Lecture 10: Chemical Evolution of Galaxies

Metalicity evolution $Z(t)$ (vs galaxy type)

Processes that alter the metalicity:

1. Type-II SNe enrich the ISM.
2. Low-mass stars form from enriched ISM and "lock-up" metals.
3. Primordial gas falls in from IGM.
4. ISM ejected into IGM. (e.g. SN explosions, galaxy collisions)

Closed Box model: 1 and 2 only.
Accreting Box: 1,2,3. **Leaky Box**: 1,2,4.

Metalicity Evolution: Z(t)

M_0 = total mass

$M_G(t)$ = mass of gas in ISM

$M_Z(t)$ = mass of metals in ISM

$M_*(t)$ = mass locked up in stars and remnants

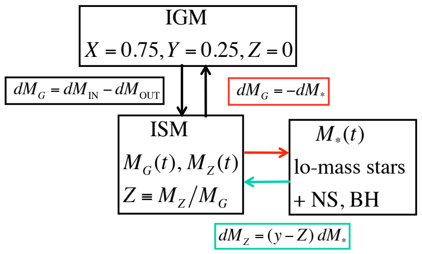
Mass conservation: $M_*(t) = M_0 - M_G(t)$

We also know: $\mu(t) \equiv \dfrac{M_G(t)}{M_0}$ $\quad \mu(0) = 1$

To derive: $Z(t) \equiv \dfrac{M_Z(t)}{M_G(t)}$ $\quad Z(0) = 0$

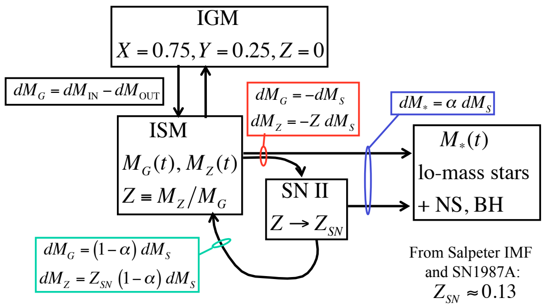
We will find: $Z(\mu(t))$

ISM Recycling Model



Yield: y = mass of metals returned to ISM per mass turned into low-mass stars and remnants

ISM Recycling Model



From Salpeter IMF and SN1987A:
$Z_{SN} \approx 0.13$
$\alpha \approx 0.93$

The "Yield"

Mass is conserved (gas ⇒ stars)

$$dM_G = -dM_* = -\alpha\, dM_S$$

Metals are lost to stars, but enriched gas is returned by SNe:

$$dM_Z = -Z\, dM_S + Z_{SN}(1-\alpha)\, dM_S$$

$$= \left[(-\alpha Z + \alpha Z) - Z + Z_{SN}(1-\alpha)\right]\left(\frac{dM_*}{\alpha}\right)$$

$$= \left[\frac{(Z_{SN}-Z)(1-\alpha)}{\alpha} - Z\right] dM_* \quad \boxed{\equiv (y - Z)\, dM_*}$$

$$\boxed{\text{Yield: } y = (Z_{SN} - Z)\left(\frac{1-\alpha}{\alpha}\right)}$$

Initial yield: $y_0 = Z_{SN}\left(\dfrac{1-\alpha}{\alpha}\right) = (0.13)\dfrac{0.07}{0.93} = 0.01$

From Salpeter IMF and SN1987A:
$Z_{SN} \approx 0.13$
$\alpha \approx 0.93$

Metalicity Evolution Z(t)

Differentiate: $Z(t) \equiv \dfrac{M_Z(t)}{M_G(t)}$

$$\delta Z = \delta\left(\frac{M_Z}{M_G}\right) = \frac{\delta M_Z}{M_G} + M_Z\, \delta\left(\frac{1}{M_G}\right)$$

$$= \frac{\delta M_Z}{M_G} + M_Z\left(-\frac{\delta M_G}{M_G^2}\right)$$

$$= \frac{1}{M_G}\left(\delta M_Z - \frac{M_Z}{M_G}\delta M_G\right)$$

$$= \frac{1}{M_G}\left((Z-y)\,\delta M_G - Z\,\delta M_G\right)$$

$$= -y\frac{\delta M_G}{M_G} = -y\,\delta\left(\ln(M_G)\right)$$

$y = 1/x$
$\dfrac{\delta y}{\delta x} = -1/x^2$
$\delta y = -\delta x / x^2$

Definition of yield:
$\delta M_Z = (y - Z)\,\delta M_*$
$= (Z - y)\,\delta M_G$

Closed Box with constant Yield

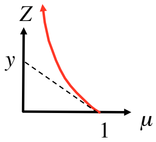
Integrate $\delta Z = -y \frac{\delta M_G}{M_G}$ (with y = constant) :

$$Z = -y \ln(M_G) + C$$

At $Z = 0$, $M_G = M_0$:

$$0 = -y \ln(M_0) + C \quad \Rightarrow \quad C = y \ln(M_0)$$

$$\therefore \; Z = -y \ln\left(\frac{M_G}{M_0}\right) = -y \ln(\mu)$$



Note that as $\mu \Rightarrow 0$, $Z \Rightarrow \infty$

Impossible ! :-(

What went wrong? Yield is not quite constant.

