Lecture 6: Jeans mass & length

Anisotropies in the CMB temperature

→ density ripples $$\frac{\Delta\rho}{\rho} \sim \frac{\Delta T}{T} \sim 10^{-5}$$

at the time of decoupling (z = 1100) .

These are the seeds that evolve (gravitational collapse) to form the structured distribution of galaxies we see around us today:

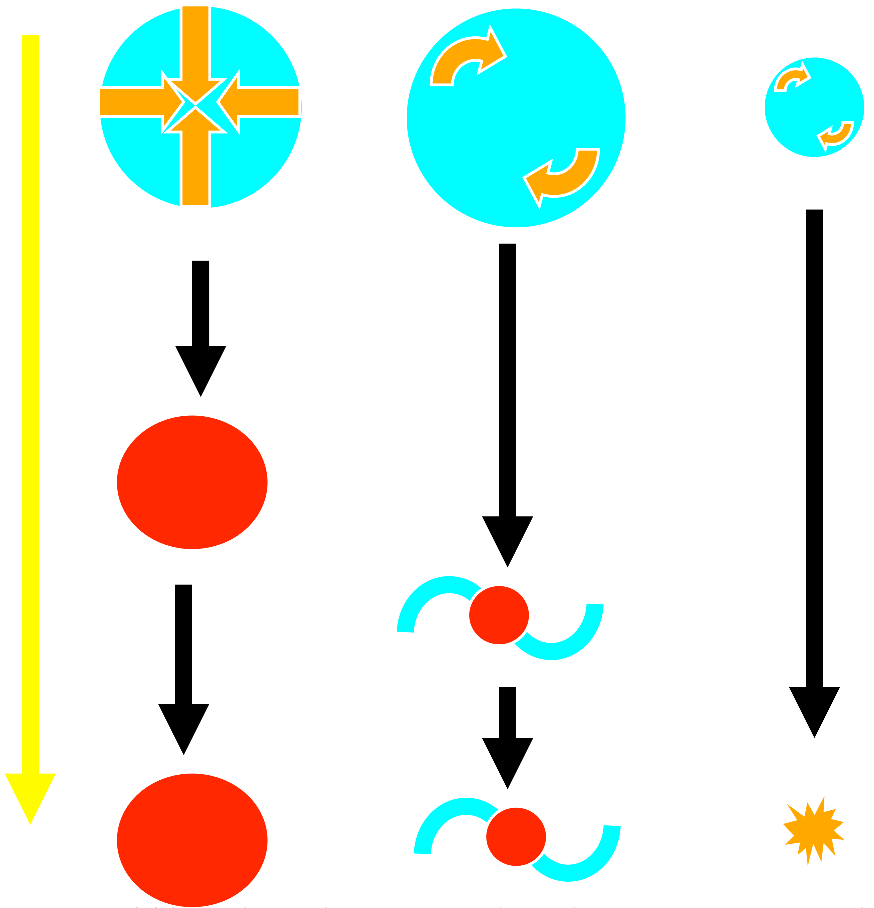
voids, walls, filaments, clusters, galaxies, …

How did Galaxies Form ?

TWO COMPETING SCENARIOS

Initial Collapse:

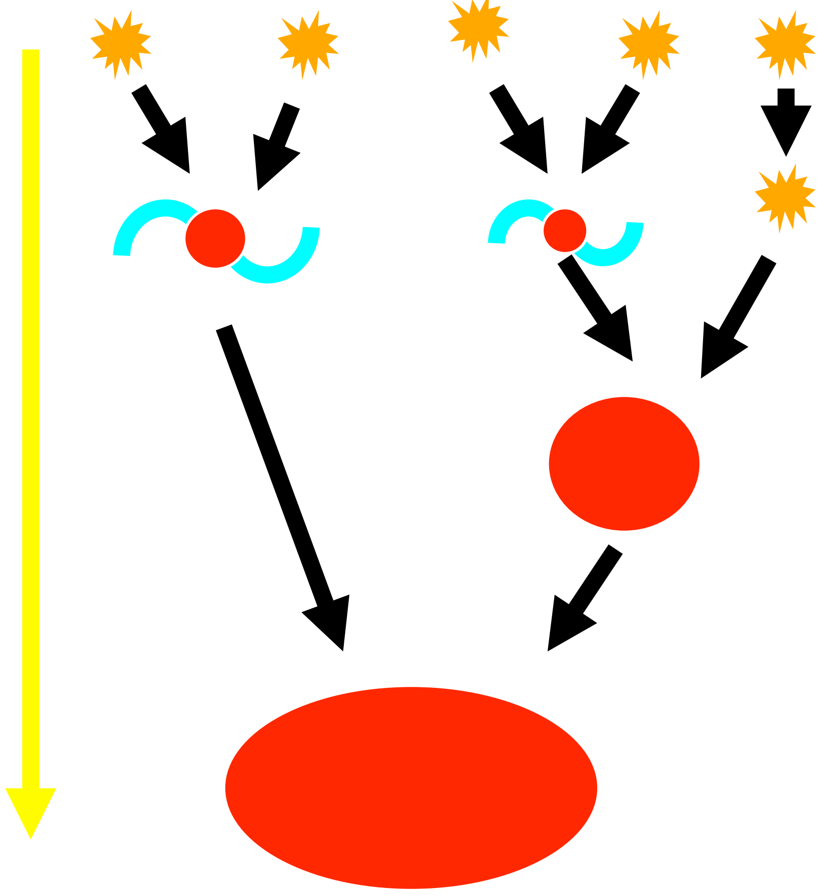
(rotation => slower collapse)



Elliptical Spiral Irregular

Hierarchical Merging:

(many small high-z galaxies)



(fewer and larger low-z galaxies)

How did Galaxies Form ?

Did over-dense regions collapse directly to form galaxies ?

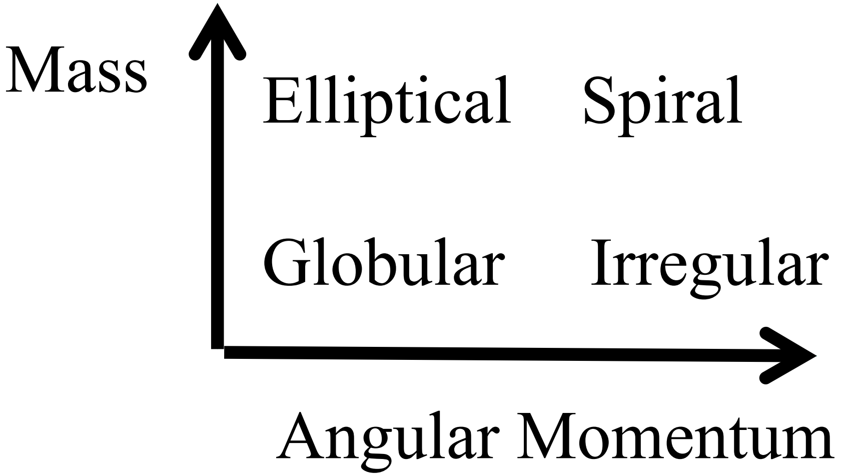
or

Did small “building blocks” form first and then merge?

Both processes clearly occur.

Initial conditions important:

Mass and Angular Momentum conserved during collapse.



