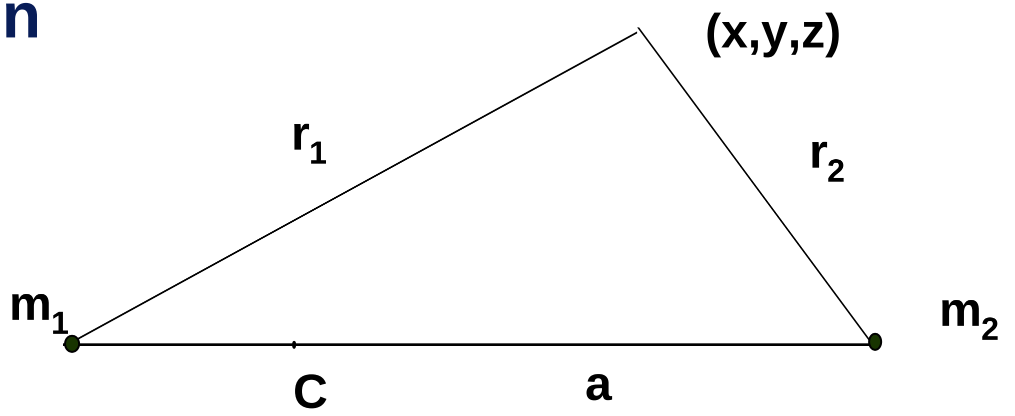
Roche Model

- **Stars deform in close binary systems**
  - **tides**
  - **rotation**
- **Observations**
  - **light curve effects from aspherical distortions**
- **Small perturbations -- Legendre Polynomials**
- **Large deformations-- Roche Model**

Roche Potential

Assumes:

- **synchronous rotation**
- **circular orbit**
- **2 point masses**
- **rotating frame**



$$w^2 = \left(\frac{2p}{P}\right)^2 = \frac{G\,M}{a^3}$$

$$r_1^{\,2} = x^2 + y^2 + z^2 \qquad r_2^{\,2} = (\,x - a\,)^2 + y^2 + z^2$$

$$\frac{x_c}{a} = \frac{m_2}{M} = \frac{q}{1+q} \qquad q \equiv \frac{m_2}{m_1} \leq 1$$

$$\Phi = -\frac{G\,m_1}{r_1} - \frac{G\,m_2}{r_2} - \frac{w^2}{2}\left[(\,x - x_c\,)^2 + y^2\right]$$

Dimensionless Roche Potential

$$\text{factor out } -\frac{w^2}{2} = -\frac{G\,M}{2\,a^3} \quad \text{and} \quad x \rightarrow \frac{x}{a}, \quad \text{etc.}$$

$$\Phi(x,y,z) = -\frac{w^2}{2}\,\Phi_N\left(\frac{x}{a}, \frac{y}{a}, \frac{z}{a},\right)$$

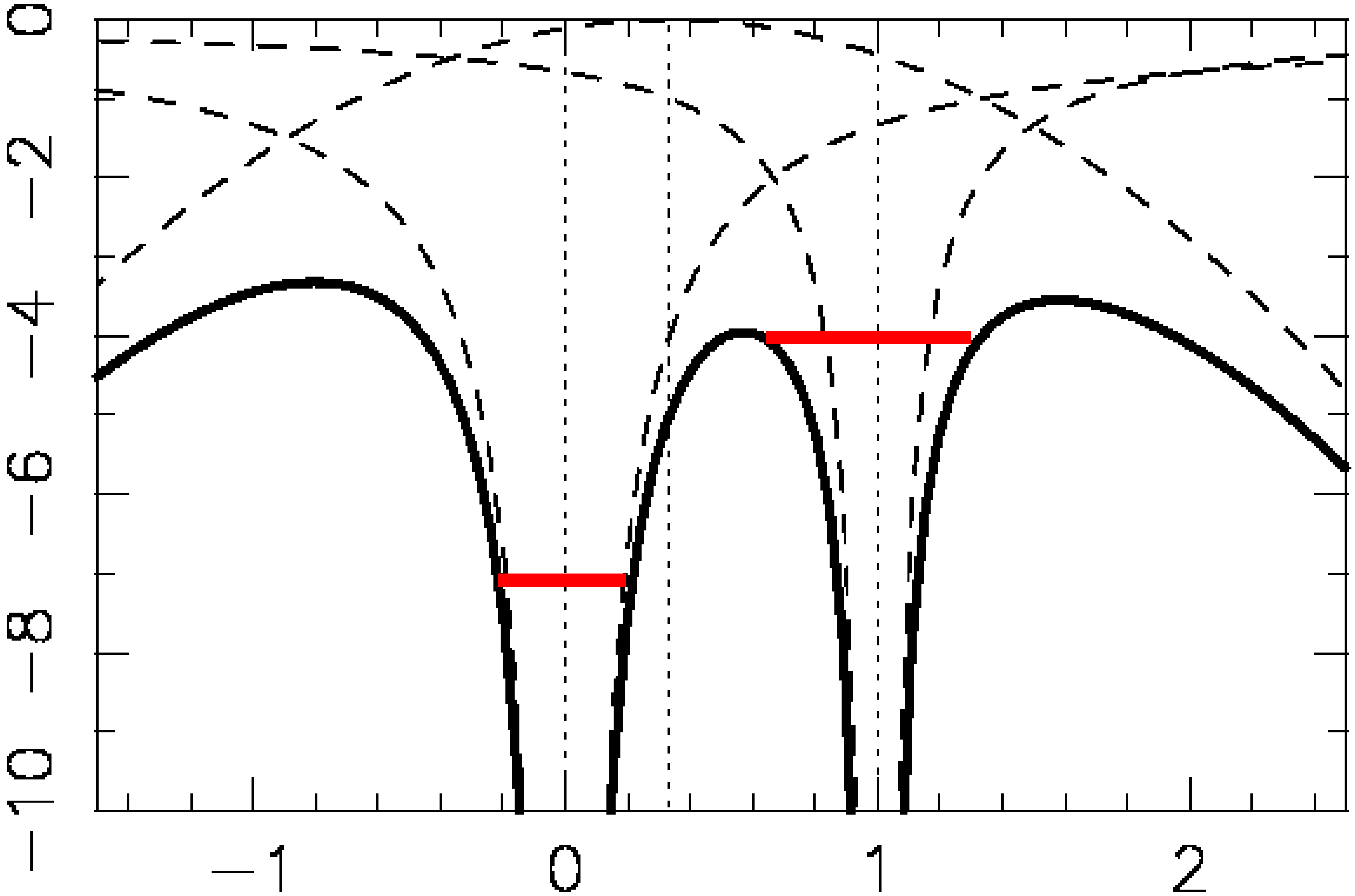
dimensionless Roche Potential:

