Lecture 2

Astronomical Distances

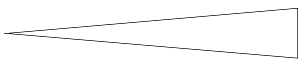
Astronomical Distances

- Size of Earth
- Distance to the Moon (1 sec)
- Distance to the Sun (8 min)
- Distance to other stars (years)
- Distance to centre of our Galaxy (30,000 yr to centre)
- Distances to other Galaxies (2 million years to Andromeda)
- Size of the Universe (13 billion years)

Distance Methods

- Standard Rulers ==> Angular Size Distances

$$\theta = \frac{l}{D}$$

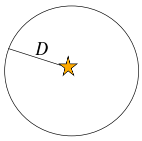


$$D_A = \frac{l}{\theta}$$

(for small angles << 1 radian)

- Standard Candles ==> Luminosity Distances

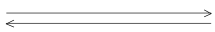
$$F = \frac{\text{energy/time}}{\text{area}} = \frac{L}{4\pi D^2}$$



$$D_L = \left(\frac{L}{4\pi F}\right)^{1/2}$$

- Light Travel Time

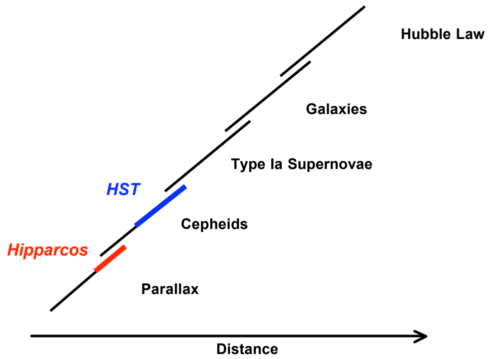
$$t = \frac{\text{distance}}{\text{velocity}} = \frac{2D}{c}$$



$$D_t = \frac{c}{2t}$$

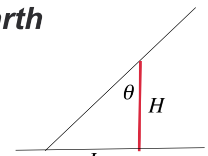
(e.g. within solar system)

Cosmic Distance Ladder

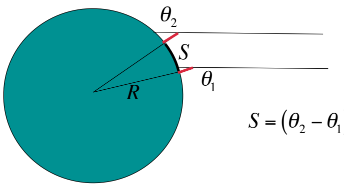


Size of Earth

- Earth radius R
- Two poles, height: H
- North-South separation: S
- Shadow length at noon: L



$$\tan\theta = \frac{L}{H}$$



$$S = (\theta_2 - \theta_1)\,R$$

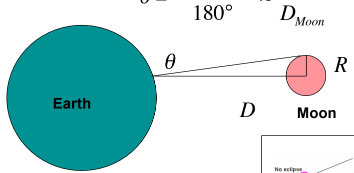
To find R, measure H and L at 2 latitudes separated by S.

Ancient Greeks used Athens to Alexandria, finding R ~ 6300 km

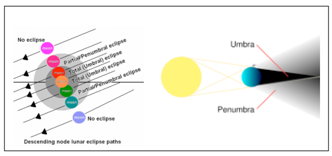
Size of and Distance to the Moon

- Lunar Eclipse gives $R_{moon} \sim R_{Earth}/ 3.5$
- Angular Diameter Distance

$$\theta = \frac{0.25^\circ \times \pi}{180^\circ} \approx \frac{R_{Moon}}{D_{Moon}}$$



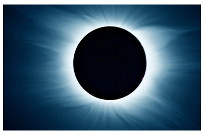
- Laser ranging
  - 1.2 light seconds
  - cm accuracy

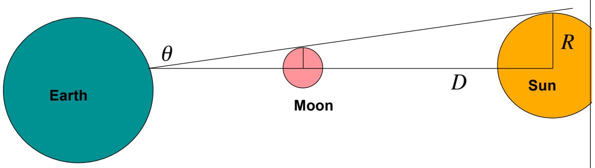


Size of and Distance to the Sun

- Same angular diameter as Moon.
- Moon closer -- by what factor?