Mass

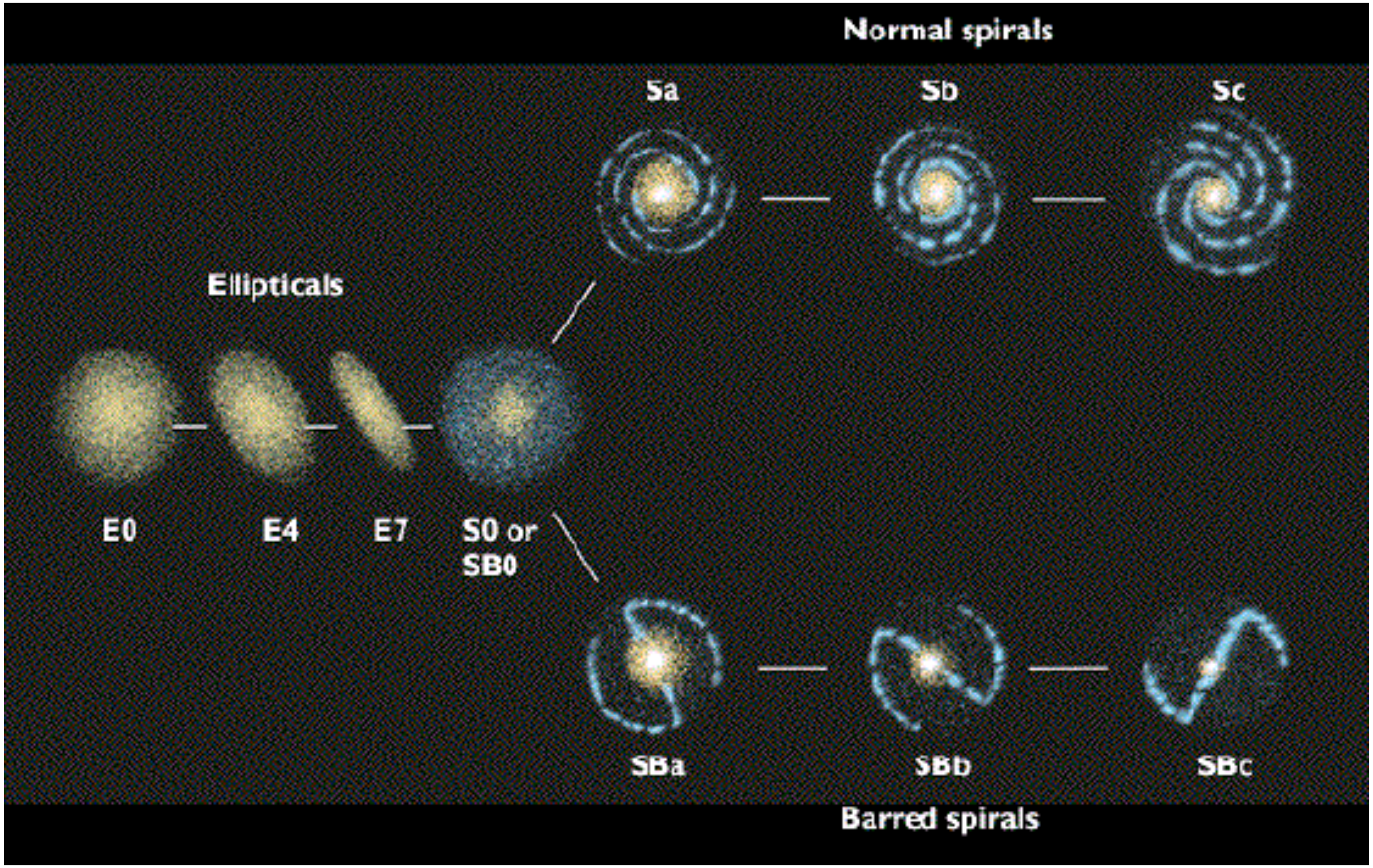
Elliptical Spiral

Globular Irregular

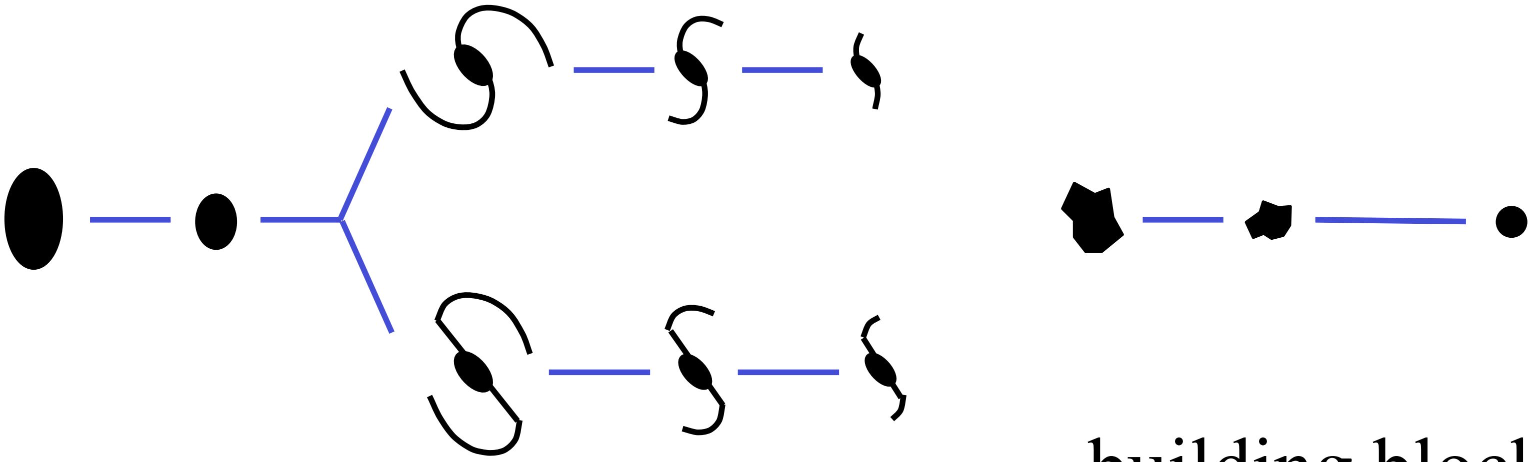
Angular Momentum

Galaxy Morphology

- Hubble’s “Tuning Fork” classification scheme.



Extended Hubble Sequence



Ellipticals

Spirals

Irregulars

Globular Clusters

low a.m.

high a.m.

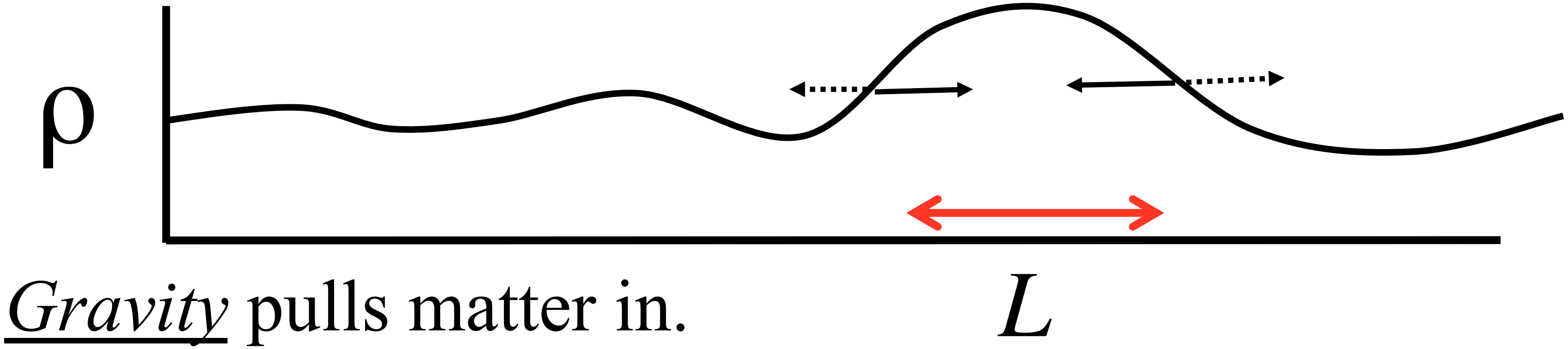
building blocks?

Galaxy formation is a topic of active research.

(We don’t yet have a complete understanding.)

Jeans' Analysis of Gravitational Stability

Which ripples will collapse ?



Gravity pulls matter in.

Pressure pushes it back out.

When pressure wins -> *oscillations* (sound waves).

When gravity wins -> *collapse*.

Cooling lowers pressure, triggers collapse.

Applies to both *Star Formation* and *Galaxy Formation*.

When does Gravity win?

N molecules of mass m in box of size L at temp T.

- Gravitational Energy: $E_G \sim -\frac{G\,M\,M}{L}$

$$M = N\,m$$
$$\sim L^3\,\rho$$

- Thermal Energy: $E_T \sim N\,k\,T$
- Ratio: $\frac{E_G}{E_T} \sim \frac{G\,M^2}{L\,N\,k\,T} \sim \frac{G\left(\rho\,L^3\right)m}{L\,k\,T} = \left(\frac{L}{L_J}\right)^2$

End up with L^2 on top. For units to balance the bottom must have same units, call this L_J^2. To collapse top must be larger than bottom.