$$\Phi_N(x,y,z) = \frac{2}{(1+q)}\frac{1}{r_1} + \frac{2\,q}{(1+q)}\frac{1}{r_2} + \left(x - \frac{q}{(1+q)}\right)^2 + y^2$$

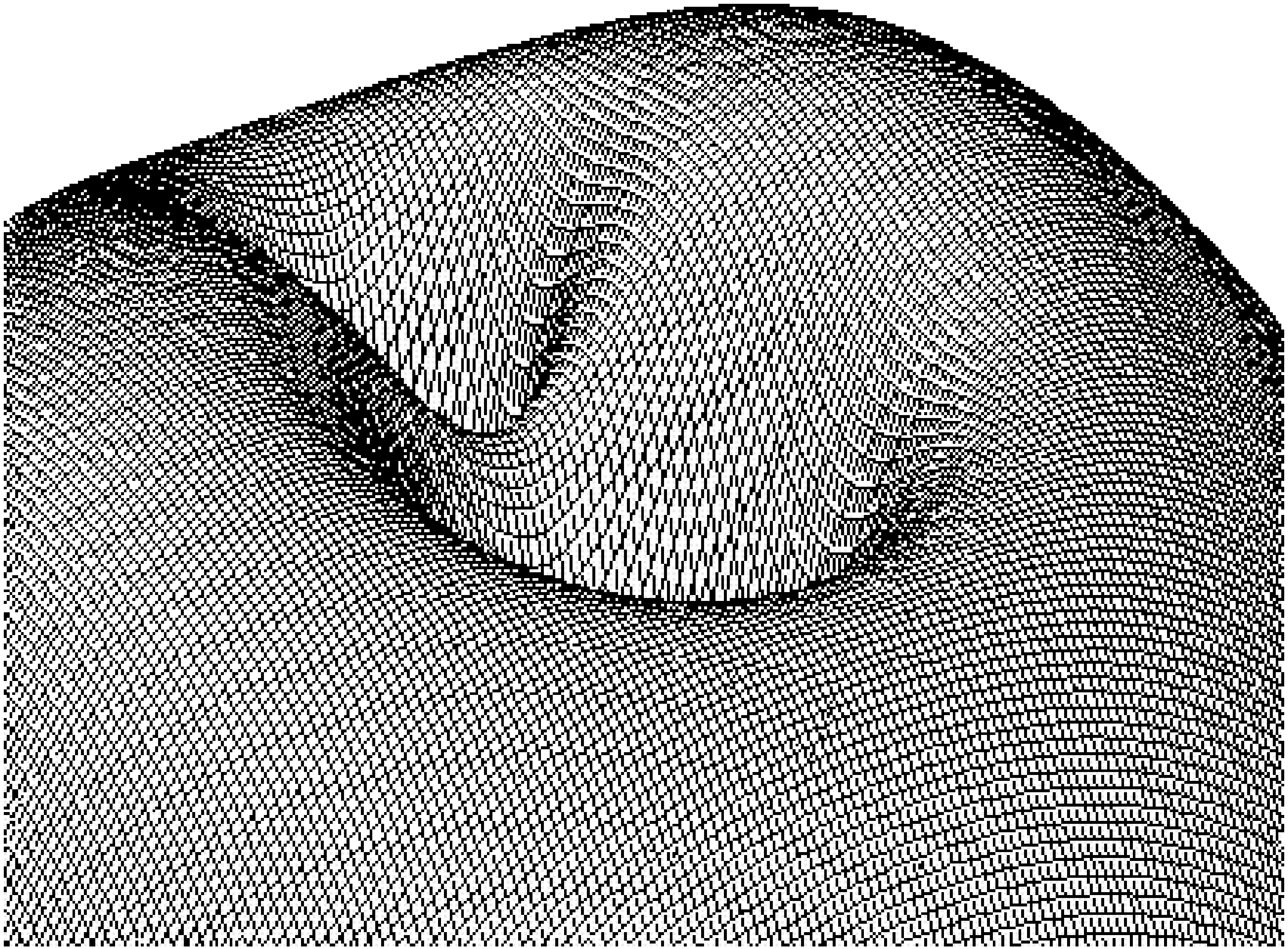
Describes shape of potential surfaces independently of the mass and size of the system.