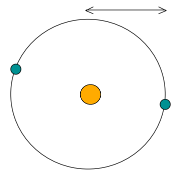
$$\theta \approx \frac{R_{moon}}{D_{moon}} = \frac{R_{sun}}{D_{sun}}$$



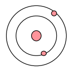


Earth's Orbit size from Jupiter's Moons

a_E = 1 AU = 8 light minutes
= 1.5 x 10^{13} cm

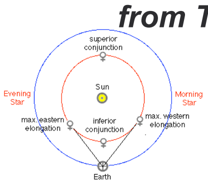


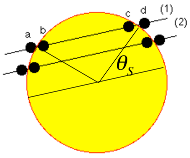
Jupiter + orbiting moons



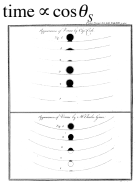
Due to light travel time across Earth's orbit, Jupiter's moons appear to orbit up to 8 minutes ahead or behind schedule.

Sun's Size and Distance from Transits of Venus





$\text{time} \propto \cos\theta_S$



Relative size of orbits:

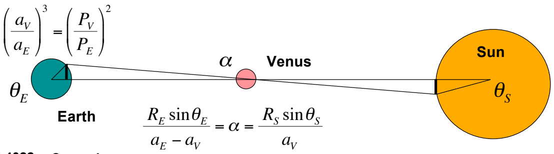
$$\sin(44^\circ) \approx \frac{a_{Venus}}{a_{Earth}} \approx 0.69$$

or, from Kepler's law:

$$\left(\frac{a_V}{a_E}\right)^3 = \left(\frac{P_V}{P_E}\right)^2$$

Method by Halley 1716

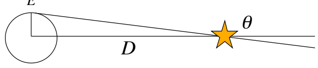
Observed 1761 and 1769
(e.g. by Capt. Cook in Tahiti)



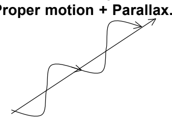
$$\frac{R_E \sin\theta_E}{a_E - a_V} = \alpha = \frac{R_S \sin\theta_S}{a_V}$$

Stellar Parallaxes

$a_E \equiv 1$ AU



Motion in the sky combines
Proper motion + Parallax.



$$\frac{D}{1\ \text{AU}} = \frac{1\ \text{radian}}{\theta}$$

1 parsec = 1 "parallax arcsec"

$$\frac{D}{\text{pc}} = \frac{1\ \text{arcsec}}{\theta}$$

1 radian = 206265 arcsec

1 parsec = 206265 AU

1 pc = 206265 x 8 light minutes

= 3.3 light years = 3.1x10^{18} cm

Imaging from ground:
0.02 arcsec => 50 pc

Hipparcos satellite

0.003 arcsec => 300 pc

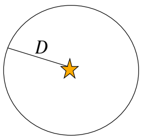
GAIA satellite (2012?):

10^{-4} arcsec => 10 kpc

Luminosity Distances

Use the inverse-square law:

$$F = \frac{L}{4\pi D^2} = \frac{energy/time}{area}$$



Luminosity Distance:

Apparent magnitude: $m \equiv -2.5\log(F/F_{Vega})$

e.g. 5 mags = 100x fainter = 10x farther away

0.1 mag = 10% fainter = 5% farther away

$$(1+x)^2 \approx 1+2x$$
$$(1.05)^2 \approx 1.1$$

Absolute magnitude M
= apparent magnitude m at standard distance 10 pc

$$m = M + 5\log(D/10\ \text{pc}) \qquad \text{since} \quad F \propto D^{-2}$$

Distance Modulus (ignoring dust extinction):

$$m - M = 5\log(D/\text{pc}) - 5$$

How Far are the Stars ?

