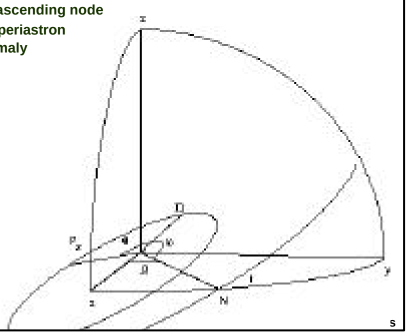
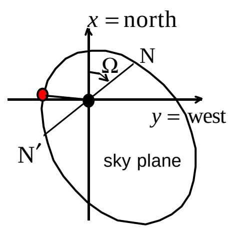
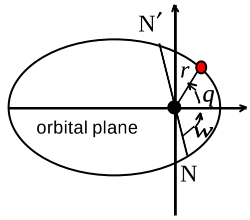
Orbit in Space

- x, y = north, west on sky plane thru m1 at x,y=0, observer at -z.
- i = inclination between sky and orbit planes
- N = ascending node, m2 crosses x-y plane in +z direction
  - Ω = longitude of ascending node
  - ω = longitude of periastron
- theta = true anomaly



Relative orbit on the Sky





$$x = x'\cos\Omega + y'\sin\Omega\cos i$$
$$y = x'\sin\Omega - y'\cos\Omega\cos i$$
$$z = y'\sin i$$

$$x' = r\cos(\theta+\omega)$$
$$y' = r\sin(\theta+\omega)$$

Orbital Elements

specific angular momentum :

$$L_x = L\sin i\sin\Omega$$
$$L_y = -L\sin i\cos\Omega$$
$$L_z = -L\cos i$$

7 orbital elements :

size (a), shape (e),

orientation in space (i, ω, Ω)

and in time (T, P).

Spectroscopic Binaries

- m2 at (r, $\theta + \omega$)
  - project along the line of nodes: r cos($\theta + \omega$)
  - perpendicular to line of nodes: r sin($\theta + \omega$)
    - project along line of sight: z = r sin($\theta + \omega$) sin i
- radial velocities along line-of sight

$$V_{rad} = \dot{z} = \sin i\left[\dot{r}\sin(\theta+\omega) + r\dot{\theta}\cos(\theta+\omega)\right]$$

use $$r = \frac{a(1-e^2)}{1+e\cos\theta} \rightarrow \dot{r} = \frac{e\sin(\theta)\, r\dot{\theta}}{1+e\cos\theta}$$

and Kepler's 2nd Law $$r^2\dot{\theta} = \frac{2\pi a^2(1-e^2)^{1/2}}{P}$$

$$V_{rad} = \frac{2\pi a\sin i}{P\sqrt{1-e^2}}\left[\cos(\theta+\omega) + e\cos(\omega)\right]$$

Spectroscopic Orbital Velocities

$$V_{rad} = K\left[\cos(\theta+\omega) + e\cos\omega\right] + \gamma$$
$$K = \frac{2\pi a\sin i}{P\sqrt{1-e^2}}$$

ascending node: $\cos(\theta+\omega) = +1$

$$V_{max} = K\left[e\cos\omega + 1\right] + \gamma$$

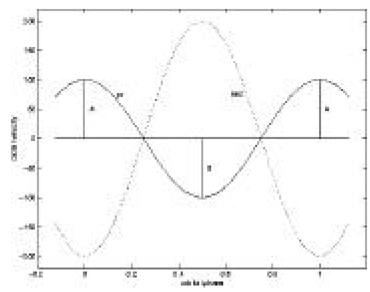
descending node: $\cos(\theta+\omega) = -1$

$$V_{min} = K\left[e\cos\omega - 1\right] + \gamma$$

semi-amplitude:

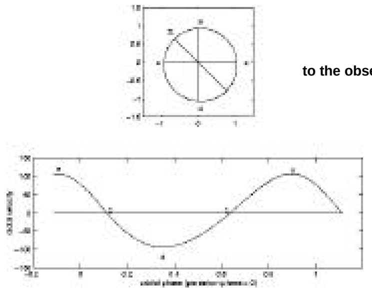
$$K \equiv (V_{max} - V_{min})/2$$

Circular Orbit



- $K_1 = 100$ km/s $K_2 = 200$ km/s $q = m_2 / m_1 = 0.5$

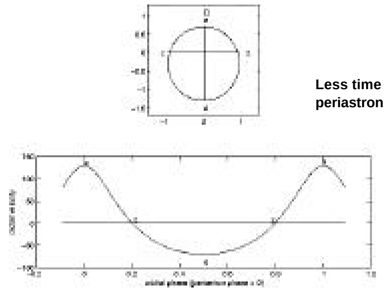
Radial Velocities



to the observer -->

e = 0.1, ω = 45°

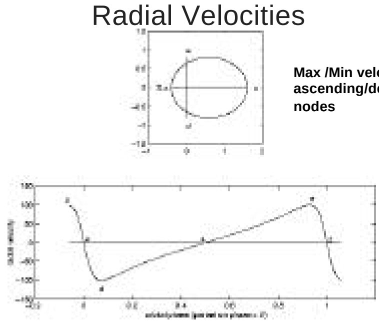
Radial Velocities



Less time near periastron

e = 0.3, ω = 0°

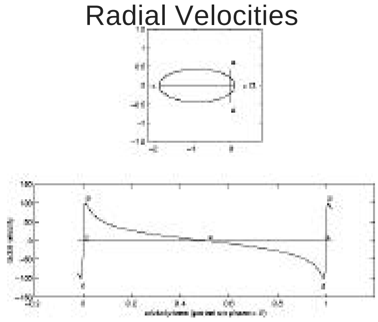
Radial Velocities



Max /Min velocity at ascending/descending nodes

- ***e=0.6 , ω= 90°***

Radial Velocities



- ***e=0.9 , ω= 270°***

Orbits from radial velocities

- **Observations *n* radial velocity measurements:**
  - $V_r(i)$ at times $t(i)$ $i = 1 \dots n$
- **Elliptical orbit model:**

$$E - e\sin E = \frac{2\pi}{P}(t-T)$$

$$\tan\left(\frac{\theta}{2}\right) = \left(\frac{1+e}{1-e}\right)^{1/2} \tan\left(\frac{E}{2}\right)$$

$$V_r = K\left(\cos(\theta+\omega) + e\cos\omega\right) + \gamma$$

- **Best fit determines *(K, e, ω, γ, T, P)***
- **e.g. using a least squares procedure**

Minimum masses

SB2 : measure P, K_1, K_2, e

$$K_i = \frac{2\pi\ a_i \sin i}{P\sqrt{1-e^2}} \quad \rightarrow \quad a_i \sin i = \frac{\sqrt{1-e^2}}{2\pi} K_i\ P$$

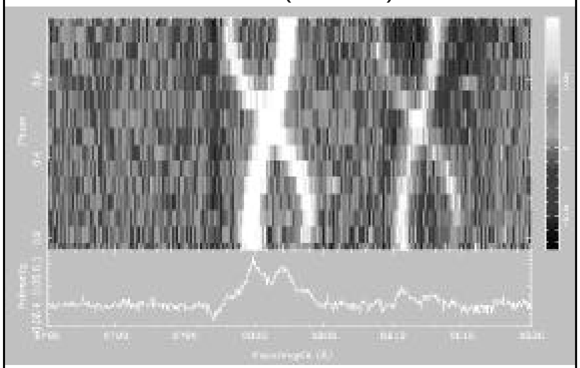
$$m_1 = M\ \frac{a_2}{a} = \frac{4\pi^2 a^2\ a_2}{G\ P^2}$$

$$m_1 \sin^3 i = \frac{4\pi^2 a^2\ a_2 \sin^3 i}{G\ P^2} = \frac{\left(1-e^2\right)^{3/2}(K_1+K_2)^2 K_2\ P}{2\pi\ G}$$

SB1 : measure P, K_1, e, calculate the <u>mass function</u> :

