Lecture 6: Jeans mass & length

Anisotropies in the CMB temperature

→ density ripples $\frac{\Delta\rho}{\rho} \sim \frac{\Delta T}{T} \sim 10^{-5}$

at the time of decoupling (z = 1100) .

These are the seeds that evolve (gravitational collapse) to form the structured distribution of galaxies we see around us today:

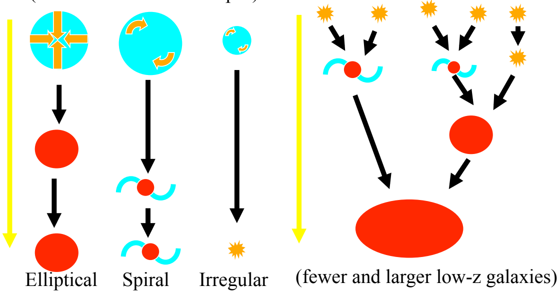
voids, walls, filaments, clusters, galaxies, …

How did Galaxies Form ?

TWO COMPETING SCENARIOS

Initial Collapse: (rotation => slower collapse)

Hierarchical Merging: (many small high-z galaxies)



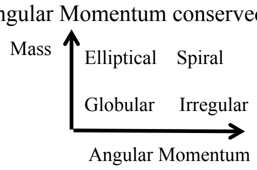
How did Galaxies Form ?

Did over-dense regions collapse directly to form galaxies ?
or
Did small "building blocks" form first and then merge?

Both processes clearly occur.

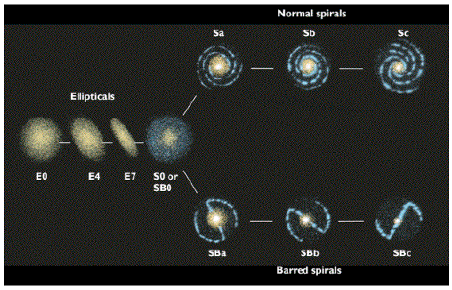
Initial conditions important:

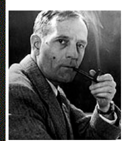
Mass and Angular Momentum conserved during collapse.



Galaxy Morphology

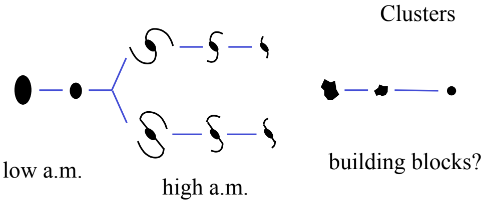
- Hubble's "Tuning Fork" classification scheme.





Extended Hubble Sequence

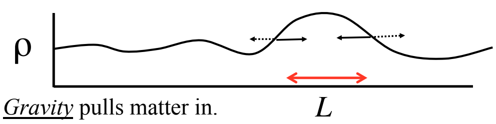
Ellipticals Spirals Irregulars Globular Clusters



Galaxy formation is a topic of active research.
(We don't yet have a complete understanding.)

Jeans' Analysis of Gravitational Stability

Which ripples will collapse ?



Gravity pulls matter in.
Pressure pushes it back out.
When pressure wins -> *oscillations* (sound waves).
When gravity wins -> *collapse*.
Cooling lowers pressure, triggers collapse.
Applies to both *Star Formation* and *Galaxy Formation*.

When does Gravity win?

N molecules of mass *m* in box of size *L* at temp *T*.

- Gravitational Energy: $E_G \sim -\frac{G\,M\,M}{L}$ $\quad M = N\,m \sim L^3\,\rho$
- Thermal Energy: $E_T \sim N\,k\,T$
- Ratio: $\frac{E_G}{E_T} \sim \frac{G\,M^2}{L\,N\,k\,T} \sim \frac{G\left(\rho\,L^3\right)m}{L\,k\,T} = \left(\frac{L}{L_J}\right)^2$

  End up with L^2 on top. For units to balance the bottom must have same units, call this L_J^2. To collapse top must be larger than bottom.
- ***Jeans Length***: $L_J \sim \left(\frac{k\,T}{G\,\rho\,m}\right)^{1/2}$
- Gravity wins when $L > L_J$.

Gravity tries to pull material in.

Pressure tries to push it out.

Gravity wins for $L > L_J$

----> *large regions collapse*.

Pressure wins for $L < L_J$

----> *small regions oscillate*.

Jeans Length: $L_J \sim \left(\frac{k\,T}{G\,\rho\,m}\right)^{1/2}$

Large cool dense regions collapse.

Collapse Timescale

Ignore Pressure. Time to collapse = free fall time, t_G.

Gravitational acceleration:

$$g \sim \frac{GM}{L^2} \sim \frac{L}{t_G^2} \qquad M \sim L^3\,\rho$$

Time to collapse:

$$t_G \sim \sqrt{\frac{L}{g}} \sim \sqrt{\frac{L^3}{G\,M}} \sim \frac{1}{\sqrt{G\,\rho}}$$

Gravitational timescale, or dynamical timescale.

Note: *denser regions collapse faster.*

same collapse time for all sizes.

Oscillation Timescale

Ignore Gravity.

Pressure waves travel at *sound speed*.

$$c_S \sim \left(\frac{P}{\rho}\right)^{1/2} \sim \left(\frac{k\,T}{m}\right)^{1/2}$$

Ideal Gas

Sound crossing time:

$$t_S \sim \frac{L}{c_S} \sim L\left(\frac{m}{k\,T}\right)^{1/2}$$

Aside: before decoupling,
radiation pressure >> gas pressure
$c_S \sim \frac{1}{\sqrt{3}}\,c$

Small hot regions oscillate more rapidly.