Sun : m_V = -24 mag

Brightest stars (about 10) :

m_V < +1 mag

Faintest (naked-eye) stars (about 6000) :

m_V < + 6 mag

Relative distances :

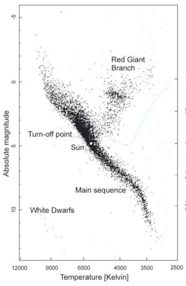
5 mag = 100 x fainter = 10 x farther away

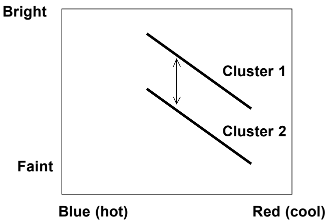
25 mag = 10^{10} x fainter = 10^5 x farther away

Distance to a sun-like m_V = +1 mag star:

8 x 10^5 light minutes = 1.5 light years

Main-Sequence Fitting





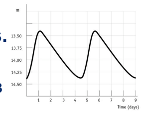
Stars with Hipparcos parallaxes calibrate the Colour-Magnitude (Hertzsprung-Russel) diagram.

Magnitude shift gives ratio of star cluster distances:

$$m_1 - m_2 = 5\log(D_1 / D_2)$$

Cepheid Variable Stars

- H ionisation instability drives pulsations.
- Pulsation period ~ sound travel time
- Period-Luminosity relationship L ~ $P^{1.3}$

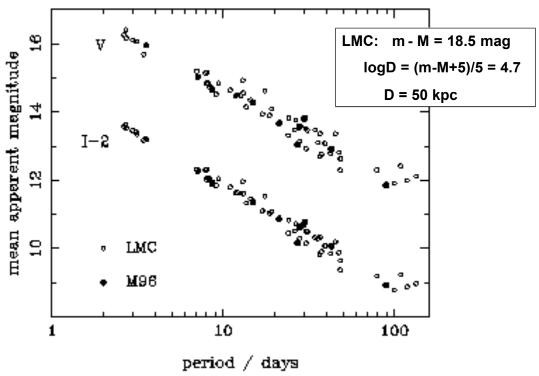


- Calibrate using parallax, main-sequence fitting.
- Also from Supernova 1987A, light travel time to circumstellar ring --> D_{LMC} = 51 kpc +/- 6%.

- Hubble used Cepheids in Local Group D < 2 Mpc.

- HST sees Cepheids in Virgo Cluster D < 20 Mpc.

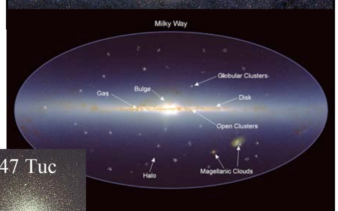
Cepheid Period-Luminosity



Distance to the Galactic Centre

$$D(\text{ Galactic Centre }) = 8.5 \text{ kpc}$$

- Dust in Galactic Disk
  $A_V \approx 1$ mag / kpc



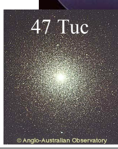
- RR Lyr variables in Galactic Bulge

M_V(RR Lyr) ~ +0.5 mag

- Globular Clusters in Galactic Halo

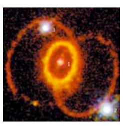
Cepheids

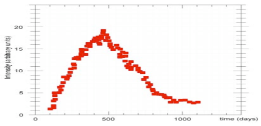
Main Sequence

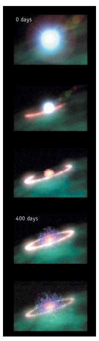


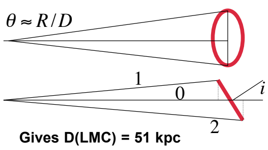
Distance to Large Magellenic Cloud

- SN 1987a explosion illuminates circumstellar gas ring.
- Light travel time gives linear size.
- Observed angular size then gives distance.









$\theta \approx R / D$

$$c\,t_0 = D$$
$$c\,t_1 \approx D + R - R\sin i$$
$$c\,t_2 \approx D + R + R\sin i$$
$$c\,(t_2 - t_1) = 2R\sin i$$
$$= 400 \text{ light days}$$

Gives D(LMC) = 51 kpc

Checks the Cepheid distances

Distances to Galaxies

Standard Candles ?

