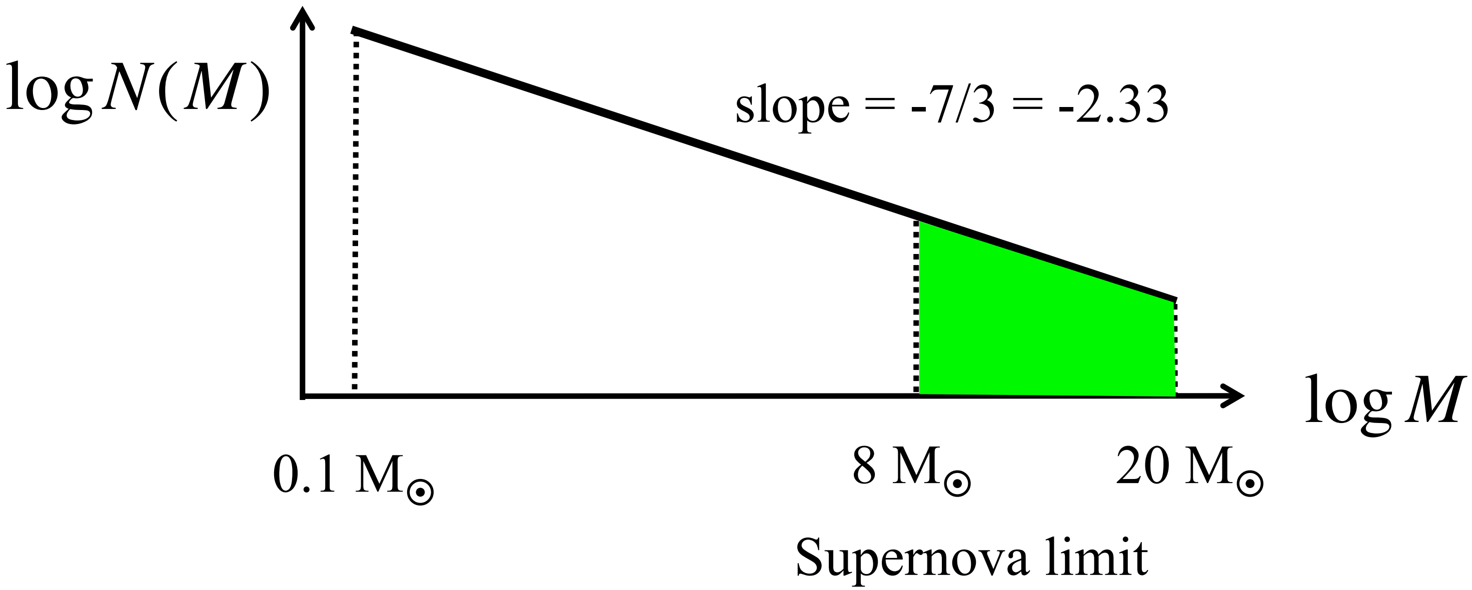
# *Lecture 9: Supernova Rates*
# *Star-Formation Efficiency, Yield*