single parameter : q

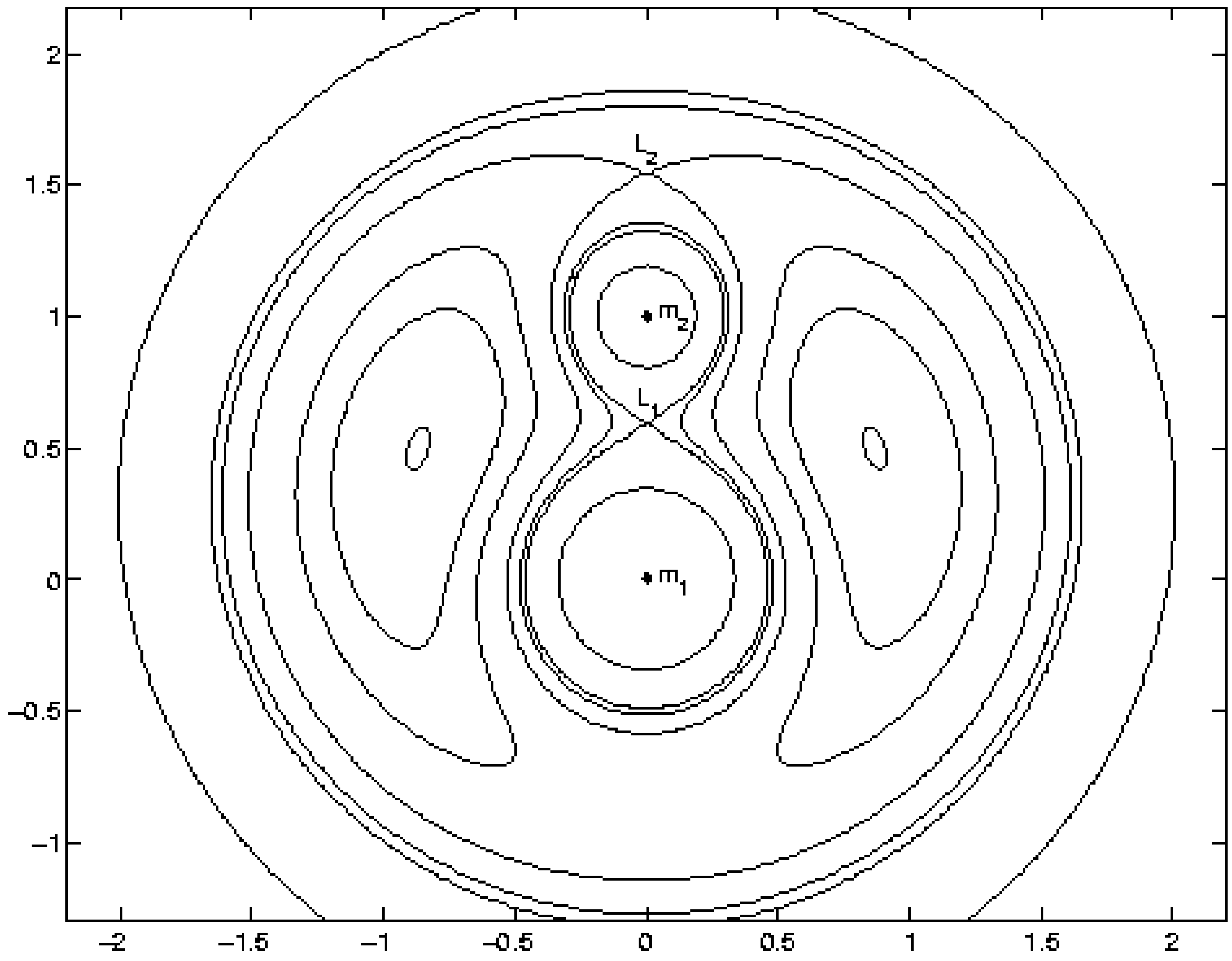
Slice along X axis



X-Y Plane



Roche Lobes

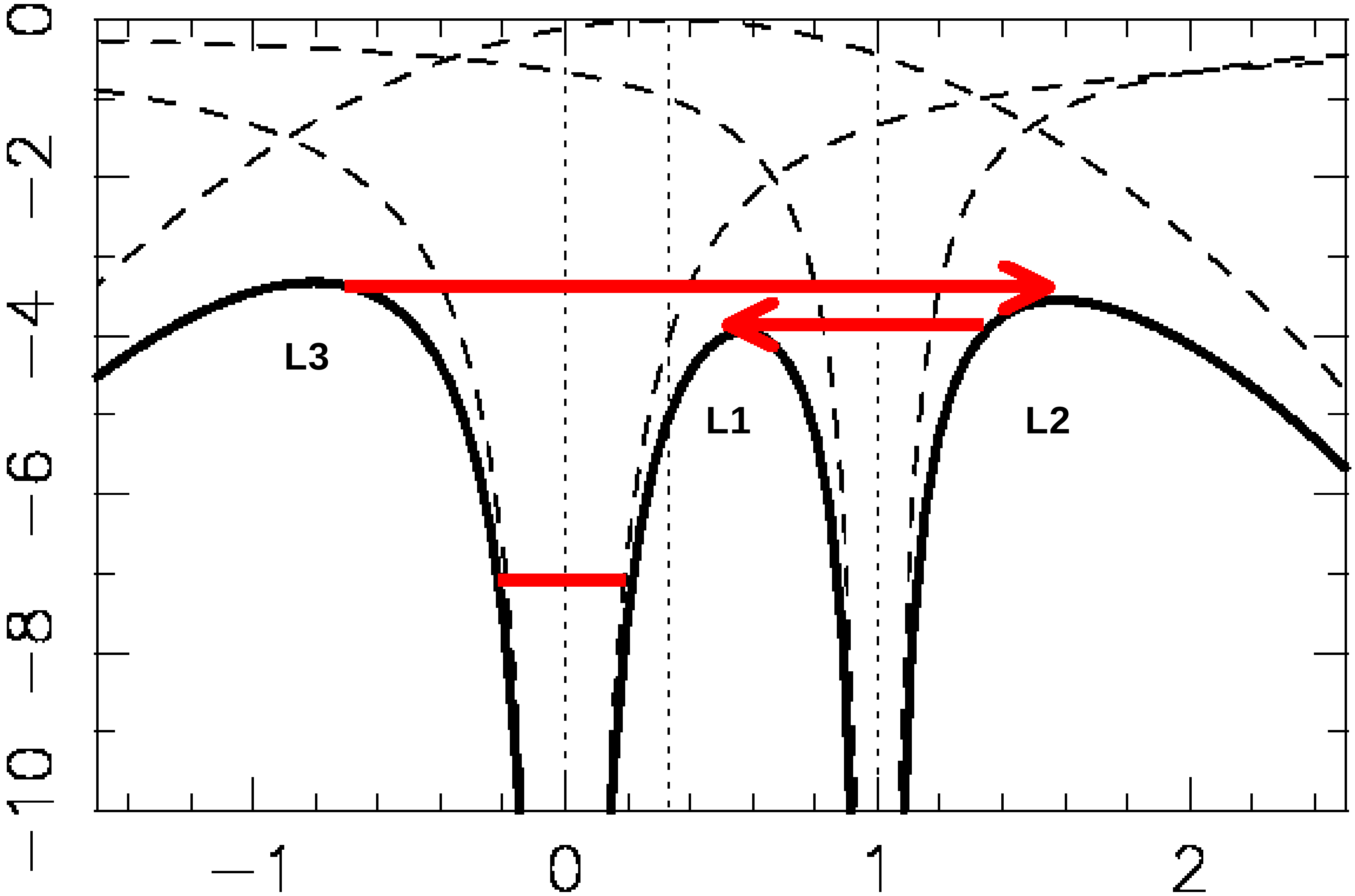


- **Lagrange points L_1, L_2, L_3, and L_4, L_5**

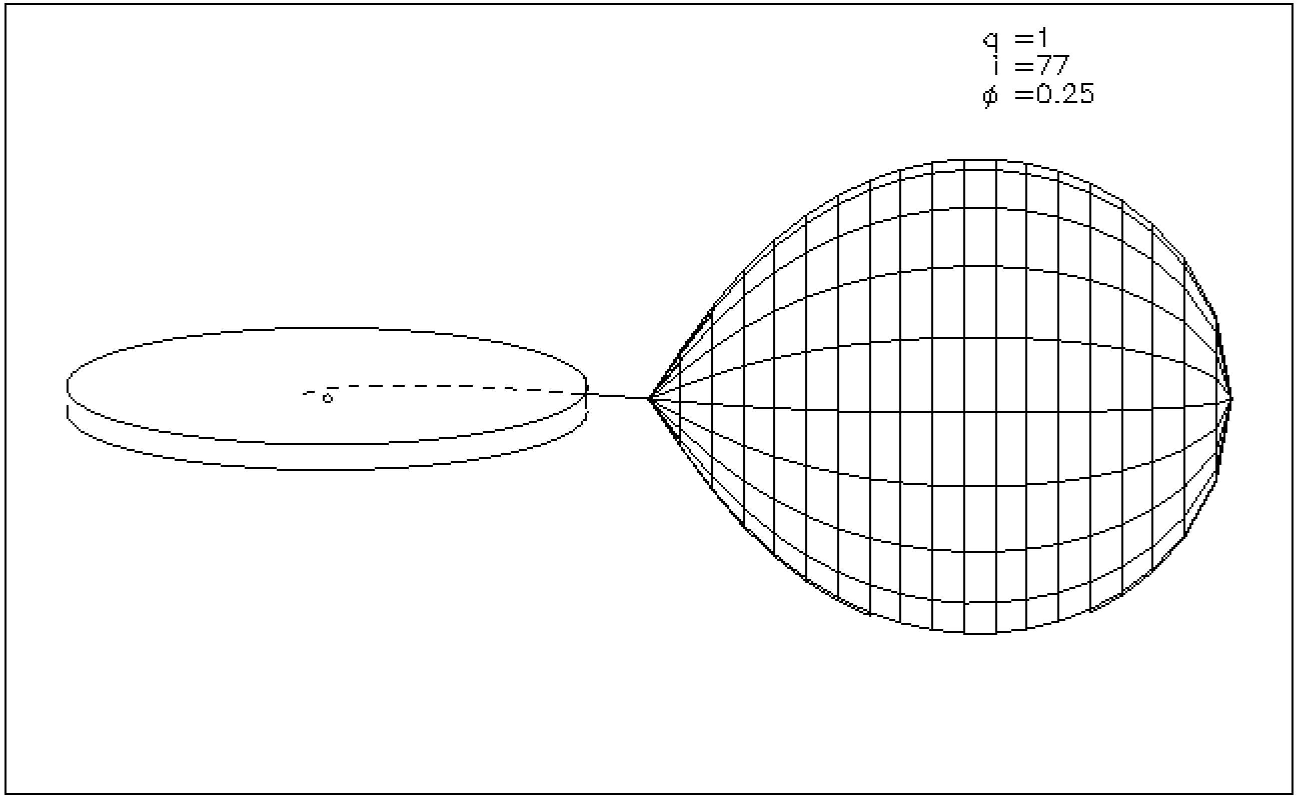
Lagrange points

- **Points where** $$\nabla \Phi_n = 0$$
- **L_1 - *Inner Lagrange Point***
  - in between two stars
  - matter can flow freely from one star to other
  - mass exhange
- **L_2 - on opposite side of secondary**
  - matter can most easily leave system
- **L_3 - on opposite side of primary**
- **L_4, L_5 - in lobes perpendicular to line joining binary**
  - form equilateral triangles with centres of two stars
- **Roche-lobes:: surfaces which just touch at L_1**
  - maximum size of non-contact systems

