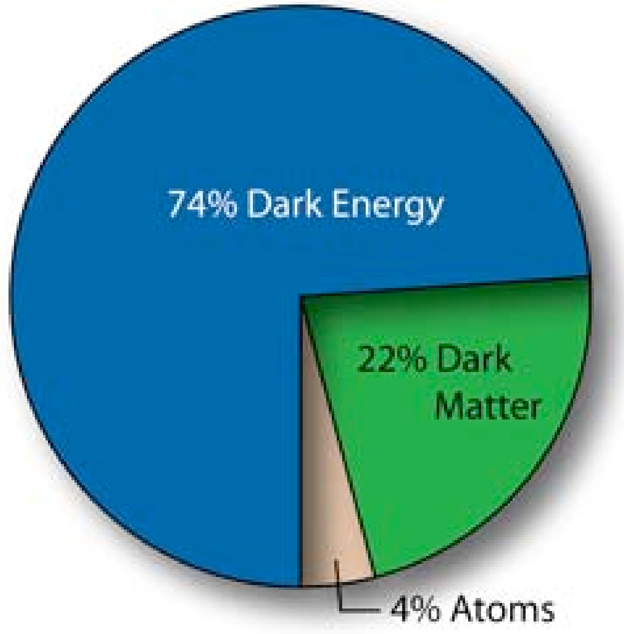
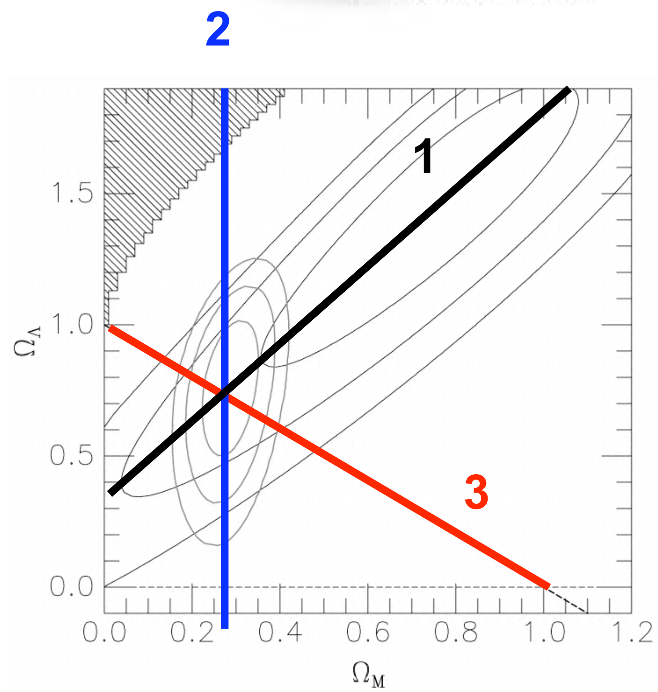
Cosmic Microwave Background

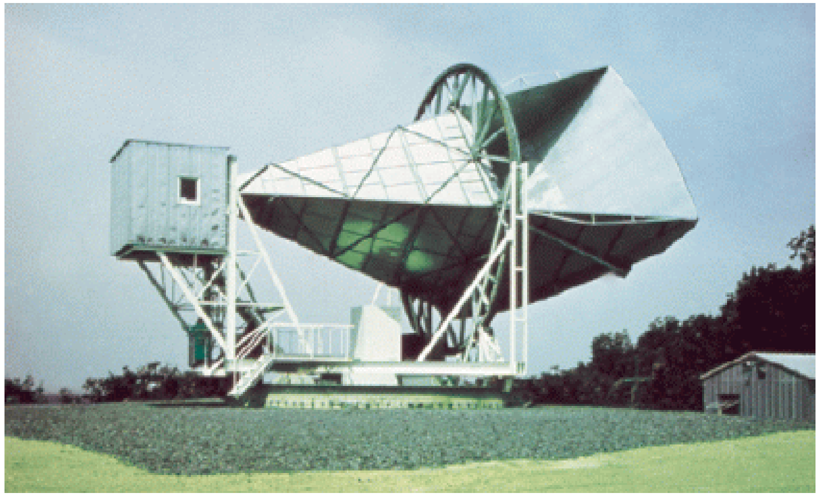
Flat Geometry

$$\Omega_0 = \Omega_M + \Omega_\Lambda$$
$$\approx 1.0$$





1965 -- Penzias + Wilson



Bell Labs telecommunications engineers
find excess microwave noise from the sky.

~1% of thermal (T ~ 300° K) noise ---> T ~ 3° K

Afterglow of the Big Bang

CMB = Cosmic Microwave Background

Confirms a forgotten 1948 prediction by Gamow.

