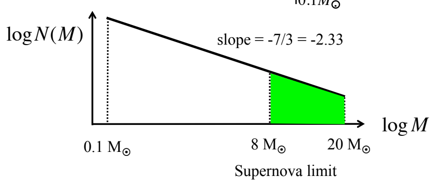
Lecture 9: Supernova Rates
Star-Formation Efficiency, Yield

How many supernovae per year for each galaxy type ?

Use power-law IMF, Salpeter slope -7/3 = -2.33

$$N(M) \propto M^{-7/3} \Big|_{0.1 M_\odot}^{20 M_\odot}$$

Limits of validity, not well known



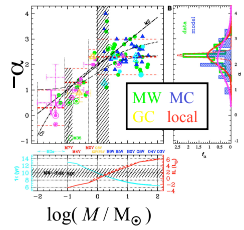
"Universal" IMF (Kroupa 2002)

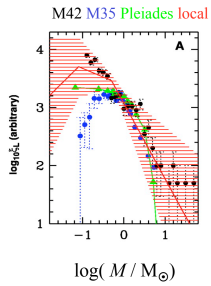
$\boxed{N(M) \propto M^{\alpha}}$

α ~ - 7/3 $\quad M > 1$ M$_\odot$

- 4/3 $\quad$ 0.1 - 1 M$_\odot$

- 1/3 $\quad M < 0.1$ M$_\odot$





Integrating a Power-Law IMF

Number of stars :

$$N = \int N(M)\, dM = A \int M^B\, dM = \frac{A}{B+1} M^{B+1} \quad (\text{ if } \; B \neq -1)$$

Fraction of stars with $M > 8$ M$_\odot$ (for B = -7/3)

$$f_N \equiv \frac{\text{number of SNe}}{\text{number of stars}} = \frac{\int_8^{20} M^B\, dM}{\int_{0.1}^{20} M^B\, dM}$$

$$f_N = \frac{\frac{A}{B+1} M^{B+1} \Big|_8^{20}}{\frac{A}{B+1} M^{B+1} \Big|_{0.1}^{20}} = \frac{M^{-4/3}\Big|_8^{20}}{M^{-4/3}\Big|_{0.1}^{20}} = \frac{0.018 - 0.063}{0.018 - 21.544}$$

Most stars at low-mass end!

500 stars --> 1 supernova!

$\Rightarrow \quad \boxed{f_N = 0.2\%}$

SN Mass Fraction

Supernovae are rare, but each is very massive.

What fraction of the <u>*mass*</u> goes into SNe?

$$f_M = \frac{\int_8^{20} M \times M^{-7/3}\, dM}{\int_{0.1}^{20} M \times M^{-7/3}\, dM}$$

$$= \frac{M^{-1/3}\Big|_8^{20}}{M^{-1/3}\Big|_{0.1}^{20}} = \frac{0.37\text{-}0.50}{0.37 - 2.15}$$

Most of mass is in low-mass stars.

$\Rightarrow \quad \boxed{f_M = 7.2\%}$

"Typical" SN Mass

Median mass:

$$\frac{1}{2} = \frac{\int_8^{\bar{M}_{SN}} M \times M^{-7/3}\, dM}{\int_8^{20} M \times M^{-7/3}\, dM} = \frac{\bar{M}_{SN}^{-1/3} - 0.50}{0.37 - 0.50}$$

$\Rightarrow \quad \boxed{\bar{M}_{SN} = 12.2\,\mathrm{M}_\odot}$

Mean mass:

$$\langle M \rangle = \frac{\int_8^{20} M \times M^{-7/3}\, dM}{\int_8^{20} M^{-7/3}\, dM} = \frac{\frac{1}{-1/3} M^{-1/3}\Big|_8^{20}}{\frac{1}{-4/3} M^{-4/3}\Big|_8^{20}}$$

$$= \frac{4 \times (20^{-1/3} - 8^{-1/3})}{20^{-4/3} - 8^{-4/3}} = \frac{4 \times (0.37 - 0.50)}{0.018 - 0.062} = \boxed{12\,\mathrm{M}_\odot}$$

SN Rates vs Galaxy Type

Spiral Galaxy: SFR: ~ 8 M$_\odot$ yr^{-1}. 7.2% have $M > 8$ M$_\odot$.

$\Rightarrow$ (8 M$_\odot$ yr^{-1}) x 0.072 ~ 0.6 M$_\odot$ yr^{-1} go into SNe

SN rate:

$$\frac{0.6\ \mathrm{M}_\odot \mathrm{yr}^{-1}}{12.2\ \mathrm{M}_\odot} \sim \frac{1}{20}\ \mathrm{yr}^{-1}$$ (fewer seen due to dust)

Irregular Galaxy: ~10x this rate during bursts (1 SN per 2 yr)!

No SNe between bursts.