**How many supernovae per year** for each galaxy type ?

Use power-law IMF, Salpeter slope -7/3 = -2.33

$$N(M) \propto M^{-7/3} \Big|_{0.1 M_\odot}^{20 M_\odot}$$

Limits of validity, not well known



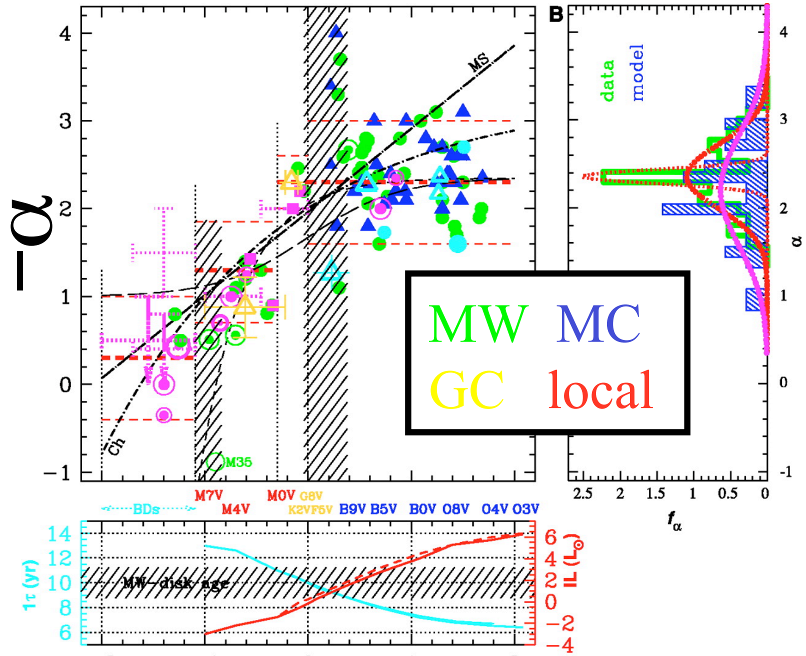
# “Universal” IMF (Kroupa 2002)

$$N(M) \propto M^{\alpha}$$

$\alpha \sim$ - 7/3 $\quad M > 1\ M_\odot$

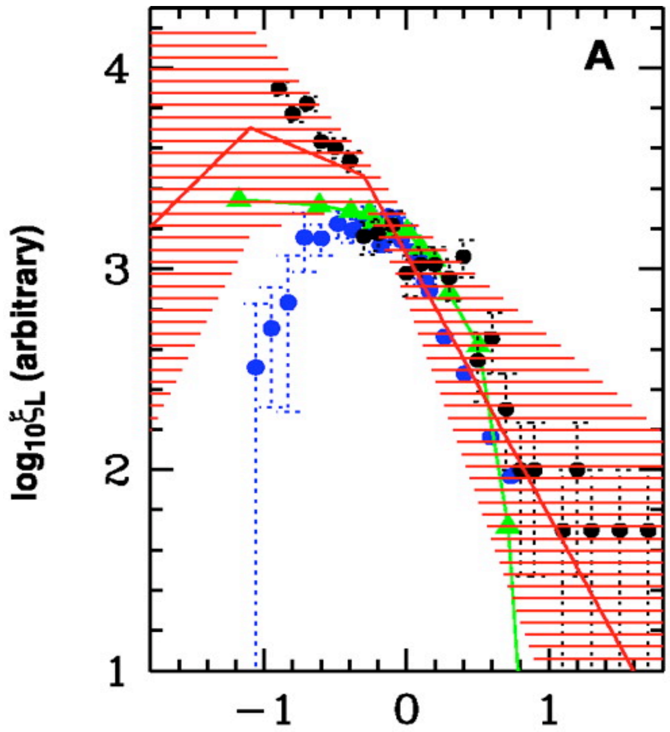
- 4/3 $\quad 0.1 - 1\ M_\odot$

- 1/3 $\quad M < 0.1\ M_\odot$




log( $M$ / $M_\odot$ )

M42 M35 Pleiades local




log( $M$ / $M_\odot$)

# *Integrating a Power-Law IMF*

**Number of stars :**

$$N = \int N(M)\, dM = A \int M^{B}\, dM = \frac{A}{B+1} M^{B+1} \quad (\text{ if } \; B \neq -1)$$

**Fraction of stars with *M* > *8* M$_\odot$** ( for *B* = -7/3 )

$$f_N \equiv \frac{\text{number of SNe}}{\text{number of stars}} = \frac{\int_8^{20} M^B\, dM}{\int_{0.1}^{20} M^B\, dM}$$

$$f_N = \frac{\left.\frac{A}{B+1} M^{B+1}\right|_8^{20}}{\left.\frac{A}{B+1} M^{B+1}\right|_{0.1}^{20}} = \frac{\left. M^{-4/3}\right|_8^{20}}{\left. M^{-4/3}\right|_{0.1}^{20}} = \frac{0.018 - 0.063}{0.018 - 21.544}$$

Most stars at low-mass end!