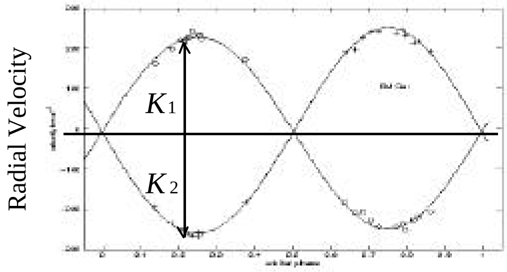
$$f(m_2) = \frac{m_2^{\ 3}\sin^3 i}{M^2} = \frac{\left(1-e^2\right)^{3/2} K_1^{\ 3} P}{2\pi\ G}$$

KV Vel (sdO+M)



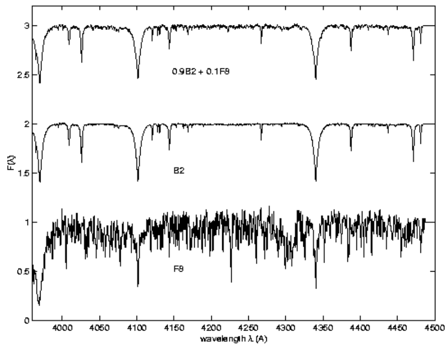
EM Car (O8V+O8V)

observe : $K = V \sin i$

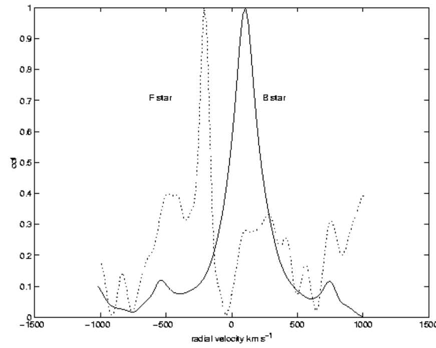


Composite spectrum

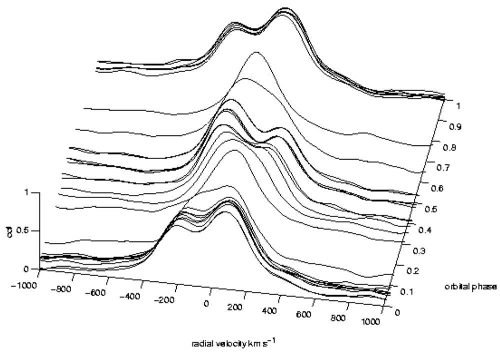
(90% B2 + 10% F8)



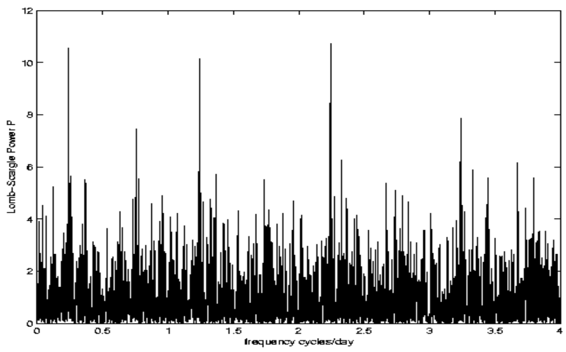
Cross-correlate with B2 and F8 template spectra



LZ Cep (O9.5V + O9.5V)

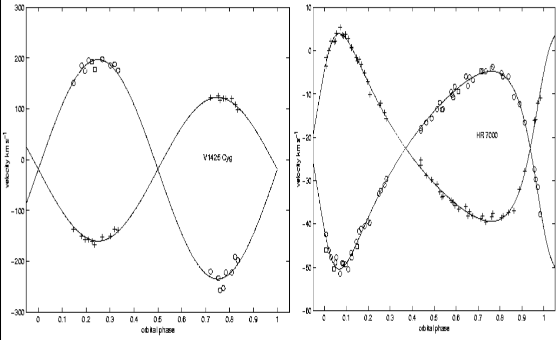


HR 1165 Periodogram



V1425 Cyg
(B5V+B9V)

HR 7000
(F4V+ F6V)



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Binary Stars and Accretion Disks