Mass transfer in binary systems

- **Mass transfer occurs when**
  - star expands to fill Roche-lobe
    - due to stellar evolution
  - orbit, and thus Roche-lobe, shrinks till $R_* < R_L$
    - due to angular momentum loss
    - e.g. magnetic braking, gravitational radiation
- **Three cases**
  - Case A: mass transfer while donor is on main sequence
  - Case B: donor star is in (or evolving to) Red Giant phase
  - Case C: SuperGiant phase
- **Mass transfer changes mass ratio**
  - changes Roche-lobe sizes
  - can drive further mass transfer

Orbit evolution

Kepler $a^3 \propto P^2 \, M \quad \rightarrow \quad 3\frac{\dot a}{a} = 2\frac{\dot P}{P} + \frac{\dot M}{M}$

$M \equiv m_1 + m_2$

orbital angular momentum

$$J = \frac{m_1 \, m_2}{M}\left(\frac{2p \, a}{P}\right)\left(1-e^2\right)^{1/2}$$

$$= m_1 \, m_2 \left(\frac{G \, a\left(1-e^2\right)}{M}\right)^{1/2}$$

$$\rightarrow \quad \frac{\dot J}{J} = \frac{\dot m_1}{m_1} + \frac{\dot m_2}{m_2} + \frac{1}{2}\frac{\dot a}{a} - \frac{1}{2}\frac{\dot M}{M} - \frac{1}{2}\frac{2\, e \, \dot e}{1-e^2}$$

$$\boxed{\frac{\dot a}{a} = \frac{\dot M}{M} + 2\frac{\dot J}{J} - 2\left(\frac{\dot m_1}{m_1} + \frac{\dot m_2}{m_2}\right) - \frac{e\,\dot e}{1-e^2}}$$

orbit size

$$\frac{\dot a}{a} = \frac{\dot M}{M} + 2\frac{\dot J}{J} - 2\left(\frac{\dot m_1}{m_1} + \frac{\dot m_2}{m_2}\right) - \frac{e\,\dot e}{1-e^2}$$

period

$$\frac{\dot P}{P} = \frac{3}{2}\frac{\dot a}{a} - \frac{1}{2}\frac{\dot M}{M}$$

$$= \frac{\dot M}{M} + 3\frac{\dot J}{J} - 3\left(\frac{\dot m_1}{m_1} + \frac{\dot m_2}{m_2}\right) - \frac{3}{2}\frac{e\,\dot e}{1-e^2}$$

Conservative mass exchange

$$\frac{\dot a}{a} = \frac{\dot M}{M} + 2\frac{\dot J}{J} - 2\left(\frac{\dot m_1}{m_1} + \frac{\dot m_2}{m_2}\right) - \frac{e\,\dot e}{1-e^2}$$

circular orbit — conservative mass exchange:

$e = 0 \quad \dot e = 0 \qquad \dot M = 0 \quad \dot J = 0 \quad \dot m_1 = -\dot m_2 > 0$

$$\frac{\dot a}{a} = -2\left(\frac{-\dot m_2}{m_1} + \frac{\dot m_2}{m_2}\right) \;=\; 2\frac{-\dot m_2}{m_2}\left(1 - \frac{m_2}{m_1}\right) > 0$$

$$\frac{\dot P}{P} = \frac{3}{2}\frac{\dot a}{a} > 0$$

Orbit expands
period increases
if m1>m2

Shrinks if m1<m2

Roche Lobe size

Eggelton 1983 — Paczynski $0.1 < q < 0.8$

$$\frac{R_L}{a} \approx \frac{0.49\, q^{2/3}}{0.69\, q^{2/3} + \ln\left(1 + q^{1/3}\right)} \approx 0.462\left(\frac{q}{1+q}\right)^{1/3}$$

Star 2 fills Roche Lobe:

$$\frac{\dot R_L}{R_L} = \frac{\dot a}{a} + \frac{1}{3}\frac{\dot m_2}{m_2}$$

$$\frac{\dot a}{a} = 2\frac{-\dot m_2}{m_2}\left(1 - \frac{m_2}{m_1}\right)$$

$$= 2\frac{-\dot m_2}{m_2}\left(\frac{5}{6} - \frac{m_2}{m_1}\right)$$

Critical mass ratio:

Lobe shrinks if q = m2 / m1 > 5 / 6

expands if q < 5 / 6

Timescales

- **Dynamical timescale**
  - timescale for star to establish hydrostatic equilibrium

$$t_{dyn} \sim \left(\frac{R^3}{Gm}\right)^{1/2} \sim 30\,\mathrm{min}\left(\frac{R}{R_\odot}\right)^{3/2}\left(\frac{m}{M_\odot}\right)^{-1/2}$$

