## Lecture 8

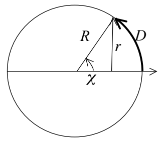
## Observational Cosmology

## Parameters of Our Universe

## The Concordance Model



## Time and Distance vs Redshift




$$\frac{d}{dt}\left(x = 1+z = \frac{R_0}{R}\right) \quad \rightarrow \quad dt = \frac{-dx}{x\,H(x)}$$

Friedmann: $H(x) = H_0\sqrt{\Omega_M\, x^3 + \Omega_\Lambda + (1-\Omega_0)\,x^2}$

$\Omega_0 \equiv \Omega_M + \Omega_\Lambda \quad (\Omega_R = 0)$

look - back time:

$$t(z) = \int_{t_e}^{t_0} dt = \int_1^{1+z} \frac{dx}{x\,H(x)} = \frac{1}{H_0}\int_1^{1+z} \frac{dx}{x\sqrt{\Omega_M\, x^3 + \Omega_\Lambda + (1-\Omega_0)\,x^2}}$$

radial distance:

$$D_0(z) = R_0\,\chi = \int_{t_e}^{t_0} \frac{R_0}{R(t)}\,c\,dt = \int_1^{1+z} \frac{x\,c\,dx}{x\,H(x)} = \frac{c}{H_0}\int_1^{1+z} \frac{dx}{\sqrt{\Omega_M\, x^3 + \Omega_\Lambda + (1-\Omega_0)\,x^2}}$$

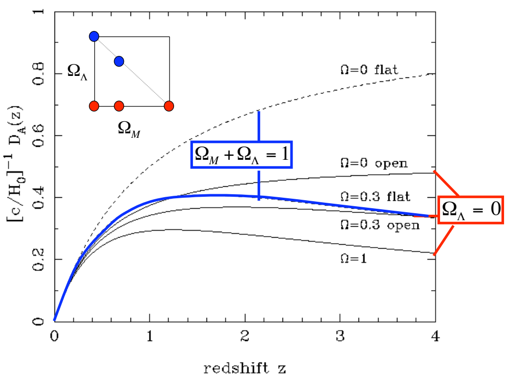
circumferencial distance: $r_0 = R_0\, S_k(\chi) \quad R_0 = \frac{c}{H_0}\left(\frac{k}{\Omega_0 - 1}\right)^{1/2}$

angular diameter distance: $D_A = r_0 / (1+z)$

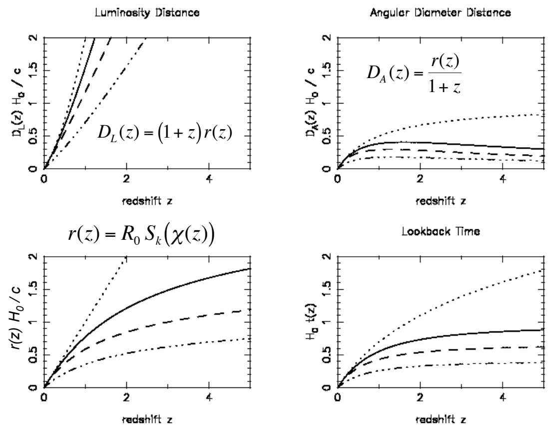
luminosity distance: $D_L = (1+z)\, r_0$



## Angular Diameter Distance








## "Concordance Model" Parameters

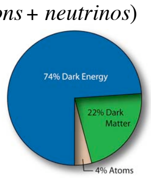
$H_0 \equiv 100\, h\, \frac{\text{km/s}}{\text{Mpc}} \approx 70\, \frac{\text{km/s}}{\text{Mpc}} \qquad h \approx 0.7$

$\Omega_R \approx 4.2\times10^{-5} h^{-2} \approx 8.4\times10^{-5}$ (*CMB photons + neutrinos*)

$\Omega_B \sim 0.02\, h^{-2} \sim 0.04$ (*baryons*)

