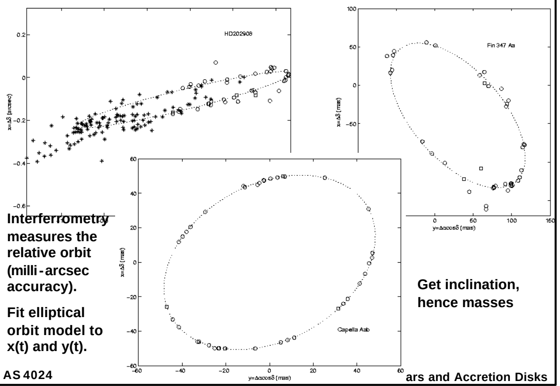
Inclinations

- **Radial velocities give minimum masses**

SB2: $m\sin^3 i$ SB1: $\dfrac{m^3\sin^3 i}{M^2}$

- **need inclinations**
  - astrometric orbits
  - eclipsing binaries
  - polarimetry
  - apsidal motion

Astrometric Orbits

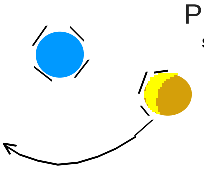


Interferrometry measures the relative orbit (milli-arcsec accuracy).

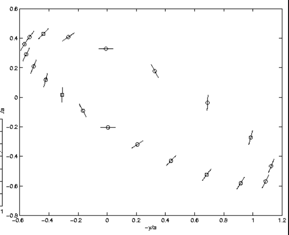
Fit elliptical orbit model to x(t) and y(t).

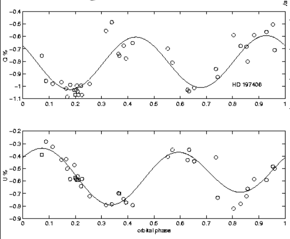
Get inclination, hence masses

Polarimetry



Scattered light is polarised (e.g. our sky)

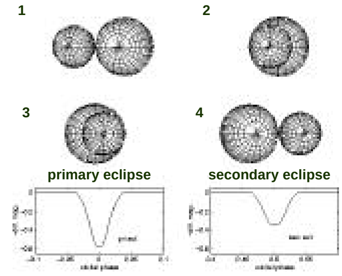




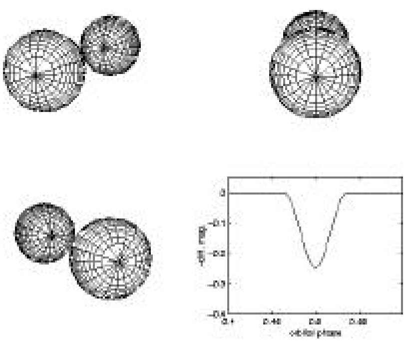
Get inclination, hence masses

Eclipsing Binaries

- **Binary properties from eclipses**
  - sizes, shapes of stars, inclination,
  - temperatures, limb darkening, apsidal motion
  - 4 contact phases:



Partial Eclipse



Eclipses

$R_1 > R_2 \quad \theta \equiv 2\pi\phi$

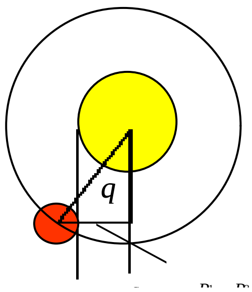
edge-on ($i = 90^\circ$):

4 contact phases:

$R_1 \pm R_2 = a\sin\theta$

$$\frac{R_2}{a} = \frac{\sin\theta_1 - \sin\theta_2}{2}$$

$$\frac{R_1}{a} = \frac{\sin\theta_1 + \sin\theta_2}{2}$$



Eclipses

4 contact phases

$R_1 \pm R_2 = a\sqrt{\sin^2 q + \cos^2 q \cos^2 i}$

$= a\sqrt{1 + \cos^2 q \sin^2 i}$

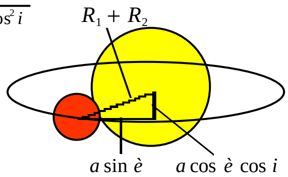
4 measurements: $f_1 \; f_2 \; f_3 \; f_4$

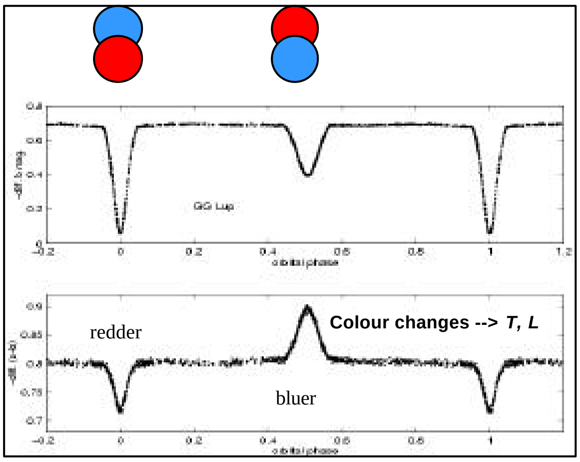
4 parameters $\frac{R_1}{a} \; \frac{R_2}{a} \; i \; f_0$

mid eclipse $q = 0$

total eclipse $a \cos i < R_1 - R_2$

partial eclipse $R_1 - R_2 < a \cos i < R_1 + R_2$



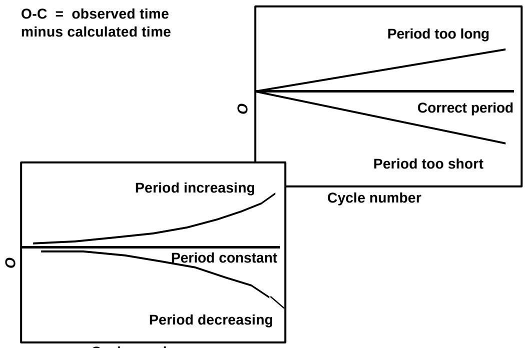


Application to Binary Pulsars

- binary system where one star is a pulsar
  - emits 'pulses' of radiation
  - accurate timing possible (accurate clocks)
  - need narrow pulses
    - radio signals from neutron stars
- solitary pulsar
  - if at 0 velocity relative to us
    - time between pulses, dt = constant (unless being spun up/down)
  - if at V_{rel} relative velocity
    - dt = constant x pulse number
  - if pulsar spins up, dt decreases with pulse number
    - concave curve
  - if pulsar spins down, dt increases with pulse number
    - convex curve