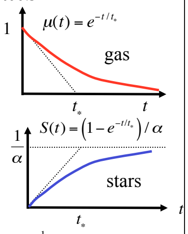
SN Rates: Ellipticals

$t_* = 1$ Gyr e-folding time
$t = 10$ Gyr age
$\alpha = 0.95$ efficiency
$M_0 = 10^{11}\,M_\odot$ total mass = initial gas mass

Gas consumption:

$$M_G(t) = M_0\,e^{-t/t_*} = M_0 - \alpha\,M_S(t)$$

Star formation: $M_S(t) = \dfrac{M_0}{\alpha}\left(1 - e^{-t/t_*}\right)$



$$\frac{dM_S}{dt} \equiv \dot{M}_S = \frac{M_0\,e^{-t/t_*}}{\alpha\quad t_*} = \frac{\left(10^{11} M_\odot\right) e^{-10}}{(0.95)\,(10^9\,\mathrm{yr})} = 5\times10^{-3}\ M_\odot\ \mathrm{yr}^{-1}$$

SN rate: $\dfrac{(0.072)\left(5\times10^{-3} M_\odot \mathrm{yr}^{-1}\right)}{12.2\ M_\odot} \approx 3\times10^{-5}\ \mathrm{yr}^{-1}$

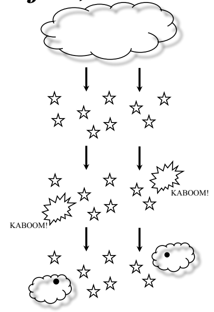
3 SN per 10^5 yr. Negligible!

What Star Formation Efficiency α and Yields of H, He and Metals ?

$M_G = M_0$
$M_S = 0$

X = 0.75
Y = 0.25
Z = 0.00

$M_G = 0$
$M_S = M_0$



X = ?
Y = ?
Z = ?

$M_G = (1-\alpha)\,M_0$
$M_S = \alpha\,M_0$
$\alpha = ?$

Estimates for efficiency α , yield in X, Y, Z

Assume:

1. Type-II SNe enrich the ISM.
   (Neglect: Type-I SNe, stellar winds, PNe, ...)
2. Closed Box Model:
   (Neglect: Infall from the IGM, outflow to the IGM)
3. SN 1987A is typical Type-II SN.

Better models include these effects.

What do we know about SN 1987A?

SN 1987A

23 Feb 1987 in LMC

Brightest SN since 1604!

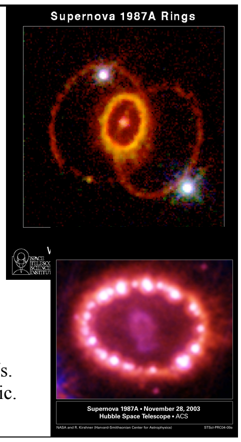
First SN detected in neutrinos.

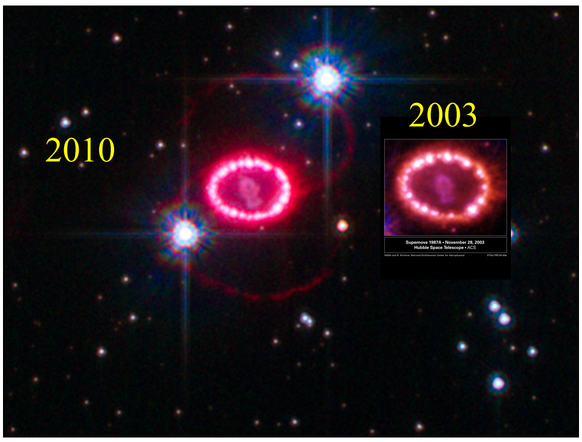
Visible (14 --> 4.2 mag) to naked eye, in southern sky.

Progenitor star visible:
~20 Msun blue supergiant.

3- ring structure (pre-SN wind)

UV flash reached inner ring in 80 d.
Fastest ejecta reached inner ring in ~6 yr.
Fast ejection velocity v~c/30~11,000 km/s.
Slower (metal-enriched) ejecta asymmetric.





Star Formation Efficiency

Use SN 1987A to calculate α and yield.

SN 1987A: progenitor star mass = 20 $M_\odot$
remnant neutron star mass = 1.6 $M_\odot$
mass returned to the ISM = 18.4 $M_\odot$

From IMF, 7.2% of M_S is in stars with $M > 8\ M_\odot$

β = Fraction of M_S returned to ISM:

$$\beta = \frac{\text{mass returned to gas}}{\text{mass turned into stars}} = 0.072 \times \frac{18.4}{20} \approx 6.6\%$$

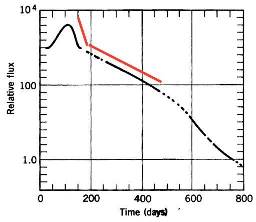
Star Formation Efficiency
α = fraction of M_S retained in stars:

$$\boxed{\alpha = 1 - \beta = 93\%}$$

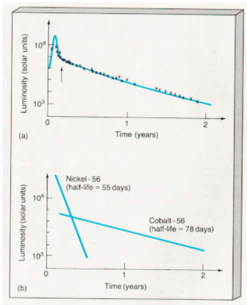
SN 1987A Lightcurve

Powered by radioactive decay of r-process nuclei.
Use to measure metal abundances in ejected gas.

$^{56}Ni \Rightarrow {}^{56}Co$ 6d half-life
$^{56}Co \Rightarrow {}^{56}Fe$ 78d half-life



Light curve of the Supernova from the first day of the visible outburst until May 16, 1988, some 800 days later.



X, Y, Z of ejecta from SN1987A

Initial mass	$\sim 20\ M_\odot$
NS mass	$\sim 1.6\ M_\odot$
Mass ejected	$\sim 18.4\ M_\odot$

$$\left.\begin{array}{rl} \text{in H} & 9.0\ M_\odot \\ \text{He} & 7.0\ M_\odot \\ Z & 2.4\ M_\odot \end{array}\right\} = 18.4\ M_\odot$$

$$\Rightarrow \quad X = \frac{9}{18.4} \approx 0.49 \qquad Y = \frac{7}{18.4} \approx 0.38$$

$$Z = \frac{2.4}{18.4} \approx 0.13$$