$\Omega_M \sim 0.3$ (*Dark Matter*)

$\Omega_\Lambda \sim 0.7$ (*Dark Energy*)




$\Omega_0 \equiv \Omega_R + \Omega_M + \Omega_\Lambda = 1.0 \quad \rightarrow \quad$ *Flat Geometry*



## Our (Crazy?) Universe

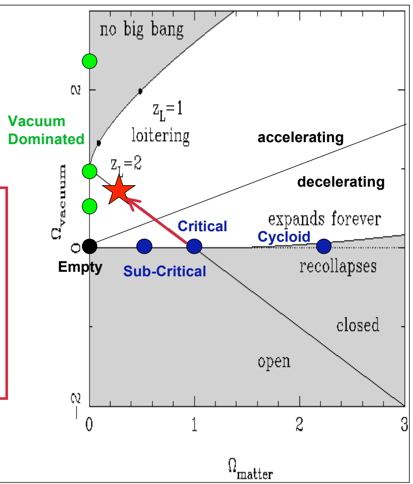
$H_0 \approx 70\, \frac{\text{km/s}}{\text{Mpc}}$

$\Omega_M \sim 0.3$

$\Omega_\Lambda \sim 0.7$

$\Omega_R \sim 8\times10^{-5}$

$\Omega_0 = 1.0$



## *Evolution of Ω*

Density at a past/future epoch in units of $\rho_c = 3\,H_0^2/8\pi\,G$

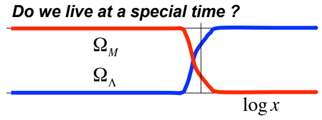
$$\Omega \equiv \frac{\rho}{\rho_c} = \sum_w \Omega_w x^{3(1+w)} = \Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda \qquad x \equiv 1+z = R_0/R$$

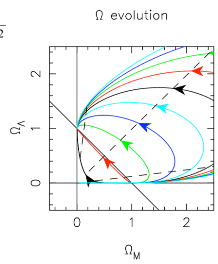
in units of critical density $3\,H^2/8\pi\,G$ at the past/future epoch:

$$\Omega_M(x) = \frac{H_0^2}{H^2}\Omega_M x^3 = \frac{\Omega_M x^3}{\Omega_M x^3 + \Omega_\Lambda + (1-\Omega_0)x^2}$$

$$\Omega_\Lambda(x) = \frac{H_0^2}{H^2}\Omega_\Lambda = \frac{\Omega_\Lambda}{\Omega_M x^3 + \Omega_\Lambda + (1-\Omega_0)x^2}$$

***Do we live at a special time ?***

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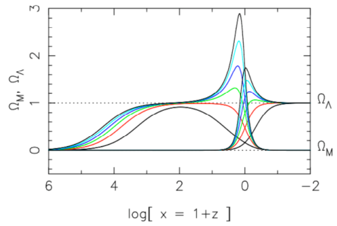
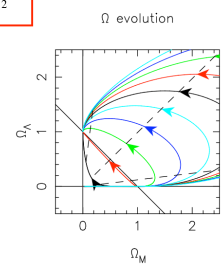
## *Evolution of Ω*

$$\Omega_M(x) = \frac{\Omega_M x^3}{\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda + (1-\Omega_0)x^2}$$

$$\Omega_\Lambda(x) = \frac{\Omega_\Lambda}{\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda + (1-\Omega_0)x^2}$$

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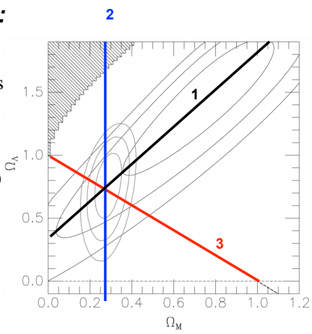
## *"Concordance" Model*