Ratio of Timescales

Collapse time: $t_G = \frac{1}{\sqrt{G\,\rho}}$

Sound crossing time: $t_S = \frac{L}{c_S}$ $\quad c_s \sim \left(\frac{k\,T}{m}\right)^{1/2}$

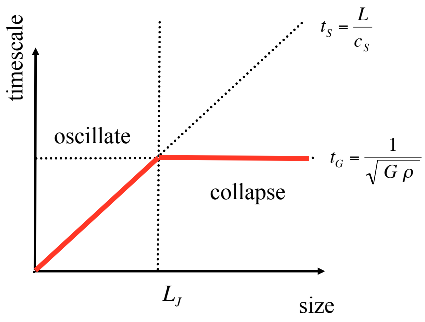
Ratio of timescales:

$$\frac{t_S}{t_G} \sim \frac{L\sqrt{G\,\rho}}{c_S} \sim L\left(\frac{G\,\rho\,m}{k\,T}\right)^{1/2} \sim \frac{L}{L_J}$$

Jeans length (again!)

$$L_J \sim \frac{c_S}{\sqrt{G\,\rho}}$$

Size Matters !



Jeans Mass and Length

Jeans Length : (smallest size that collapses)

$$L_J \sim \left(\frac{k\,T}{G\,\rho\,m}\right)^{1/2}$$

Jeans Mass: (smallest mass that collapses)

$$M_J \sim \rho\,L_J^{\;3} \sim \rho\left(\frac{k\,T}{G\,\rho\,m}\right)^{3/2} \propto T^{3/2}\rho^{-1/2}$$

- Need cool dense regions to collapse stars,
- But galaxy-mass regions can collapse sooner.

Conditions at Decoupling

Today:

$$T_0 = 2.7\text{ K} \qquad \rho_0 = 10^{-28}\text{ kg m}^{-3}$$

Expanding Universe (matter dominated):

$$T \propto R^{-1} \qquad \rho \propto R^{-3} \propto T^3$$

At decoupling: $T = 3000\text{ K}$

$$\rho = 10^{-28}\left(\frac{3000}{2.7}\right)^3 = 1.4\times10^{-19}\text{ kg m}^{-3}$$

$$\Rightarrow 2\text{ M}_{\text{sun}}\text{ pc}^{-3}$$

Size and Mass of first Galaxies

$T = 3000$ K

$\rho = 1.4\times10^{-19}\text{ kg m}^{-3} \;\Rightarrow\; 2\text{ M}_{\text{sun}}\text{ pc}^{-3}$

Jeans Length :

$$L_J \sim \left(\frac{k\,T}{G\,\rho\,m}\right)^{1/2} = \left(\frac{\left(1.4\times10^{-23}\,\text{J K}^{-1}\right)\left(3000\text{K}\right)}{\left(6.7\times10^{-11}\text{m}^3\text{ kg}^{-1}\text{ s}^{-2}\right)\left(1.4\times10^{-19}\text{kg m}^{-3}\right)\left(1.7\times10^{-27}\text{kg}\right)}\right)^{1/2}$$

$$= \frac{1.6\times10^{18}\text{ m}}{3.2\times10^{16}\text{ m/pc}} = \underline{50\text{ pc}}$$

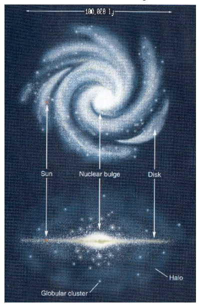
Jeans Mass:

$$M_J \sim \rho\,L_J^{\;3} \sim \left(2\text{ M}_{\text{sun}}\text{ pc}^{-3}\right)\left(50\text{ pc}\right)^3$$

$$= \underline{3\times10^5\text{ M}_{\text{sun}}}$$

More than a star, less than a galaxy,
close to a globular cluster mass.

Globular clusters in the Milky Way



Hold the oldest stars. Orbit in the Halo.

Time to form first galaxies

At decoupling:

$$\rho = 1.4\times10^{-19}\text{ kg m}^{-3}$$

Collapse timescale:

$$t_G \sim \frac{1}{\sqrt{G\rho}} = 3.3\times10^{14}\text{ s} = 10^7\text{ yr}$$

Expect first "galaxies" ($M > 3\text{x}10^5\text{ M}_{\text{sun}}$)
to form $\sim10^7$ yr after decoupling.

Jeans Analysis: Scaling with Expansion factor

$T \propto R^{-1} \qquad \rho \propto R^{-3} \propto T^3 \qquad R \propto t^{2/3}$ From earlier lectures

$$\text{Jeans mass}: \quad M_J \propto \left(\frac{T^3}{\rho}\right)^{1/2} \propto R^0$$

$$\text{Jeans length}: \quad L_J \propto \left(\frac{T}{\rho}\right)^{1/2} \propto R^{+1} \propto t^{2/3}$$

$$\text{Collapse time}: \quad t_J = \left(\frac{1}{G\,\rho}\right)^{1/2} \propto R^{+3/2} \propto t$$

Collapse time scales with expansion time,
so actual collapse takes longer.

Summary

Over-dense regions collapse after decoupling

IF large enough i.e. $L > L_J$ $M > M_J$

Large mass --> Giant Elliptical

Smaller mass --> Dwarf Galaxy

Smallest that collapse: globular clusters

Tiny regions stable: can't form stars (yet).

We enter the "Dark Ages"