- ***Jeans Length***: $\boxed{L_J \sim \left(\frac{k\,T}{G\,\rho\,m}\right)^{1/2}}$
- Gravity wins when $L > L_J$.

Gravity tries to pull material in.

Pressure tries to push it out.

Gravity wins for $L > L_J$

----> large regions collapse.

Pressure wins for $L < L_J$

----> small regions oscillate.

Jeans Length: $\boxed{L_J \sim \left(\frac{k\,T}{G\,\rho\,m} \right)^{1/2}}$

Large cool dense regions collapse.

Collapse Timescale

Ignore Pressure. Time to collapse = free fall time, t_G.

Gravitational acceleration:

$$g \sim \frac{GM}{L^2} \sim \frac{L}{t_G^2} \qquad\qquad M \sim L^3\ \rho$$

Time to collapse:

$$t_G \sim \sqrt{\frac{L}{g}} \sim \sqrt{\frac{L^3}{G\,M}} \sim \frac{1}{\sqrt{G\,\rho}}$$

<u>Gravitational timescale</u>, or <u>dynamical timescale</u>.

Note: *denser regions collapse faster.*

same collapse time for all sizes.

Oscillation Timescale

Ignore Gravity.

Pressure waves travel at *sound speed*.

$$c_s \sim \left(\frac{P}{\rho}\right)^{1/2} \sim \left(\frac{k\,T}{m}\right)^{1/2}$$

Ideal Gas

Sound crossing time:

$$t_s \sim \frac{L}{c_s} \sim L\left(\frac{m}{k\,T}\right)^{1/2}$$

Aside: before decoupling,

radiation pressure
>> gas pressure

$$c_s \sim \frac{1}{\sqrt{3}}\,c$$

Small hot regions oscillate more rapidly.

Ratio of Timescales

Collapse time:

$$t_G = \frac{1}{\sqrt{G\rho}}$$

Sound crossing time:

$$t_S = \frac{L}{c_S} \qquad c_s \sim \left(\frac{kT}{m}\right)^{1/2}$$

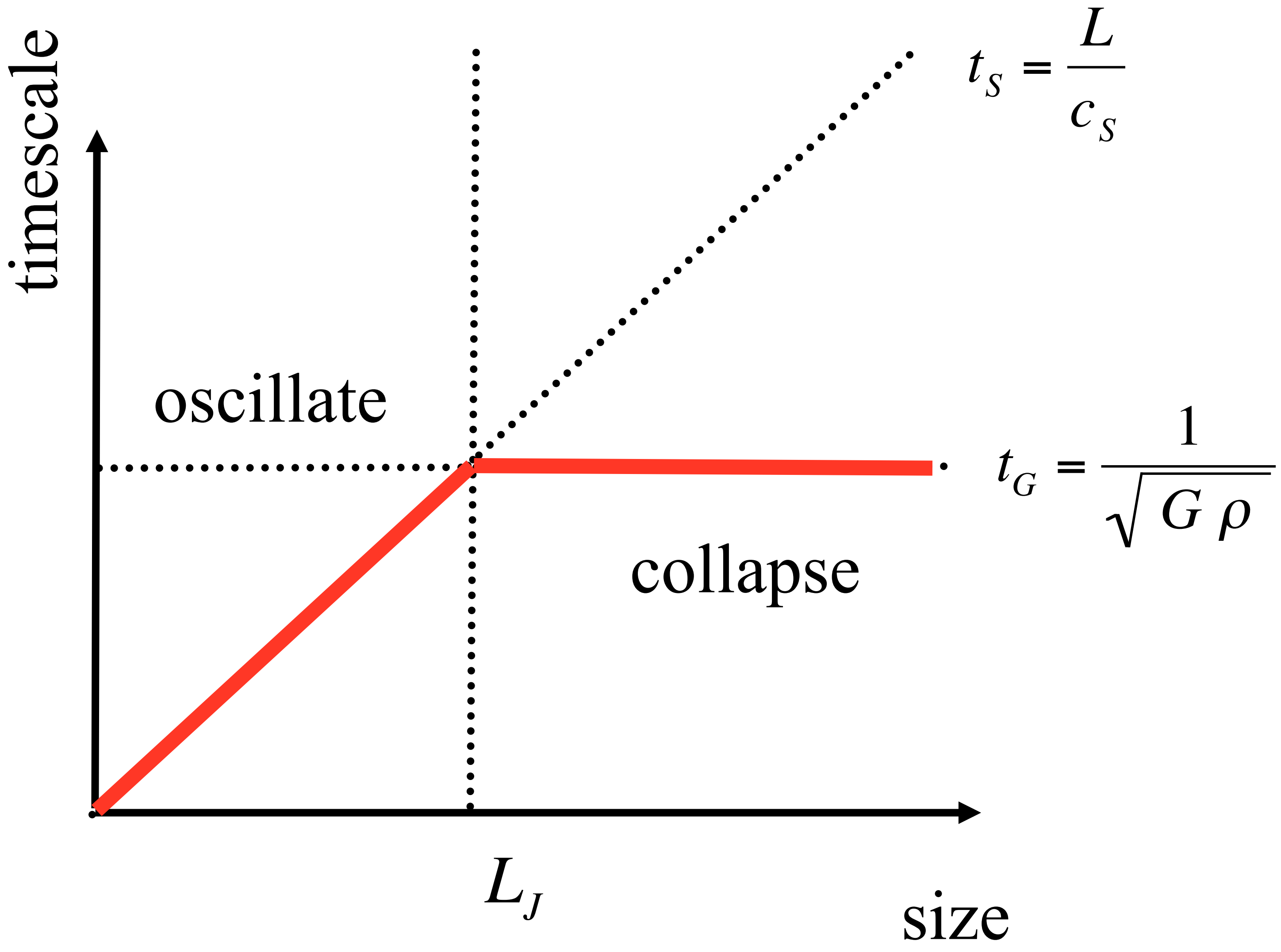
Ratio of timescales:

$$\frac{t_S}{t_G} \sim \frac{L\sqrt{G\rho}}{c_S} \sim L\left(\frac{G\rho m}{kT}\right)^{1/2} \sim \frac{L}{L_J}$$

Jeans length (again!)

$$\boxed{L_J \sim \frac{c_S}{\sqrt{G\rho}}}$$

Size Matters !



Jeans Mass and Length

Jeans Length : (smallest size that collapses)

$$L_J \sim \left(\frac{k\,T}{G\,\rho\,m} \right)^{1/2}$$

Jeans Mass: (smallest mass that collapses)

$$M_J \sim \rho\, L_J^{\ 3} \sim \rho \left(\frac{k\,T}{G\,\rho\,m} \right)^{3/2} \propto T^{3/2} \rho^{-1/2}$$

- Need cool dense regions to collapse stars,
- But galaxy-mass regions can collapse sooner.

Conditions at Decoupling

Today:

$$T_0 = 2.7\ \mathrm{K} \qquad \rho_0 = 10^{-28}\ \mathrm{kg\ m^{-3}}$$

Expanding Universe (matter dominated):

$$T \propto R^{-1} \qquad \rho \propto R^{-3} \propto T^3$$

At decoupling: $T = 3000\ \mathrm{K}$

$$\rho = 10^{-28}\left(\frac{3000}{2.7}\right)^3 = 1.4 \times 10^{-19}\ \mathrm{kg\ m^{-3}}$$

$$\Rightarrow 2\ \mathrm{M_{sun}\ pc^{-3}}$$

Size and Mass of first Galaxies

$$T = 3000 \text{ K}$$

$$\rho = 1.4 \times 10^{-19} \text{ kg m}^{-3} \Rightarrow 2 \text{ M}_{\text{sun}} \text{ pc}^{-3}$$

Jeans Length :

$$L_J \sim \left(\frac{k\,T}{G\,\rho\,m} \right)^{1/2} = \left(\frac{\left(1.4 \times 10^{-23} \text{J K}^{-1}\right)\left(3000\text{K}\right)}{\left(6.7 \times 10^{-11} \text{m}^3 \text{ kg}^{-1} \text{ s}^{-2}\right)\left(1.4 \times 10^{-19} \text{kg m}^{-3}\right)\left(1.7 \times 10^{-27} \text{kg}\right)} \right)^{1/2}$$

$$= \frac{1.6 \times 10^{18} \text{ m}}{3.2 \times 10^{16} \text{ m/pc}} = \underline{50 \text{ pc}}$$