Mass transfer and loss



Roche Lobe



Roche Lobe Volumes

- **Effective size**
  - radius of Roche-lobe R_L
  - fit to results of numerical integration
  - Eggleton formula:

$$\frac{R_L}{a} \approx \frac{0.49q^{2/3}}{0.69q^{2/3} + \ln\left(1+q^{1/3}\right)} a$$

- **Effectively, it is a tidal radius where**
  - mean density in lobes are equal

$$\frac{R_{L,2}}{a} \approx \frac{1}{2}\left(\frac{m_2}{M}\right)^{1/3} \text{ for } 0 \le q \le 0.8$$

Inner Lagrange point

- **to find L_1**

$$\Phi_n(x,0,0) = \frac{2}{(1+q)|x|} + \frac{2q}{(1+q)|1-x|} + \left(x - \frac{q}{1+q}\right)^2$$

for $\quad 0 < x < 1,$

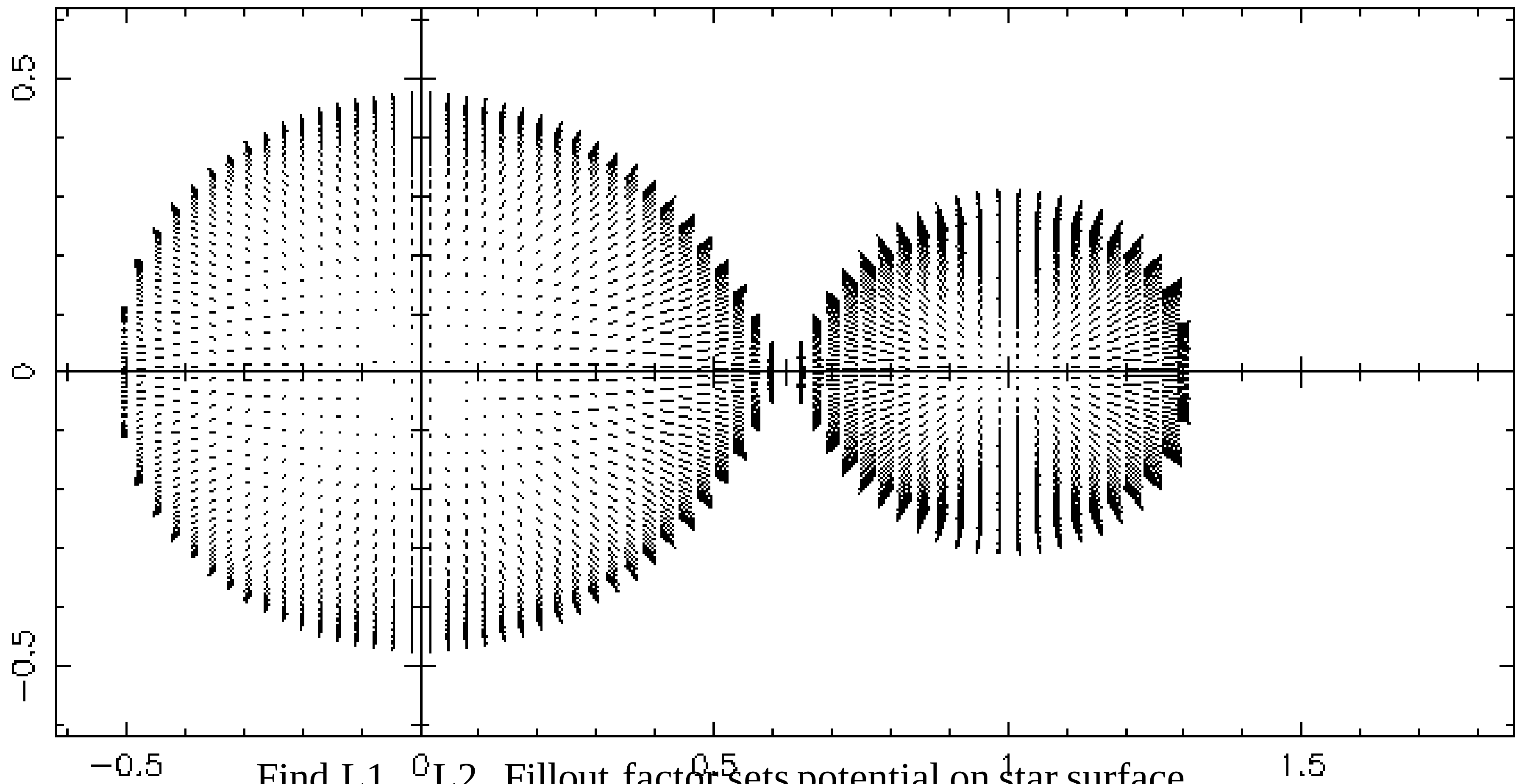
$$\frac{\partial \Phi_n}{\partial x} = \frac{-2}{(1+q)x^2} + \frac{2q}{(1+q)(1-x)^2} + 2\left(x - \frac{q}{1+q}\right)$$

maximum of $\Phi(\mathrm{x})$ at

$$0 = \frac{1}{x^2} - x + q\left((1-x) - \frac{1}{(1-x)^2}\right)$$

- – **solve numerically**

Modelling lightcurves



Find L1. L2. Fillout factor sets potential on star surface.