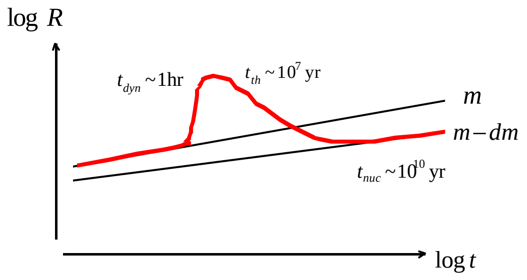
- **Thermal timescale**
  - timescale for star to establish thermal equilibrium

$$t_{th} \sim \frac{Gm^2}{R\,L} \sim 3\times10^7\ \mathrm{yr}\left(\frac{m}{M_\odot}\right)^2\left(\frac{R}{R_\odot}\frac{L}{L_\odot}\right)^{-1}$$

- **Nuclear timescale**
  - timescale of energy source of star
  - ie main sequence lifetime

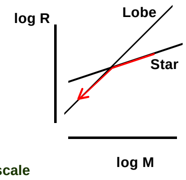
$$t_{nuclear} \approx 7\times10^9\ \mathrm{yr}\,\frac{m}{M_\odot}\left(\frac{L}{L_\odot}\right)^{-1}$$

Star reacts to mass loss



Reaction to mass loss

- **Star reacts to mass loss**
  - expands / contracts
  - Roche-lobe also expands or contracts
- **Define** $\zeta \equiv \frac{d \ln R}{d \ln m}$



If $\zeta_L > \zeta_{dyn}$
  - star transfers mass on dynamical timescale
    - star stripped down too fast to adjust

If $\zeta_{dyn} > \zeta_L > \zeta_{th}$
  - hydrostatic equilirbium easily maintained
    - star transfers mass on thermal timescale

If $\zeta_{dyn}, \zeta_{th} > \zeta_L$
  - stable on thermal timescale
    - mass transfer due to stellar evolution, nuclear timescale

Mass loss timescales

e.g. for $\dot M = 0$, $V_L \approx 2\left(q - \frac{5}{6}\right)$

thermal timescale

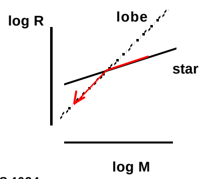
$V_{dyn} < V_L < V_{th\,L}$

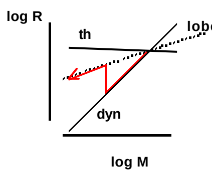
Dynamical timescale

$V_L > V_{dyn}$

nuclear timescale

$V_{dyn}\,,\ V_{th} < V_L < V_{nuc}$





q > 5/6 -- unstable mass transfer

- **Unstable (runaway) mass transfer**
- **q > 5/6**
- **Roche lobe shrinks down around the star, stripping it down.**
- **Rapid (dynamical)**
- **violent**
- **rare because very fast**
- **must occur (more massive stars evolve first)**
- **Possible example**
- **super-soft x-ray binaries:**
  - wd primary accreting from main sequence secondary with q > 5/6
  - orbit and Roche lobe shrink from orbital angular momentum loss
  - high accretion rate (e.g. 1e-4 Msun / yr) allows steady burning of H on white dwarf surface

q< 5/6 -- stable mass transfer

- **q < 5/6**
- **conservative mass transfer makes Roche lobe expand.**
  - cuts off mass transfer
- **Mass transfer if**
- **1) star expands**
  - nuclear evolution
- **2) angular momentum lost**
  - winds
  - gravitational radiation
- **donor star fills Roche lobe**
- **Case A,B,C mass transfer**
- **Many interacting binaries of this type**

$$\frac{1}{t_{nuc}} \approx \frac{\dot R_2}{R_2} = \frac{-\dot m_2}{m_2}\left(\frac{5}{3} - 2q\right)$$

$$-\frac{\dot m_2}{m_2} = \frac{1}{\left(\frac{5}{3} - 2q\right) t_{nuc}}$$

$$\frac{\dot a}{a} = \frac{2}{3}\frac{\dot P}{P} = \frac{1-q}{\left(\frac{5}{3} - 2q\right) t_{nuc}} > 0$$

Orbit expands

period increases

on nuclear timescale

Angular momentum loss

- **Magnetic braking**
- **gravitational radiation**
- **Stable q < 5/6**
- **donor stays on main-seq**
- **R ~ M**
- **Cataclysmic variables**
  - white dwarf primary
  - late K or M donors

$$\frac{\dot m_2}{m_2} \approx \frac{\dot R_2}{R_2} = 2\frac{\dot J}{J} + \frac{-\dot m_2}{m_2}\left(\frac{5}{3} - 2q\right)$$

$\dot J < 0$

$$\frac{-\dot m_2}{m_2} = \frac{-\dot J / J}{\left(\frac{4}{3} - q\right)}$$

$$\frac{\dot a}{a} = \frac{2}{3}\frac{\dot P}{P} = \frac{2\dot J / 3J}{\left(\frac{4}{3} - q\right)} < 0$$

orbit shrinks

period decreases