Jeans Mass:

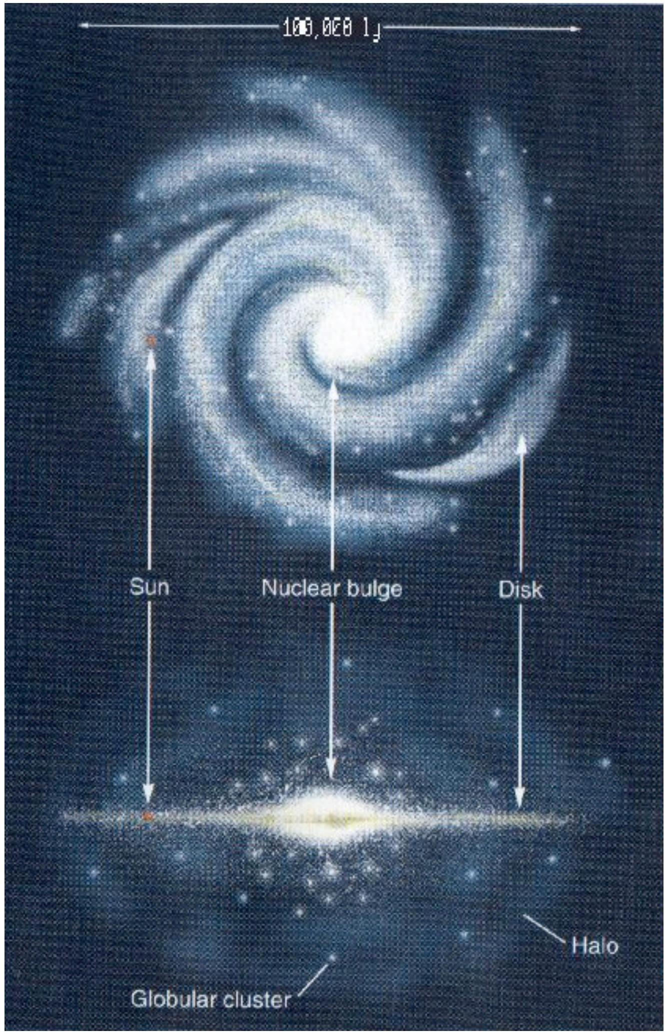
$$M_J \sim \rho\, L_J^{\,3} \sim \left(2 \text{ M}_{\text{sun}} \text{ pc}^{-3}\right)\left(50 \text{ pc}\right)^3$$

$$= \underline{3 \times 10^5 \text{ M}_{\text{sun}}}$$

More than a star, less than a galaxy,
close to a globular cluster mass.

Globular clusters in the Milky Way

Hold the oldest stars.



Orbit in the Halo.

Time to form first galaxies

At decoupling:

$$\rho = 1.4 \times 10^{-19} \text{ kg m}^{-3}$$

Collapse timescale:

$$t_G \sim \frac{1}{\sqrt{G\rho}} = 3.3 \times 10^{14} \text{ s} = 10^7 \text{yr}$$

Expect first "galaxies" ($M > 3\text{x}10^5$ M_{sun})
to form $\sim 10^7$ yr after decoupling.

Jeans Analysis: Scaling with Expansion factor

$T \propto R^{-1} \qquad \rho \propto R^{-3} \propto T^3 \qquad R \propto t^{2/3}$ From earlier lectures

Jeans mass: $$M_J \propto \left(\frac{T^3}{\rho} \right)^{1/2} \propto R^0$$

Jeans length: $$L_J \propto \left(\frac{T}{\rho} \right)^{1/2} \propto R^{+1} \propto t^{2/3}$$

Collapse time: $$t_J = \left(\frac{1}{G\,\rho} \right)^{1/2} \propto R^{+3/2} \propto t$$

Collapse time scales with expansion time,
so actual collapse takes longer.

Summary

Over-dense regions collapse after decoupling

IF large enough i.e. $L > L_J$ $M > M_J$

Large mass --> Giant Elliptical

Smaller mass --> Dwarf Galaxy

Smallest that collapse: globular clusters

Tiny regions stable: can't form stars (yet).

We enter the "Dark Ages"