***Three main constraints:***

1. Supernova Hubble Diagram
2. Galaxy Counts + M/L ratios
   $\Omega_M \sim 0.3$
3. Flat Geometry
   ( inflation, CMB fluctuations)
   $\Omega_0 = \Omega_M + \Omega_\Lambda = 1$




concordance model

$H_0 \approx 72 \quad \Omega_M \approx 0.3 \quad \Omega_\Lambda \approx 0.7$

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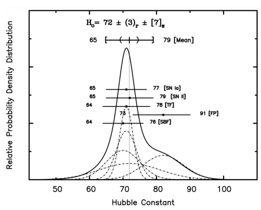
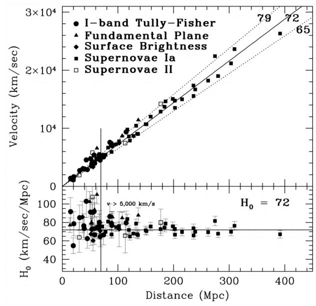
## *HST Key Project*

$$H_0 \approx 72 \pm 3 \pm 7 \quad \text{km s}^{-1}\ \text{Mpc}^{-1}$$

**Freedman, et al.**
**2001 ApJ 553, 47.**

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## *Hubble time and radius*

Hubble constant :

$$H \equiv \frac{\dot R}{R} \qquad H_0 \equiv \left(\frac{\dot R}{R}\right)_0 = 100\,h\,\frac{\text{km/s}}{\text{Mpc}} \approx 70\,\frac{\text{km/s}}{\text{Mpc}}$$

Hubble time :

$$t_H \equiv \frac{1}{H_0} = 10^{10}\,h^{-1}\ \text{yr} \approx 14\times10^9\ \text{yr}$$

$\sim$ age of Universe

Hubble radius :

$$R_H \equiv \frac{c}{H_0} = 3000\ h^{-1}\ \text{Mpc} \approx 4\times10^9\,\text{pc}$$

= distance light travels in a Hubble time.

~ distance to the Horizon.

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## *Age vs Hubble time*

$$H = \frac{\dot R}{R} \qquad H = \left.\frac{\dot R}{R}\right|_{t=t_0}$$

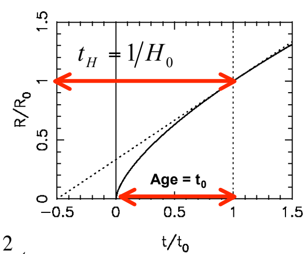
deceleration decreases age

acceleration increases age

e.g. matter dominated

$$R \propto t^{2/3} \quad \Rightarrow \quad \dot R = \frac{2}{3}\frac{R}{t}$$

$$H = \frac{\dot R}{R} = \frac{2}{3}\frac{1}{t} \quad \Rightarrow \quad t_0 = \frac{2}{3}\frac{1}{H_0} = \frac{2}{3}t_H$$

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## Beyond $H_0$

- Globular cluster ages: $t < t_0$ --> acceleration
- Radio jet lengths: $D_A(z)$ --> deceleration
- Hi-Redshift Supernovae: $D_L(z)$ --> acceleration
- Dark Matter estimates ---> $\Omega_M \sim 0.3$
- Inflation ---> Flat Geometry $\Omega_0 \approx 1.0$
- CMB power spectra
- "Concordance Model" $\Omega_M \sim 0.3 \quad \Omega_\Lambda \sim 0.7$ ; $\Omega_0 = \Omega_M + \Omega_\Lambda \approx 1.0$



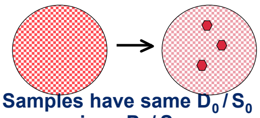
## Age Constraints

- Nuclear decay ( U, Th -> Pb )
  - Decay times for ($^{232}Th$, $^{235}U$, $^{238}U$) = (20.3, 1.02, 6.45) Gyr
  - 3.7 Gyr = oldest Earth rocks
  - 4.57 Gyr = meteorites
  - ~10 Gyr = time since supernova produced U, Th
    - ( $^{235}U$ / $^{238}U$ = 1.3 --> 0.33, $^{232}Th$ / $^{238}U$ = 1.7 --> 2.3 )
- Stellar evolution
  - 13-17 Gyr = oldest globular clusters
- White dwarf cooling
  - ~13 Gyr = coolest white dwarfs in M4