500 stars --> 1 supernova!

$$\Rightarrow \qquad \boxed{f_N = 0.2\%}$$

# *SN Mass Fraction*

Supernovae are rare, but each is very massive.

**What fraction of the *mass* goes into SNe?**

$$f_M = \frac{\int_8^{20} M \times M^{-7/3}\, dM}{\int_{0.1}^{20} M \times M^{-7/3}\, dM}$$

$$= \frac{M^{-1/3}\Big|_8^{20}}{M^{-1/3}\Big|_{0.1}^{20}} = \frac{0.37\text{-}0.50}{0.37 - 2.15}$$

Most of mass is in low-mass stars.

$$\Rightarrow \quad \boxed{f_M = 7.2\%}$$

# “Typical” SN Mass

**Median mass:**

$$\frac{1}{2} = \frac{\int_8^{\bar{M}_{SN}} M \times M^{-7/3}\, dM}{\int_8^{20} M \times M^{-7/3}\, dM} = \frac{\bar{M}_{SN}^{-1/3} - 0.50}{0.37 - 0.50}$$

$$\Rightarrow \quad \boxed{\bar{M}_{SN} = 12.2\ \mathrm{M}_\odot}$$

**Mean mass:**

$$\langle M \rangle = \frac{\int_8^{20} M \times M^{-7/3}\, dM}{\int_8^{20} M^{-7/3}\, dM} = \frac{\frac{1}{-1/3}\ M^{-1/3}\Big|_8^{20}}{\frac{1}{-4/3} M^{-4/3}\Big|_8^{20}}$$

$$= \frac{4 \times (20^{-1/3} - 8^{-1/3})}{20^{-4/3} - 8^{-4/3}} = \frac{4 \times (0.37 - 0.50)}{0.018 - 0.062} = \boxed{12\,\mathrm{M}_\odot}$$

# *SN Rates vs Galaxy Type*

**Spiral Galaxy:** SFR: $\sim 8\ M_\odot\ yr^{-1}$. 7.2% have $M > 8\ M_\odot$ .

$\Rightarrow$ $(8\ M_\odot\ yr^{-1}) \times 0.072 \sim 0.6\ M_\odot\ yr^{-1}$ go into SNe

**SN rate:**

$$\frac{0.6\ M_\odot yr^{-1}}{12.2\ M_\odot} \sim \frac{1}{20}\ yr^{-1}$$ (fewer seen due to dust)

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**Irregular Galaxy**: ~10x this rate during bursts (1 SN per 2 yr)!

No SNe between bursts.

