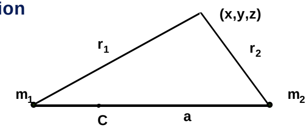
Roche Model

- Stars deform in close binary systems
  - tides
  - rotation
- Observations
  - light curve effects from aspherical distortions
- Small perturbations -- Legendre Polynomials
- Large deformations-- Roche Model

Roche Potential

Assumes:

synchronous rotation
circular orbit
2 point masses
rotating frame



$$w^2 = \left(\frac{2p}{P}\right)^2 = \frac{G\,M}{a^3}$$

$$r_1^2 = x^2 + y^2 + z^2 \qquad r_2^2 = (x-a)^2 + y^2 + z^2$$

$$\frac{x_c}{a} = \frac{m_2}{M} = \frac{q}{1+q} \qquad q \equiv \frac{m_2}{m_1} \le 1$$

$$\Phi = -\frac{G\,m_1}{r_1} - \frac{G\,m_2}{r_2} - \frac{w^2}{2}\left[(x-x_c)^2 + y^2\right]$$

Dimensionless Roche Potential

factor out $-\frac{w^2}{2} = -\frac{G\,M}{2\,a^3}$ and $x \to \frac{x}{a}$, etc.

$$\Phi(x,y,z) = -\frac{w^2}{2}\,\Phi_N\left(\frac{x}{a},\frac{y}{a},\frac{z}{a}\right)$$

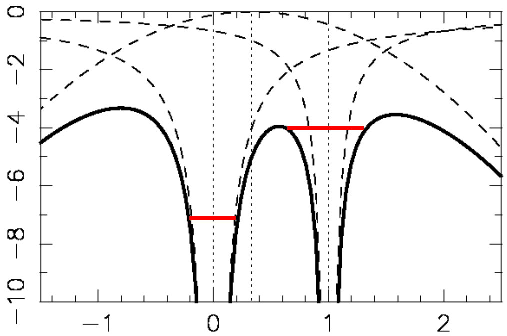
dimensionless Roche Potential:

$$\Phi_N(x,y,z) = \frac{2}{(1+q)}\frac{1}{r_1} + \frac{2\,q}{(1+q)}\frac{1}{r_2} + \left(x - \frac{q}{(1+q)}\right)^2 + y^2$$

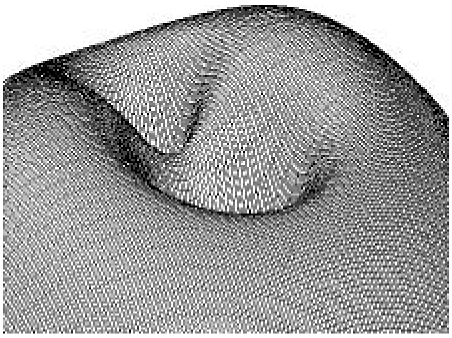
Describes shape of potential surfaces independently of the mass and size of the system.

single parameter : q

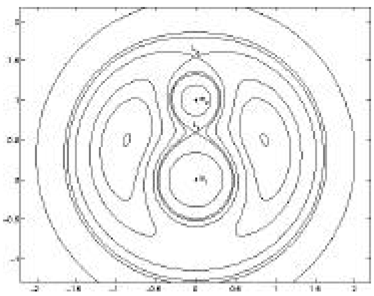
Slice along X axis



X-Y Plane



Roche Lobes

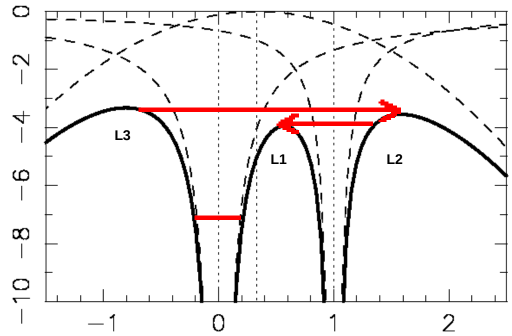


- Lagrange points L_1, L_2, L_3, and L_4, L_5

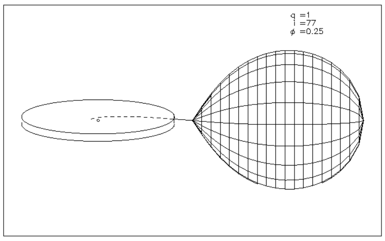
Lagrange points

- **Points where** $$\nabla \Phi_n = 0$$
- **L_1 - *Inner Lagrange Point***
  - in between two stars
  - matter can flow freely from one star to other
  - mass exhange
- **L_2 - on opposite side of secondary**
  - matter can most easily leave system
- **L_3 - on opposite side of primary**
- **L_4, L_5 - in lobes perpendicular to line joining binary**
  - form equilateral triangles with centres of two stars
- **Roche-lobes:: surfaces which just touch at L_1**
  - maximum size of non-contact systems

Mass transfer and loss



Roche Lobe



Roche Lobe Volumes

- **Effective size**
  - radius of Roche-lobe R_L
  - fit to results of numerical integration
  - Eggleton formula:

$$\frac{R_L}{a} \approx \frac{0.49\, q^{2/3}}{0.69\, q^{2/3} + \ln\left(1 + q^{1/3}\right)} a$$