Closed Box with varying Yield

$$y = (Z_{SN} - Z)\left(\frac{1-\alpha}{\alpha}\right) \qquad y_0 = Z_{SN}\left(\frac{1-\alpha}{\alpha}\right)$$

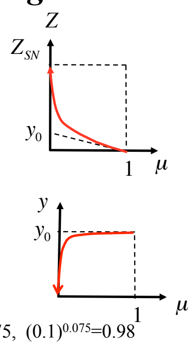
$$\delta Z = -y\,\delta(\ln\mu) = (Z - Z_{SN})\left(\frac{1-\alpha}{\alpha}\right)\delta(\ln\mu)$$

$$\frac{\delta Z}{Z - Z_{SN}} = \left(\frac{1-\alpha}{\alpha}\right)\delta(\ln\mu)$$

$$\ln(Z - Z_{SN}) = \left(\frac{1-\alpha}{\alpha}\right)\ln(\mu) + C$$

$$Z - Z_{SN} = A\,\mu^{\left(\frac{1-\alpha}{\alpha}\right)} \qquad A = e^C = -Z_{SN}$$

$$Z = Z_{SN}\left(1 - \mu^{\left(\frac{1-\alpha}{\alpha}\right)}\right) \qquad y = y_0\,\mu^{\left(\frac{1-\alpha}{\alpha}\right)}$$

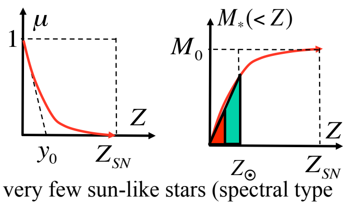


Yield is approx constant: $(1-\alpha)/\alpha \sim 0.075$, $(0.1)^{0.075} = 0.98$

But $y \Rightarrow 0$ and $Z \Rightarrow Z_{SN}$ as $\mu \Rightarrow 0$ (from the last SN).

Metalicity distribution of the Stars

Metalicity of stars = Metalicity of gas from which they formed.



"G dwarf problem": very few sun-like stars (spectral type G) have metalicity below ½ solar.

Closed Box Model FAILS: predicts that > ¼ of stars with $Z < Z_\odot$ have $Z <$ ½ $Z_\odot$

Why are there so few low-metalicity stars?

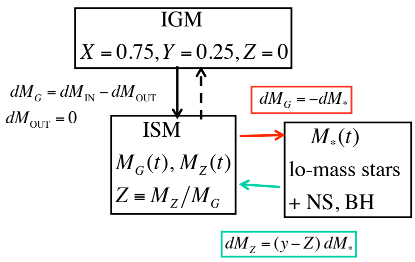
What caused the rapid initial enrichment?

What caused the initial enrichment?

IGM somehow enriched before galaxies form?

First generation (Pop III) $Z = 0$ stars all high mass?

Accreting Box model with low initial gas mass and $Z \Rightarrow y$?



Accreting Box varying Yield

$$\text{Yield} = y \equiv \frac{(Z_{SN} - Z)(1-\alpha)}{\alpha}$$

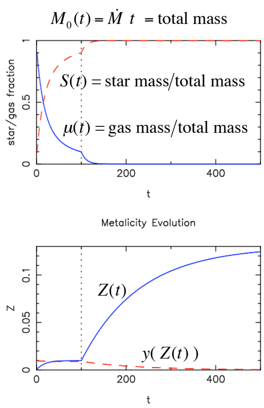
$\alpha = 0.93$ $\quad Z_{SN} = 0.13$

Assume star formation proportional to gas mass (e.g. Elliptical galaxy)

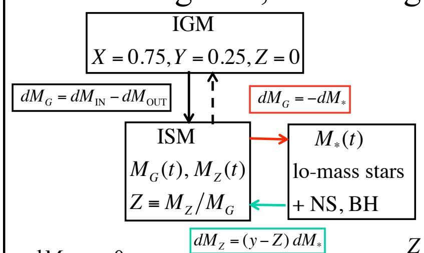
Accrete $Z = 0$ gas, constant dM_{IN}/dt , until $t = 100$.

Closed box for $t > 100$.

Result: $Z(t)$ rises until $Z \sim y(Z(t))$



Accreting Box, constant gas mass

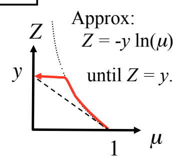


$dM_{OUT} = 0$

$dM_{IN} = dM_*$ $\Rightarrow$ M_G = constant

$dZ = dM_Z / M_G = (y - Z)\,dM_*$

Z stops increasing when $Z \Rightarrow y$



Approx: $Z = -y \ln(\mu)$ until $Z = y$.