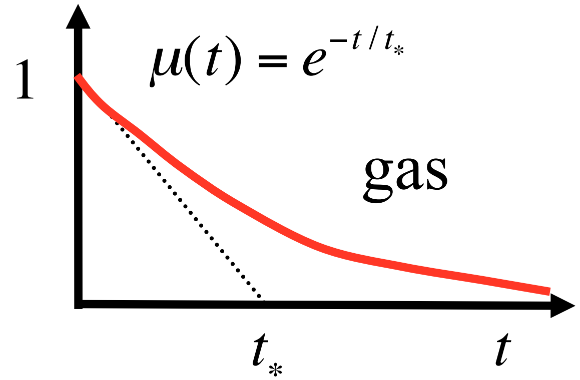
# *SN Rates: Ellipticals*

$t_* = 1$ Gyr  e-folding time

$t = 10$ Gyr  age

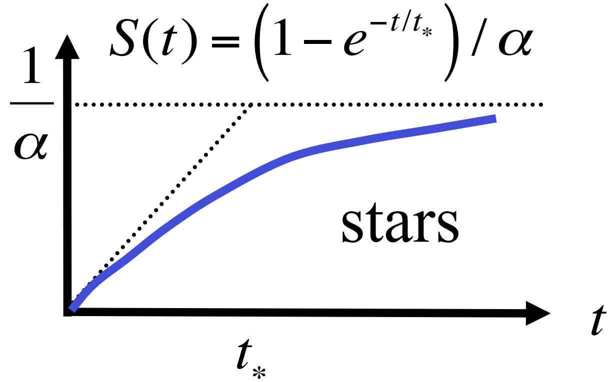
$\alpha = 0.95$  efficiency

$M_0 = 10^{11}\ M_\odot$  total mass = initial gas mass




**Gas consumption:**

$$M_G(t) = M_0\, e^{-t/t_*} = M_0 - \alpha\, M_S(t)$$




**Star formation:** $M_S(t) = \frac{M_0}{\alpha}\left(1 - e^{-t/t_*}\right)$

$$\frac{dM_S}{dt} \equiv \dot{M}_S = \frac{M_0}{\alpha}\frac{e^{-t/t_*}}{t_*} = \frac{\left(10^{11} M_\odot\right) e^{-10}}{(0.95)\,(10^9\,\mathrm{yr})} = 5 \times 10^{-3}\ M_\odot\ \mathrm{yr}^{-1}$$

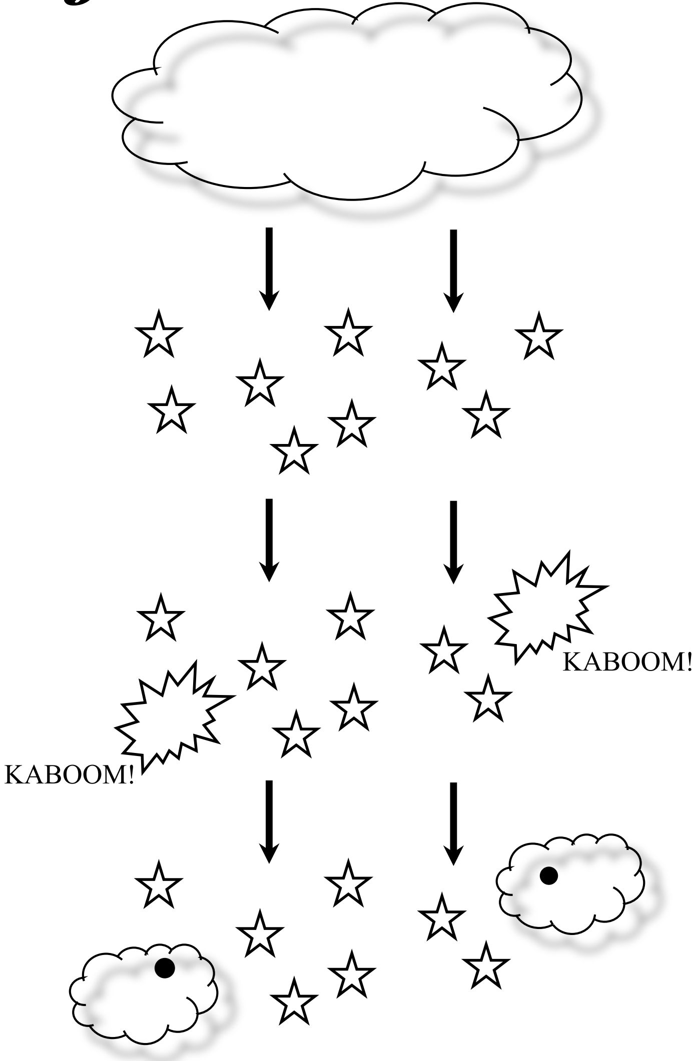
**SN rate:** $\frac{(0.072)\left(5 \times 10^{-3} M_\odot \mathrm{yr}^{-1}\right)}{12.2\ M_\odot} \approx 3 \times 10^{-5}\ \mathrm{yr}^{-1}$

3 SN per $10^5$ yr.  Negligible!

# *What Star Formation Efficiency $\alpha$ and Yields of H, He and Metals ?*

$M_G = M_0$
$M_S = 0$

$X = 0.75$
$Y = 0.25$
$Z = 0.00$




$M_G = 0$
$M_S = M_0$

$M_G = (1-\alpha)\, M_0$
$M_S = \alpha\, M_0$
$\alpha = ?$

$X = ?$
$Y = ?$
$Z = ?$

Estimates for efficiency $\alpha$ , yield in X, Y, Z

Assume:

1. Type-II SNe enrich the ISM.
   (Neglect: Type-I SNe, stellar winds, PNe, ....)
2. Closed Box Model:
   (Neglect: Infall from the IGM, outflow to the IGM)
3. SN 1987A is typical Type-II SN.