Cepheids (to 20 Mpc)

Brightest stars

Planetary nebulae

Globular Clusters

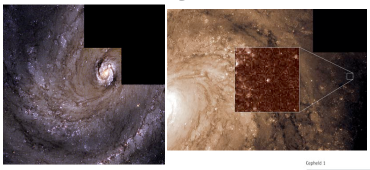
Supernovae
(e.g. Type 1a 20-400 Mpc)

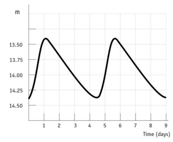
Galaxies (e.g. using Luminosity-Rotation Velocity correlations)

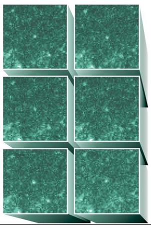
Giant Elliptical in Virgo Cluster
$\sim 10^4$ globular clusters

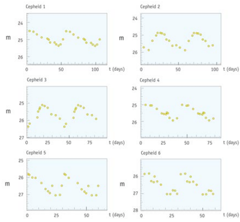
HST designed to find Cepheids in Virgo Cluster Galaxies

M100





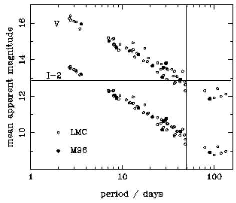




Cepheid Distance to M100

LMC: D = 50 kpc

m - M = 18.5 mag

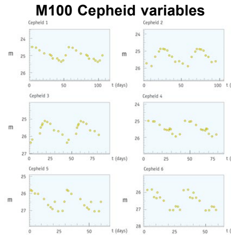


Match periods to get same luminosity.

Difference in apparent mag gives ratio of distances.

D = 50 kpc x $10^{(25-13)/5}$ = 13 Mpc

M100 Cepheid variables



$$m - M = 5\log(D/\mathrm{pc}) - 5$$

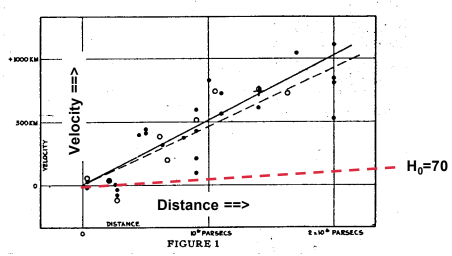
P = 50d => M_V = 13-18.5 = -5.5 mag

m_V = 25 mag m - M = 30.5 mag

log(D/pc) = (m-M+5)/5 = 7.1

D = $10^{7.1}$ = 13 Mpc

Hubble's Diagram (~1924)

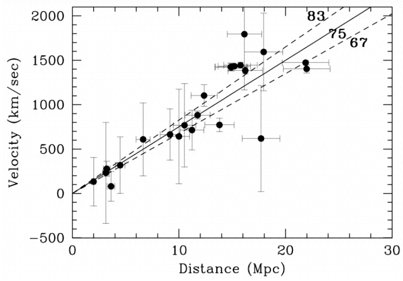


Slope = H_0 = 500 km/s/Mpc (!)
Cepheid distance calibration was wrong
(dust in Milky Way was not yet recognised).

Hubble was wrong (but his idea was good).

HST Key Project

Cepheid Distances

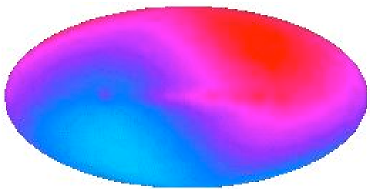


Freedman et al.
2001 ApJ 553, 47.

Why go beyond Cepheids?

- HST sees Cepheids to D = 10-20 Mpc.
- H_0 x D = 70 x 15 ~ 1000 km/s.
- not really far enough
- galaxy pecular velocities ~500 km/s.
- galaxies falling toward Virgo cluster.

CMB dipole --> Milky Way velocity



$$\frac{\Delta T}{T} \approx \frac{V}{c} \rightarrow V \approx 600 \text{ km s}^{-1}$$

Largely due to Milky Way (Local Group) falling toward Virgo Cluster.