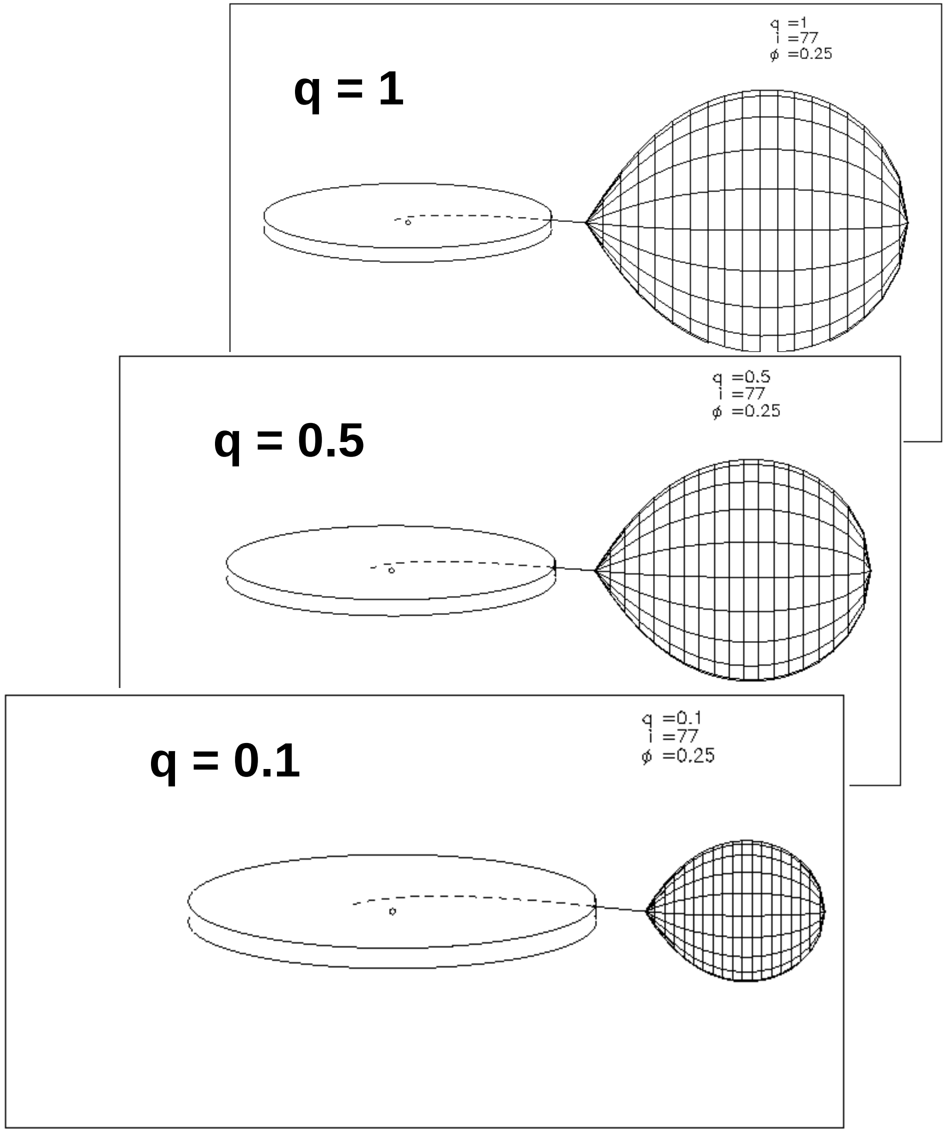
For grid of x, and theta, find r (perpendicular to x axis).

set of area elements. evaluate local gravity, temperature.

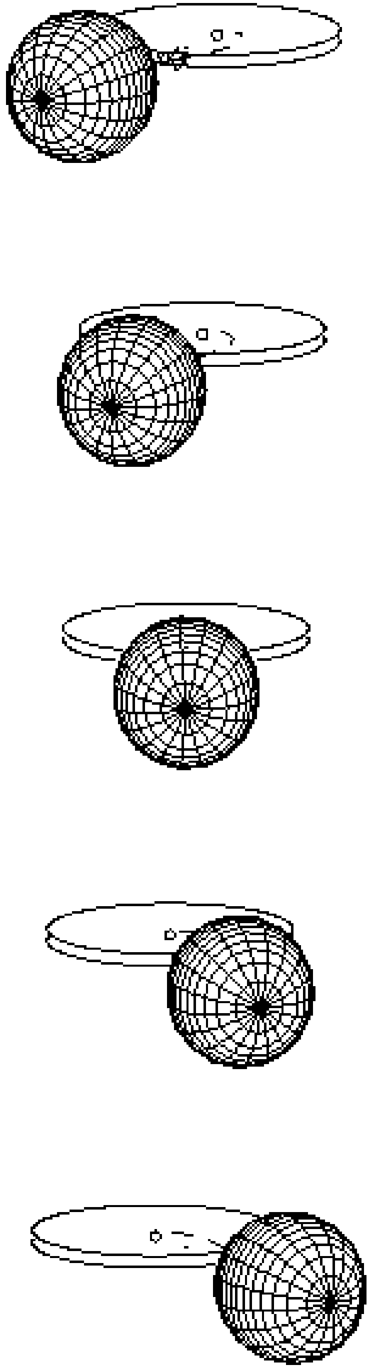
sum over other elements to account for heating.

Evaluate lightcurve by summing over visible elements.

