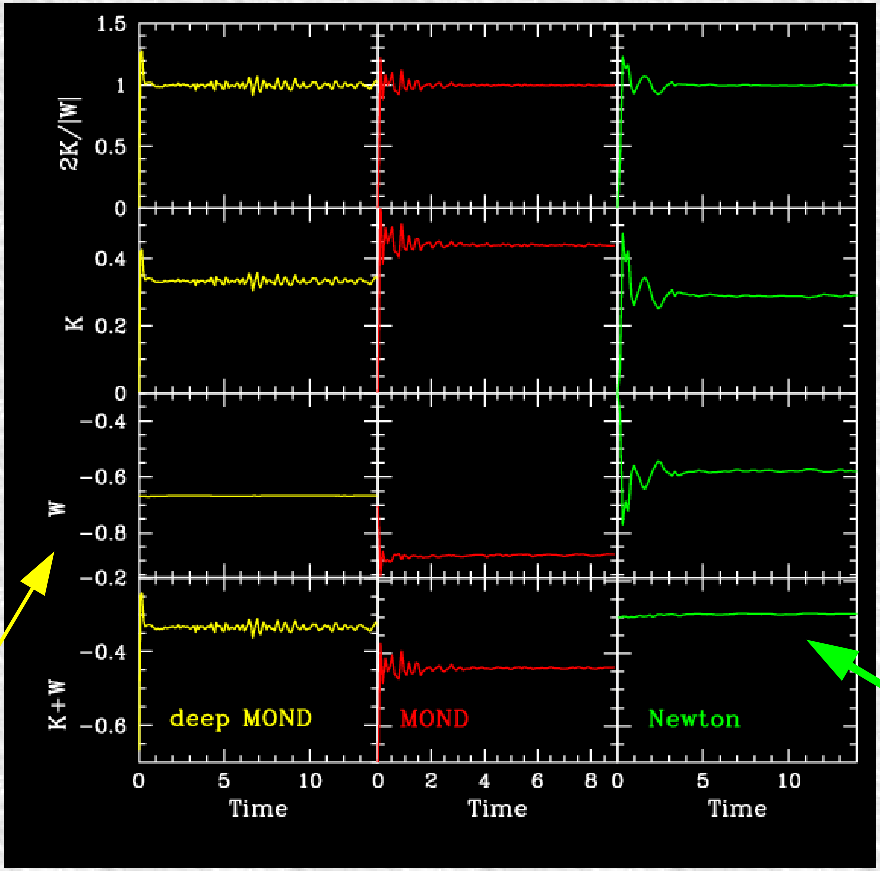
Time evolution

Virial ratio ->

Kinetic energy (K) ->

Interaction energy (W) ->

K+W ->



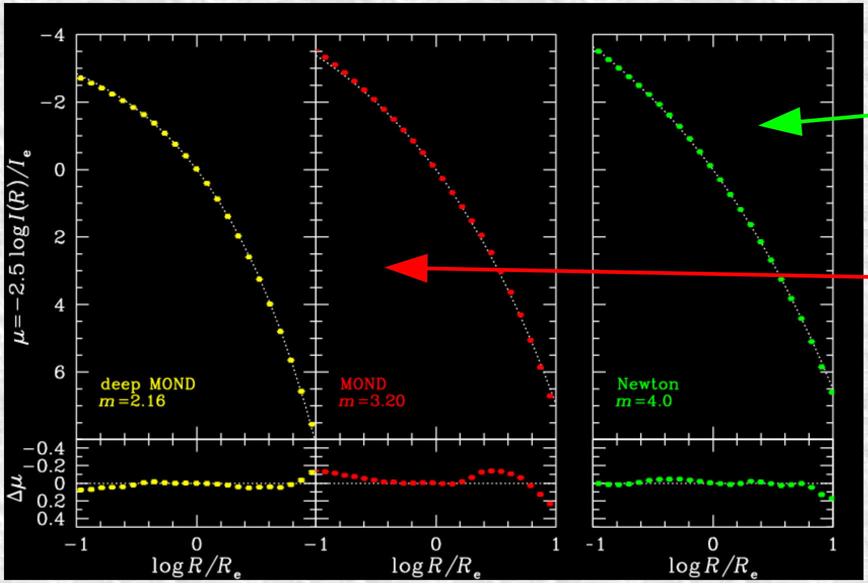
Virial Theorem holds in MOND (Gerhard & Spergel 1992)

$$W = -\int \rho(\vec{x}) < \vec{x}, \nabla \phi > d^3 \vec{x}$$

W is conserved in deep MOND. This can be proved analytically (Nipoti et al. in prep).