SN Type Ia in Virgo Galaxy NGC 4526

Supernova outshines the entire galaxy, but only for a month or so.

Type II -- massive stars ($M > 8\ M_{SUN}$) explode at end of life.

Type Ia -- white dwarf in a binary system accretes mass, collapses when $M_{WD} = 1.4\ M_{SUN}$.

Good “standard bombs”.

Calibrate SN distances using HST to see Cepheids in Virgo galaxies.

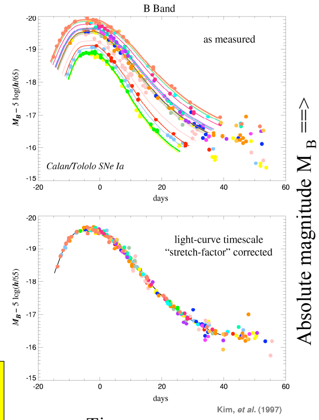
Calibrating “Standard Bombs”

1. Brighter ones decline more slowly.

2. Time runs slower by factor (1+z).

AFTER correcting: Constant peak brightness $M_B = -19.7$

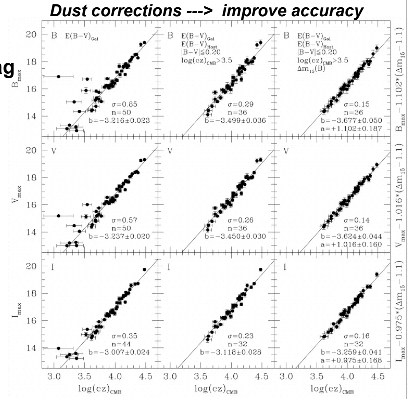
Observed peak magnitude: $m = M + 5 \log (d/\mathrm{Mpc}) + 25$ gives the distance!



HST Key Project

Dust corrections ---> improve accuracy

SN Ia distances: accuracy ~0.15 mag ~8% in distance



Freedman et al. 2001 ApJ 553, 47.

Galaxy Luminosity Calibrations

$$L = 4\pi\ D^2\ F = K\ V^4$$

Determine K using galaxies with Type Ia Supernovae.

$$D = V^2 \sqrt{\frac{K}{4\pi\ F}}$$

Measure flux F and velocity V to determine distance D.

Tully - Fisher relation

spirals : V = rotation velocity

(HI 21 cm emission line width)

Faber - Jackson relation

ellipticals : V = stellar velocity dispersion

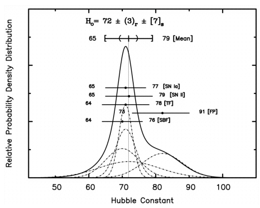
(optical absorption line widths)

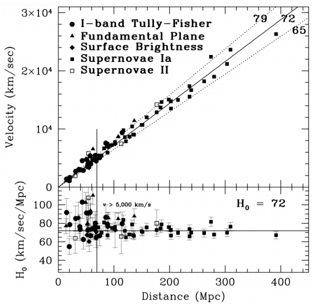
“The Fundamental Plane of Ellipticals” improves the F-J relation by including a surface brightness correction.

HST Key Project

$$H_0 \approx 72 \pm 3 \pm 7 \quad \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

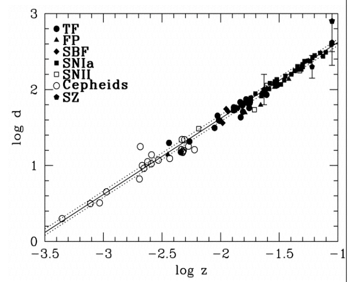
Freedman, et al. 2001 ApJ 553, 47.





Frailty of the Distance Ladder

- **Parallax**
  - 0 - 300 pc
  - (GAIA 2015 5 kpc)
- **Cepheids**
  - ~100 pc - 20 Mpc (HST)
- **Type Ia SNe**
  - 20 - 400 Mpc (8m)
  - z ~ 1.5 (HST)
- **Little overlap between Cepheids and SN Ia.**



Only 3 galaxies with both Cepheids and SN Ia