Roche Lobes



eclipses



Limb Darkening

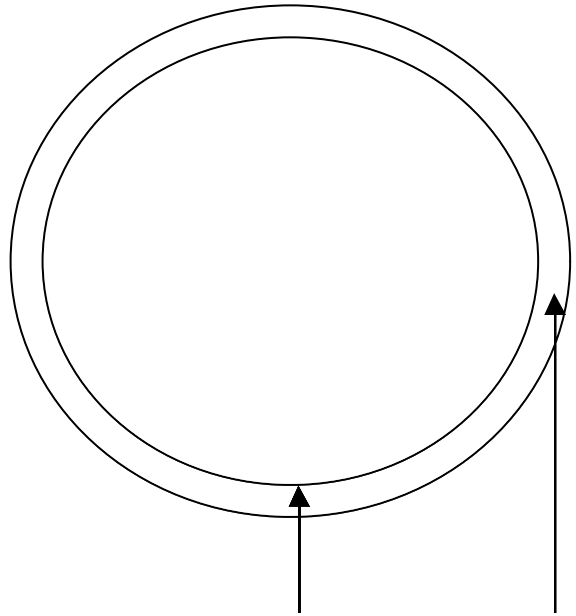
linear limb darkening law

$$I(q) = I_0 \left(1 - u + u \cos q \right)$$

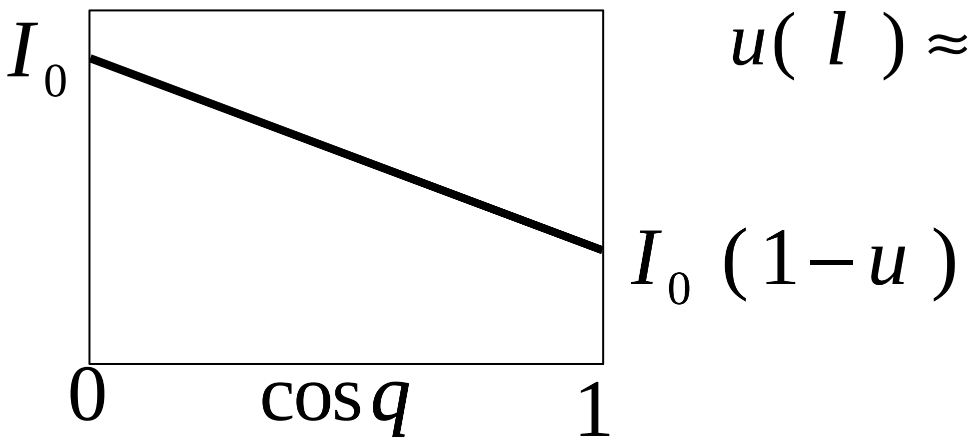
Eddington - Barbier relation

$$I_n(q, l) \approx B_n (T(q), l)$$

$$T(q) = T(t \approx \cos q)$$



See down to hotter zones

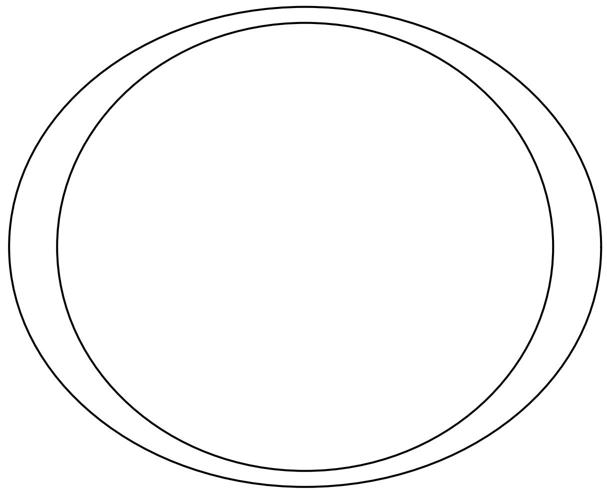


$$u(l) \approx 0.6 \text{ (e.g. for Sun)}$$

Gravity Darkening

Flux emerges more easily where local scale height is small. (same opacity, shorter thickness)