- **Effectively, it is a tidal radius where**
  - mean density in lobes are equal

$$\frac{R_{L,2}}{a} \approx \frac{1}{2}\left(\frac{m_2}{M}\right)^{1/3} \text{ for } 0 \le q \le 0.8$$

Inner Lagrange point

- **to find L_1**

$$\Phi_n(x,0,0) = \frac{2}{(1+q)|x|} + \frac{2q}{(1+q)|1-x|} + \left(x - \frac{q}{1+q}\right)^2$$

for $\quad 0 < x < 1,$

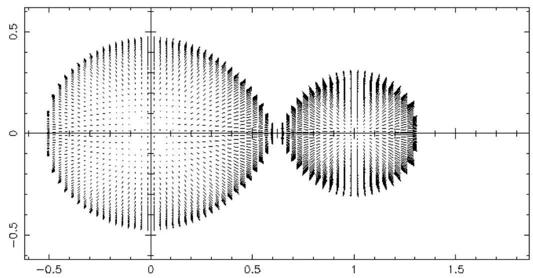
$$\frac{\partial \Phi_n}{\partial x} = \frac{-2}{(1+q)x^2} + \frac{2q}{(1+q)(1-x)^2} + 2\left(x - \frac{q}{1+q}\right)$$

maximum of $\Phi(x)$ at

$$0 = \frac{1}{x^2} - x + q\left((1-x) - \frac{1}{(1-x)^2}\right)$$

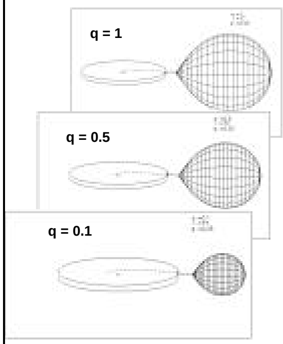
  - solve numerically

Modelling lightcurves

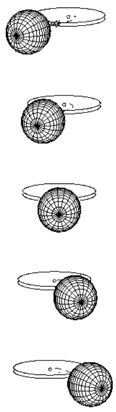


Find L1. L2. Fillout factor sets potentialon star surface.
For grid of x, and theta, find r (perpendicular to x axis).
set of area elements. evaluate local gravity, temperature.
sumover other elements toaccount for heating.

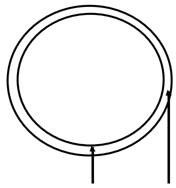
Roche Lobes



eclipses



Limb Darkening



linear limb darkening law

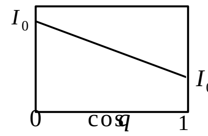
$I(\theta) = I_0\,(1 - u + u\cos\theta)$

Eddington - Barbier relation

$I_\nu(\theta, \lambda) \approx B_\nu(T(\theta), \lambda)$

$T(\theta) = T(\tau \approx \cos\theta)$

See down to hotter zones

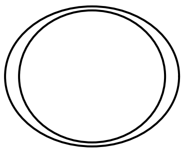


$u(\lambda) \approx 0.6$ (e.g. for Sun)

Gravity Darkening

Flux emerges more easily where local scale height is small.
(same opacity, shorter thickness)

Darker in low-gravity regions:
equator of rotating star
g=0 at L1



von Zeipel gravity darkening law

$$T \rightarrow T_0 \left(\frac{g}{g_0}\right)^{\beta}$$

$g = |\nabla\Phi|$ T_0, g_0 at pole of star

$\beta \approx 0.25$ radiative envelope (O, B, A, early F)

$\beta \approx 0.08$ convective envelope (late F, G, K, M)

Heating Effects

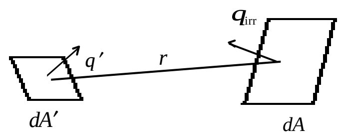
Flux from below and from above

irradiation

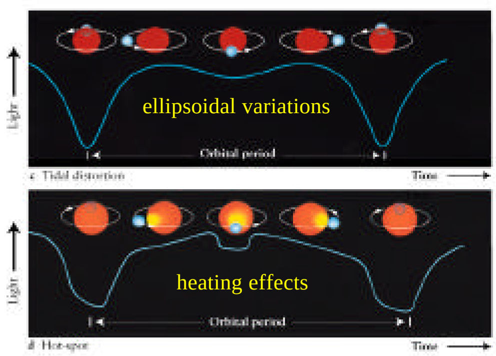
$T^4 \rightarrow T^4 + T_{irr}^{\ 4}$

$$\sigma T_{irr}^{\ 4} = F_{irr} = \int \frac{(1-A)\,dL}{4\pi r^2} \cos\theta_{irr}$$

$dL' = dA'\,\sigma T^4(\underline{r}')\cos\theta'\,(1 + u - u\cos\theta')$



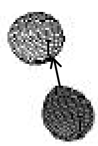
Proximity Effects



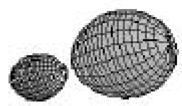
Binaries in Roche-Lobes

detached

semi-detached
(Algol)



close to contact



contact (W UMa)