Better models include these effects.

What do we know about SN 1987A?

# *SN 1987A*

23 Feb 1987 in LMC

Brightest SN since 1604!

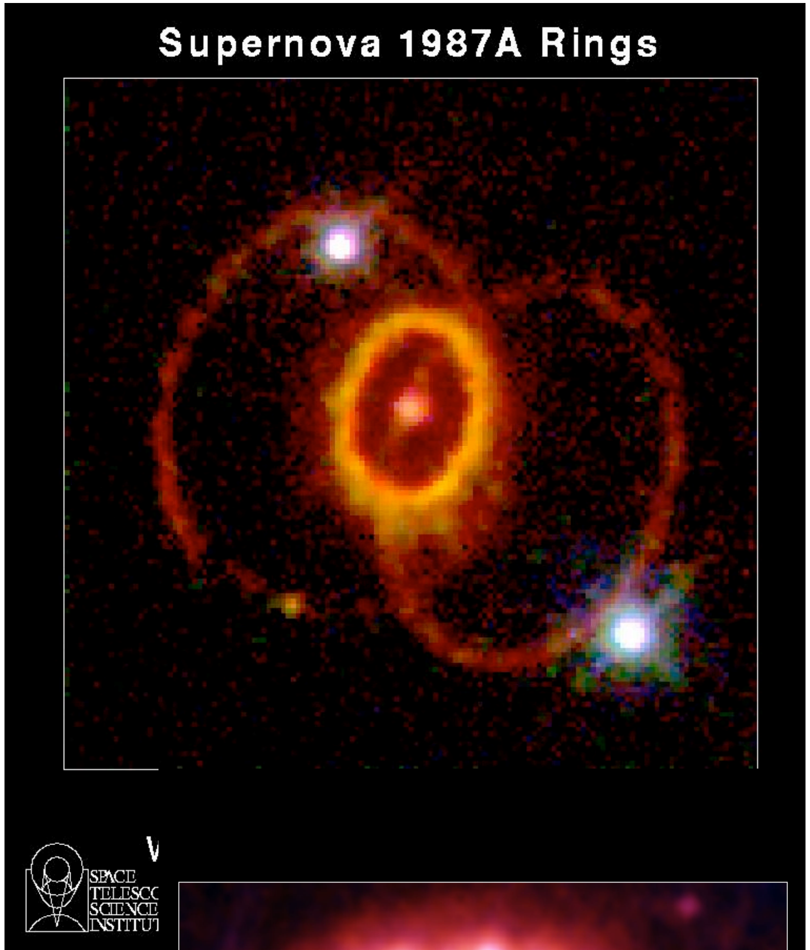
First SN detected in neutrinos.

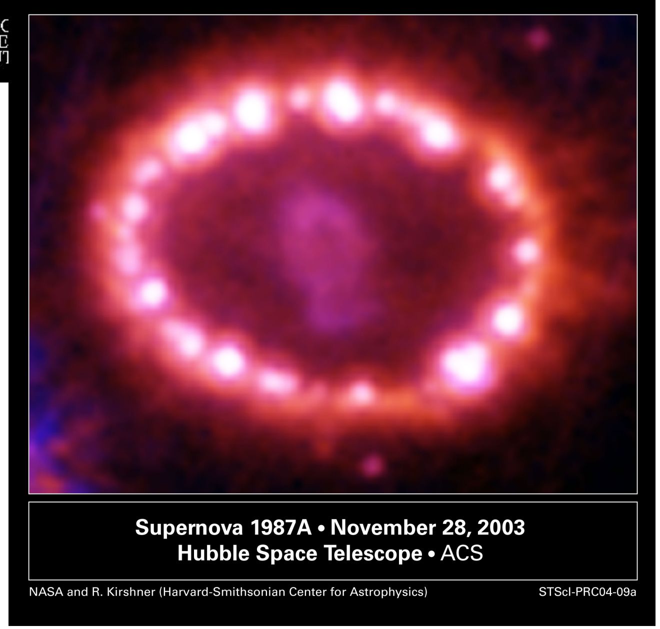
Visible (14 --> 4.2 mag) to naked eye, in southern sky.