Darker in low-gravity regions:
equator of rotating star
g=0 at L1



von Zeipel gravity darkening law

$$T \rightarrow T_0 \left(\frac{g}{g_0} \right)^b$$

$$g = \left| \nabla \Phi \right| \quad T_0,\, g_0 \text{ at pole of star}$$

$$b \approx 0.25 \quad \text{radiative envelope (O, B, A, early F)}$$

$$b \approx 0.08 \quad \text{convective envelope (late F, G, K, M)}$$

Heating Effects

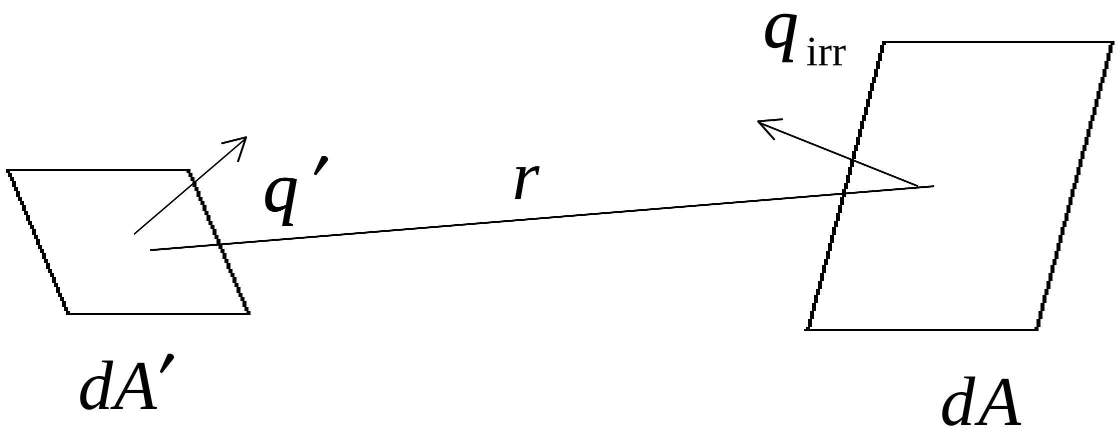
Flux from below and from above

irradiation

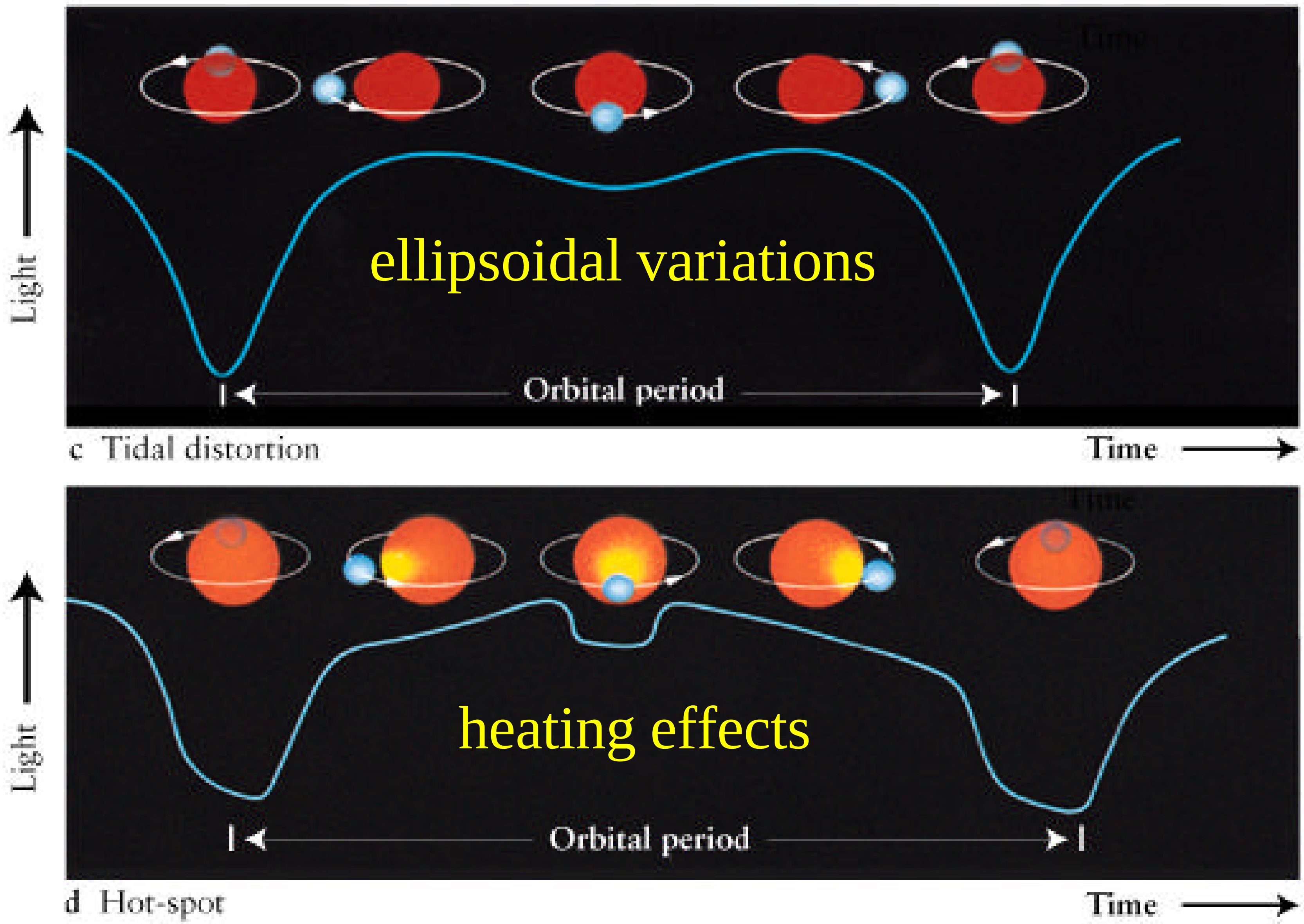
$$T^4 \rightarrow T^4 + T_{irr}{}^4$$

$$\sigma \, T_{irr}{}^4 = F_{irr} = \int \frac{(1-A)\, dL}{4\pi \, r^2} \cos\theta_{irr}$$

$$dL' = dA' \, \sigma \, T^4(\underline{r}') \cos\theta' \, (1 + u - u \cos\theta')$$



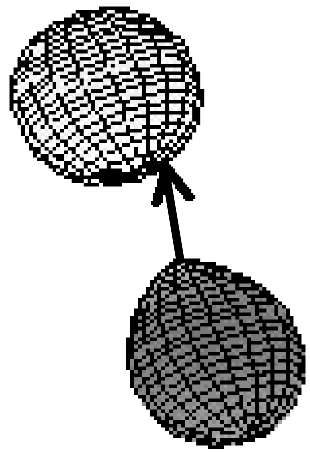
Proximity Effects



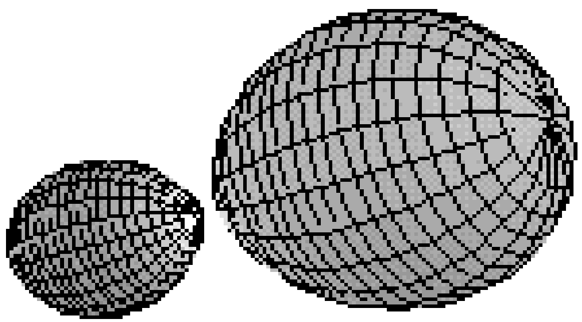
Binaries in Roche-Lobes

detached

semi-detached (Algol)



close to contact



contact (W UMa)