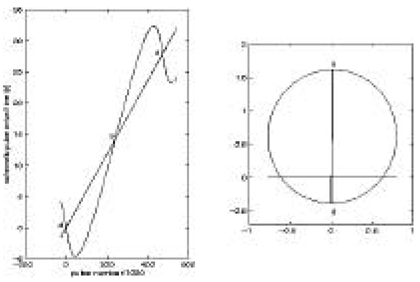
Timing Residuals

O-C = observed time minus calculated time



Binary Pulsars

- In binary system, time between pulses affected by orbital motions
  - due to light travel time (distance) changing along orbit



Observer-->

Light travel time

pulsar orbit: $r_p = \frac{a_p\,(1+e^2)}{1 + e\cos q}$

distance along line of sight:

$z_p = r_p \sin i \sin(q + w)$

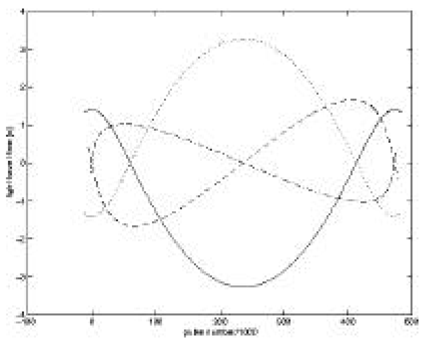
light travel time:

$$\frac{z_p}{c} = \frac{a_p \sin i}{c}(1+e^2)\frac{\sin(q+w)}{1+e\cos q}$$

circular orbit

$$\frac{z_p}{c} = \frac{a_p \sin i}{c}\sin\left(\frac{2p}{P}(t - T_0)\right)$$

light travel time



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Binary Pulsar timing residuals

- Time difference between predicted , $j_n t$, and actual (binary) pulse arrival times, t_n is

$$\Delta t = t_n - j_n t = a\,t + b \sin\left[\frac{2p\,(t - T_0)}{P}\right]$$

  - P is the orbital period, T_0 is a reference time
  - a,b are determined by the velocity of the pulsar
    - a: from systematic velocity
    - b: from Keplerian velocity
  - for circular orbits: b = (a_p/c) sini

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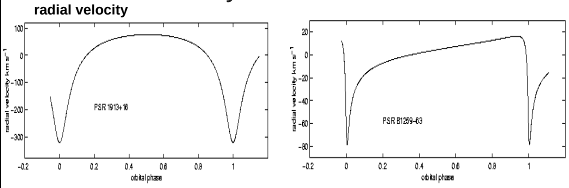
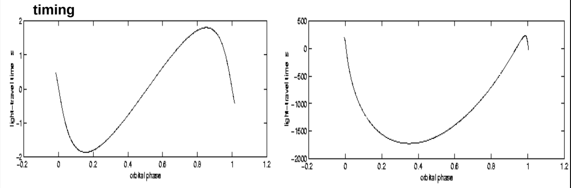
Binary Pulsar Orbits

radial velocity



timing



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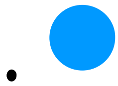
Mass determinations

- visible companion star
  - O-B star in High-Mass X-ray Binaries (HMXB)
  - A-K star in Low-Mass X-ray Binaries (LMXB)



$$a_c \sin i = \frac{(1-e^2)^{1/2} K_c\, P}{2p}$$

mass function

$$f(m_p) = \frac{m_p^{\,3} \sin^3 i}{M^2} = \frac{(1-e^2)^{3/2} K_c^{\,3}\, P}{2p\, G}$$

mass ratio, q,

$$q = \frac{m_p}{m_c} = \frac{a_c \sin i}{a_p \sin i}$$

  - If inclination, *i*, can be found, then masses follow

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Frequency shifts

- Binary orbit also affects pulsar frequency
  - radio pulsars , very narrow pulse widths
  - pulse frequency affected by orbital velocity
  - Doppler shift:

$$\Delta f = f\,\frac{V_{rad}}{c} = f\,\frac{\dot{z}}{c}$$

  - gives a phase lag of:

$$\Delta f = \int_{T_0}^{t} \Delta f\, dt \approx f_0\,\frac{\dot{z}}{c}(t - T_0)$$

$$= f_0\,\frac{\dot{z}}{c}\left[\frac{z}{c}\right]$$

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Pulsar Phase lag

- Combined phase lag is
  - from light travel time due to orbit

$$\Delta f_L = -f\,\frac{z}{c}$$

  - and from Doppler shift

$$\Delta f_D = f_0\,\frac{\dot{z}}{c}\left[\frac{z}{c}\right]$$

  - hence

$$\Delta f = \Delta f_D + \Delta f_L \approx \left[\frac{z}{c}\right]\left[f_0\frac{\dot{z}}{c} - f_0 - \dot{f}_0(t - T_0)\right]$$

  - generally $\Delta f_D \cong 0.001 \Delta f_L$
  - but measurable in radio pulsars

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