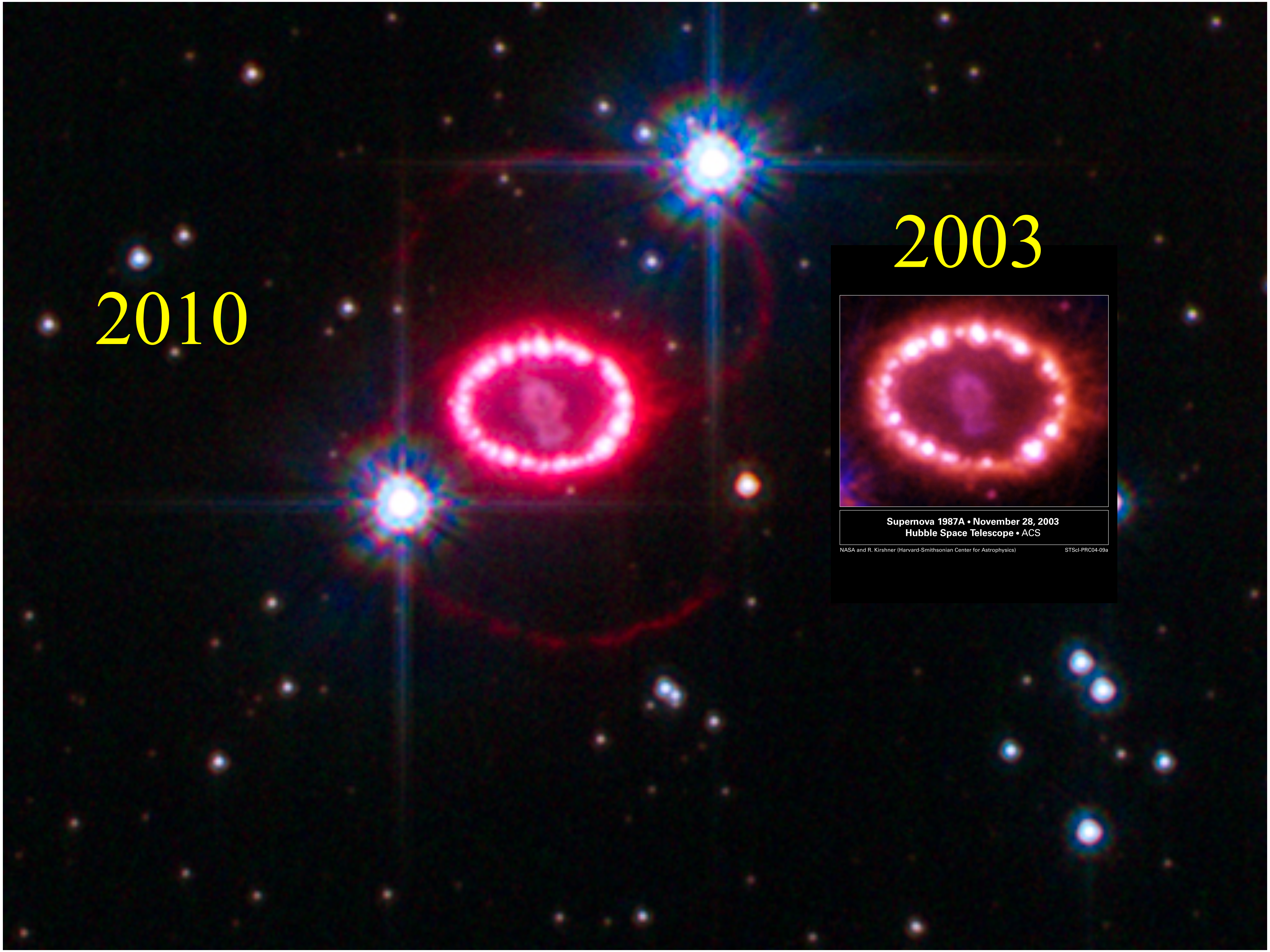
Progenitor star visible:
~20 Msun blue supergiant.