Insert $\mu(t)$ for each galaxy type into $Z(t) = -y\ln(\mu(t))$ for $Z < y$

Ellipticals:

$\mu(t) = e^{(-t/t_*)}$

$Z(t) = -y\ln\left(e^{-t/t_*}\right) = y\frac{t}{t_*}$ for $Z \le y$

$Z(t) = y$ otherwise

Spirals:

$\mu(t) = 1 - \frac{\alpha \dot{M} t}{M_0}$

$Z(t) = -y\ln\left(1 - \frac{\alpha \dot{M} t}{M_0}\right)$ for $Z \le y$

$Z(t) = y$ otherwise

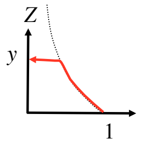
Irregulars:

$\langle \mu(t) \rangle = f\frac{\alpha M t}{M_0}$

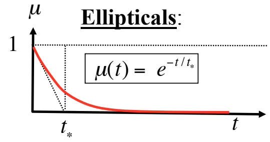
$Z(t) = -y\ln\left(1 - f\frac{\alpha \dot{M} t}{M_0}\right)$ for $Z \le y$

$Z(t) = y$ otherwise

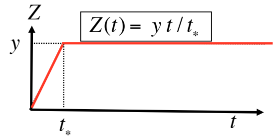
$Z(t) = -y\ln(\mu(t))$ for $Z < y$



Ellipticals: $\mu(t) = e^{-t/t_*}$

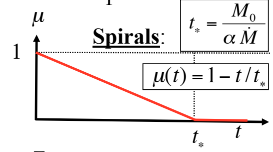


$Z(t) = y\,t/t_*$

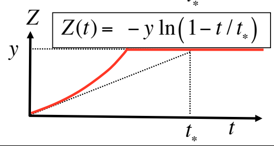


Spirals: $t_* = \frac{M_0}{\alpha \dot{M}}$

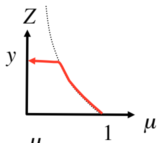
$\mu(t) = 1 - t/t_*$



$Z(t) = -y\ln(1 - t/t_*)$



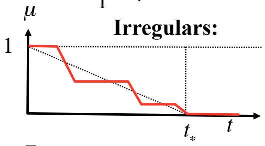
$Z(t) = -y\ln(\mu(t))$ for $Z < y$



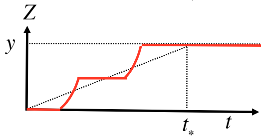
$\langle \mu(t) \rangle \approx 1 - t/t_*$

$t_* = f\frac{M_0}{\alpha \dot{M}_{\text{burst}}}$ $\quad f < 1$

Irregulars:



$Z(t) \approx -y\ln(1 - t/t_*)$



Initial and Effective Yield

$$y \equiv -\frac{\delta Z}{\delta(\ln\mu)} = (Z_{SN} - Z)\frac{1-\alpha}{\alpha}$$

First generation: $Z = 0$ later generations $Z << Z_{SN}$:

From Salpeter IMF and SN 1987A: $\alpha = 0.93$

From SN 1987A: $Z_{SN} = 0.13$

$\Rightarrow$ Initial yield $= y_0 \approx 0.13\frac{0.07}{0.93} = 0.01$

Solar metals : $Z_\odot \approx 0.02$

Milky Way has used about 90% of its gas:

$$y_{\text{eff}} \equiv \frac{Z_{\text{obs}}}{\ln(1/\mu)} \sim \frac{0.02}{\ln(10)} = 0.01$$

$$\mu \approx \frac{M_G}{M_* + M_G} \sim 0.1$$

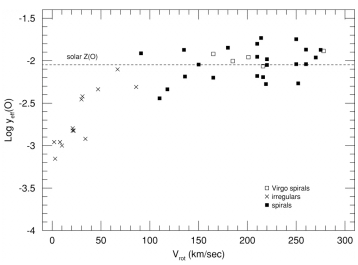
Effective Yield vs Galaxy Mass

Tully-Fisher : (M / $10^{11}M_\odot$) ~ (V_{rot} / 200 km/s)^4

Lower yield in small galaxies because SN ejecta excape.

$$y_{\text{eff}} \equiv \frac{Z_{\text{obs}}}{\ln(1/\mu)}$$

$$\mu \approx \frac{M_G}{M_* + M_G}$$



Garnett 2002

Summary

- Simple models for $Z(\mu(t))$ (Closed Box, Accreting Box, Leaky Box)
- Yield: y = mass of metals returned to ISM per mass turned into low-mass stars and remnants
  $Z = -y\ln(\mu) = y\ln(1/\mu)$
- "G dwarf problem" Closed Box model fails, predicts too many low-Z stars.
- Infall of $Z = 0$ material causes $Z \Rightarrow y$.
- $y_{\text{eff}} = Z_{\text{obs}} / \ln(1/\mu) \sim 0.01$
- 0.001 for small Galaxies (SN ejecta escape)