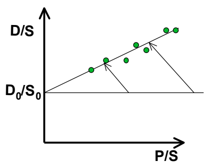


## Nuclear Decay Chronology

- P=parent D=daughter S=stable isotope of D
- Chemical fractionation changes P/S but not D/S:



e.g. concentrate D+S in crystals




- Samples have same $D_0 / S_0$ various $P_0 / S_0$
- P decays to D:

$$P(t) = P_0 e^{-t/\tau}$$

$$D(t) = D_0 + P_0\left(1 - e^{-t/\tau}\right) = D_0 + P(t)\left(e^{t/\tau} - 1\right) \qquad S(t) = S_0$$

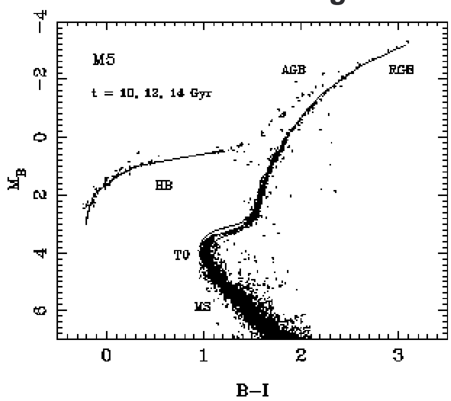
$$\frac{D(t)}{S(t)} = \frac{D_0}{S_0} + \frac{P(t)}{S_0}\left(e^{t/\tau} - 1\right)$$

Observed slope $= \left(e^{t/\tau} - 1\right)$

gives age $t/\tau$ , typically to $\sim 1\%$



## Globular Cluster Ages

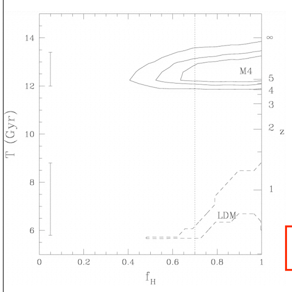


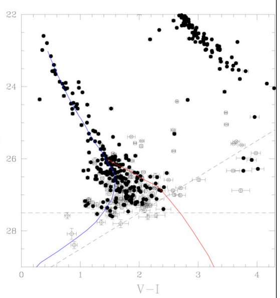



## Coolest White Dwarfs

$12.7 \pm 0.7$ Gyr

Hansen et al. 2002 ApJ 574,155



White dwarf cooling ages --> star formation at z > 5.

Cooling times have been measured using "ZZ Ceti" oscillation period changes.



## Age Crisis (~1995)

$$H_0\, t_0 = \int_1^\infty \frac{dx}{x\sqrt{\Omega_M\, x^3 + \Omega_\Lambda + (1-\Omega_0)\, x^2}}$$

observations :

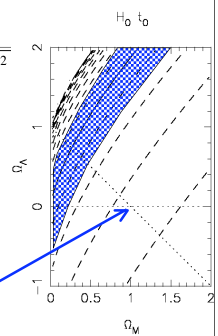
$H_0 = 72 \pm 8$ km s$^{-1}$ Mpc$^{-1}$

$t_0 \geq 14 \pm 2$ Gyr old globular clusters

$H_0\, t_0 \geq 1.0 \pm 0.15$

Globular clusters older than the Universe ?
Inconsistent with critical-density matter-only model :

$$H_0\, t_0 = \frac{2}{3} \quad \text{for} \quad (\Omega_M, \Omega_\Lambda) = (1,0)$$




Strong theoretical prejudice for inflation. $(\Omega_0 = 1)$
Doubts about stellar evolution therory (e.g. convection).