3- ring structure (pre-SN wind)

UV flash reached inner ring in 80 d.
Fastest ejecta reached inner ring in ~6 yr.
Fast ejection velocity v~c/30~11,000 km/s.
Slower (metal-enriched) ejecta asymmetric.




2010
2003
Supernova 1987A • November 28, 2003
Hubble Space Telescope • ACS
NASA and R. Kirshner (Harvard-Smithsonian Center for Astrophysics)
STScI-PRC04-09a

# *Star Formation Efficiency*

Use SN 1987A to calculate α and yield.

SN 1987A: progenitor star mass = 20 $M_\odot$
remnant neutron star mass = 1.6 $M_\odot$
mass returned to the ISM = 18.4 $M_\odot$

From IMF, 7.2% of $M_S$ is in stars with $M > 8\ M_\odot$

β = Fraction of $M_S$ returned to ISM:

$$\beta = \frac{\text{mass returned to gas}}{\text{mass turned into stars}} = 0.072 \times \frac{18.4}{20} \approx 6.6\%$$

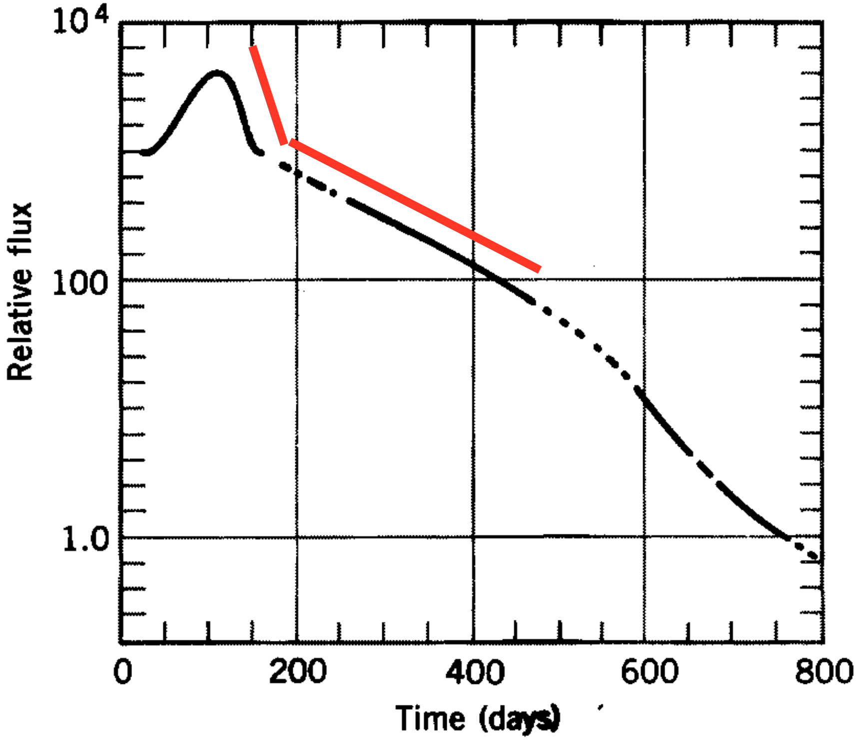
Star Formation Efficiency
α = fraction of $M_S$ retained in stars:

$$\alpha = 1 - \beta = 93\%$$

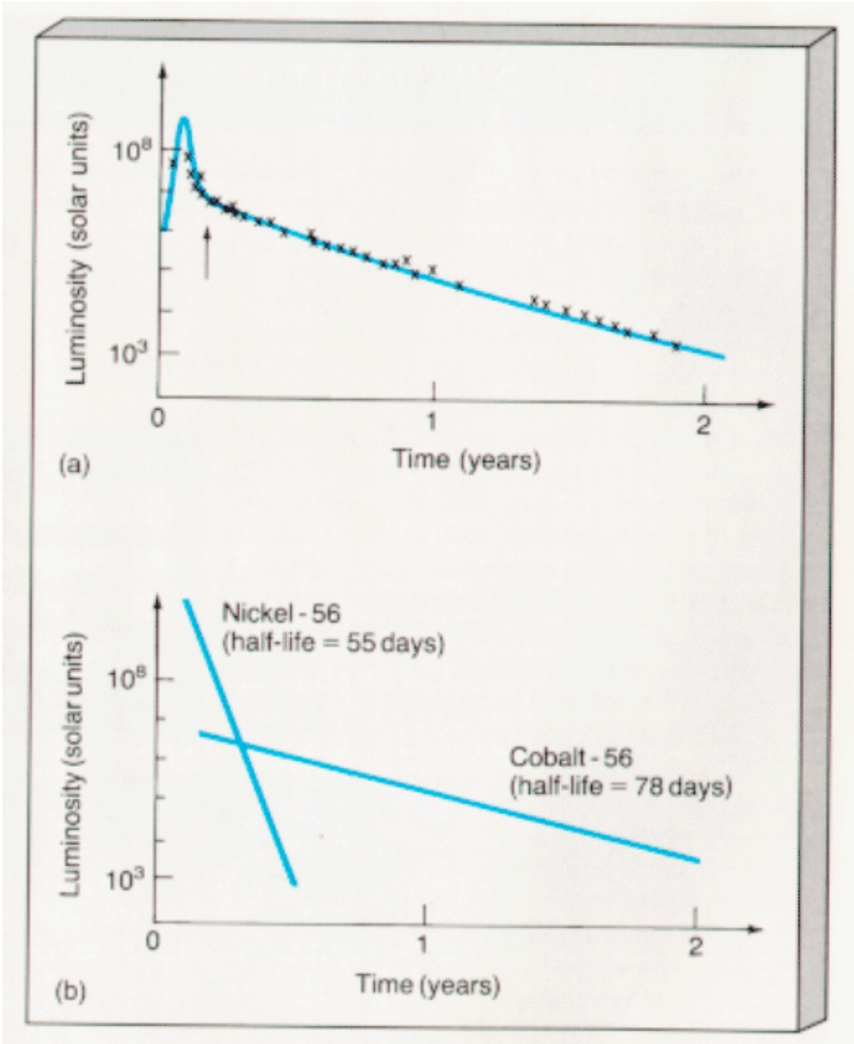
# *SN 1987A Lightcurve*

Powered by radioactive decay of r-process nuclei.
Use to measure metal abundances in ejected gas.

$^{56}Ni \Rightarrow {}^{56}Co$ 6d half-life
$^{56}Co \Rightarrow {}^{56}Fe$ 78d half-life




**Light curve of the Supernova from the first day of the visible outburst until May 16, 1988, some 800 days later.**



# *X, Y, Z of ejecta from SN1987A*

| | |
|---|---|
| Initial mass | ~ 20 $M_\odot$ |
| NS mass | ~ 1.6 $M_\odot$ |
| Mass ejected | ~ 18.4 $M_\odot$ |

$$\left.\begin{array}{ll} \text{in H} & 9.0\ M_\odot \\ \text{He} & 7.0\ M_\odot \\ \text{Z} & 2.4\ M_\odot \end{array}\right\} = 18.4\ M_\odot$$

$$\Rightarrow \quad X = \frac{9}{18.4} \approx 0.49 \qquad Y = \frac{7}{18.4} \approx 0.38$$

$$Z = \frac{2.4}{18.4} \approx 0.13$$