Conservation of total energy K+W in Newtonian gravity

End-products of dissipationless collapse in MOND



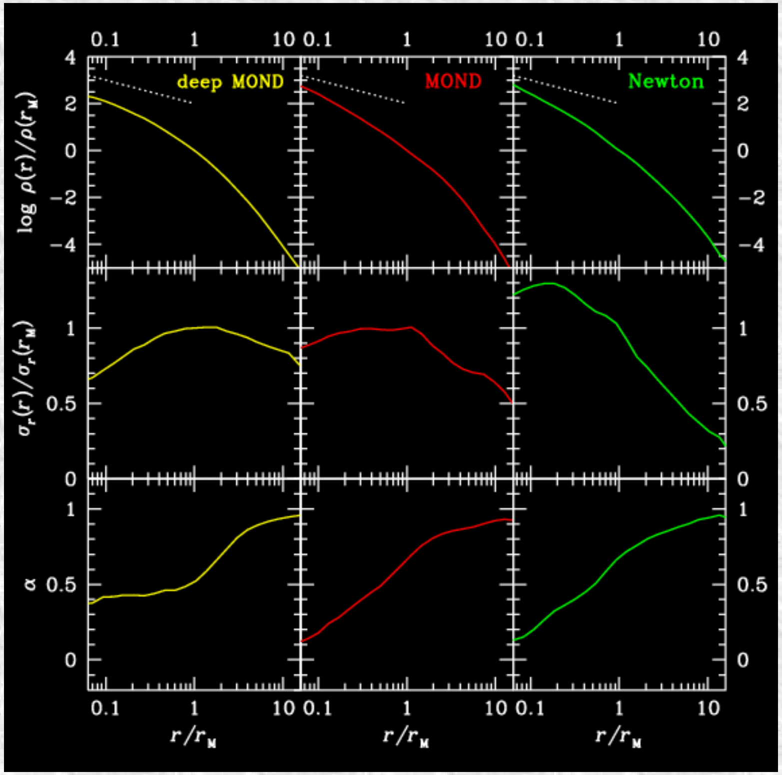
Newtonian collapses: $R^{1/4}$ de Vaucouleurs profiles (see van Albada 1982) **well reproduced** by our code

MOND collapses produce systems with **shallower inner cusps**: Sersic $R^{1/m}$ profile with m=2-3

Density ->

Velocity dispersion ->

Anisotropy parameter ->



MOND end-products have flatter velocity dispersion profile and are more radially anisotropic than Newtonian end-products

SUMMARY & CONCLUSIONS

ANALYTICAL METHODS AND NUMERICAL CODES

- We presented a flexible method to build analytical axisymmetric and triaxial MOND density-potential pairs with realistic density distributions
- We developed and tested a numerical MOND potential solver for generic density distributions
- We developed and tested a parallel particle-mesh code for MOND N-body simulations

APPLICATIONS AND FIRST RESULTS

- The (often neglected) solenoidal field S is typically small in stationary systems, BUT in some (low-surface density) systems we found S/g up to 0.6
- Preliminary results of N-body simulations suggest that the end-products of cold collapse in MOND differ structurally and kinematically from the end-products of Newtonian collapse

NOTE: here we considered the Bekenstein & Milgrom (1984) μ function but our numerical code works for all the proposed μ functions for MOND and TeVeS

WORK IN PROGRESS & FUTURE APPLICATIONS

- Vertical kinematics of disk galaxies in MOND
- Constraints on the μ function from rotation curves
- TeVes gravitational lensing from non-spherical lenses
- Stability of disks in MOND

(Also in collaboration with P. Londrillo, L. Ciotti, H. Zhao & B. Famaey)