Nobel Prize -> P+W

Recombination Epoch (z~1100)

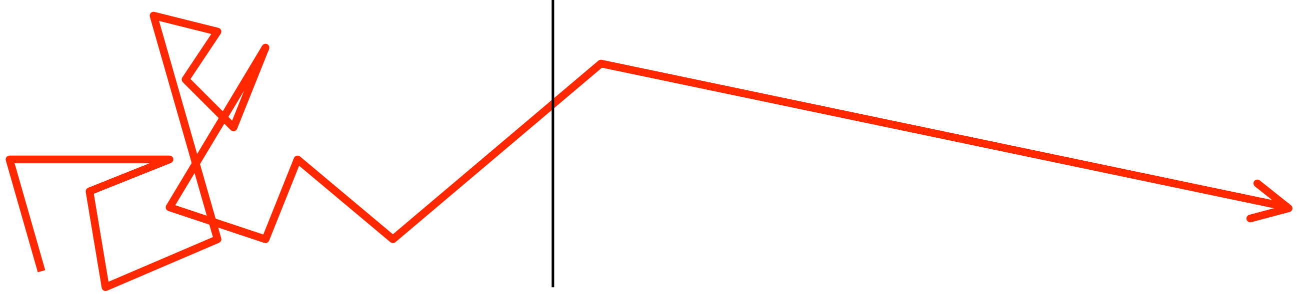
ionised plasma --> neutral gas

- Redshift z > 1100
- Temp T > 3000 K
- H ionised

- electron -- photon Thompson scattering

- z < 1100
- T < 3000 K
- H recombined

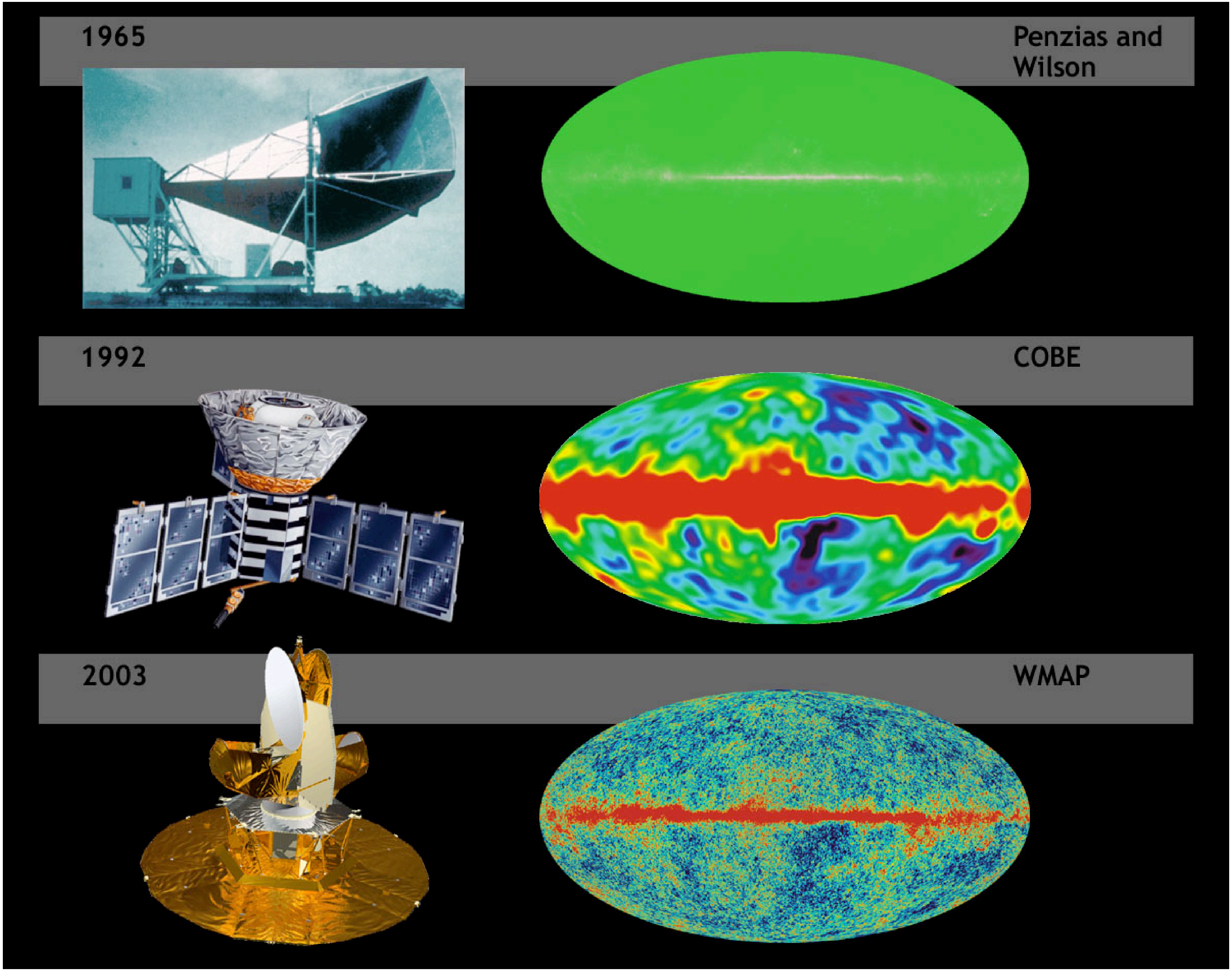
- almost no electrons
- neutral atoms
- photons set free



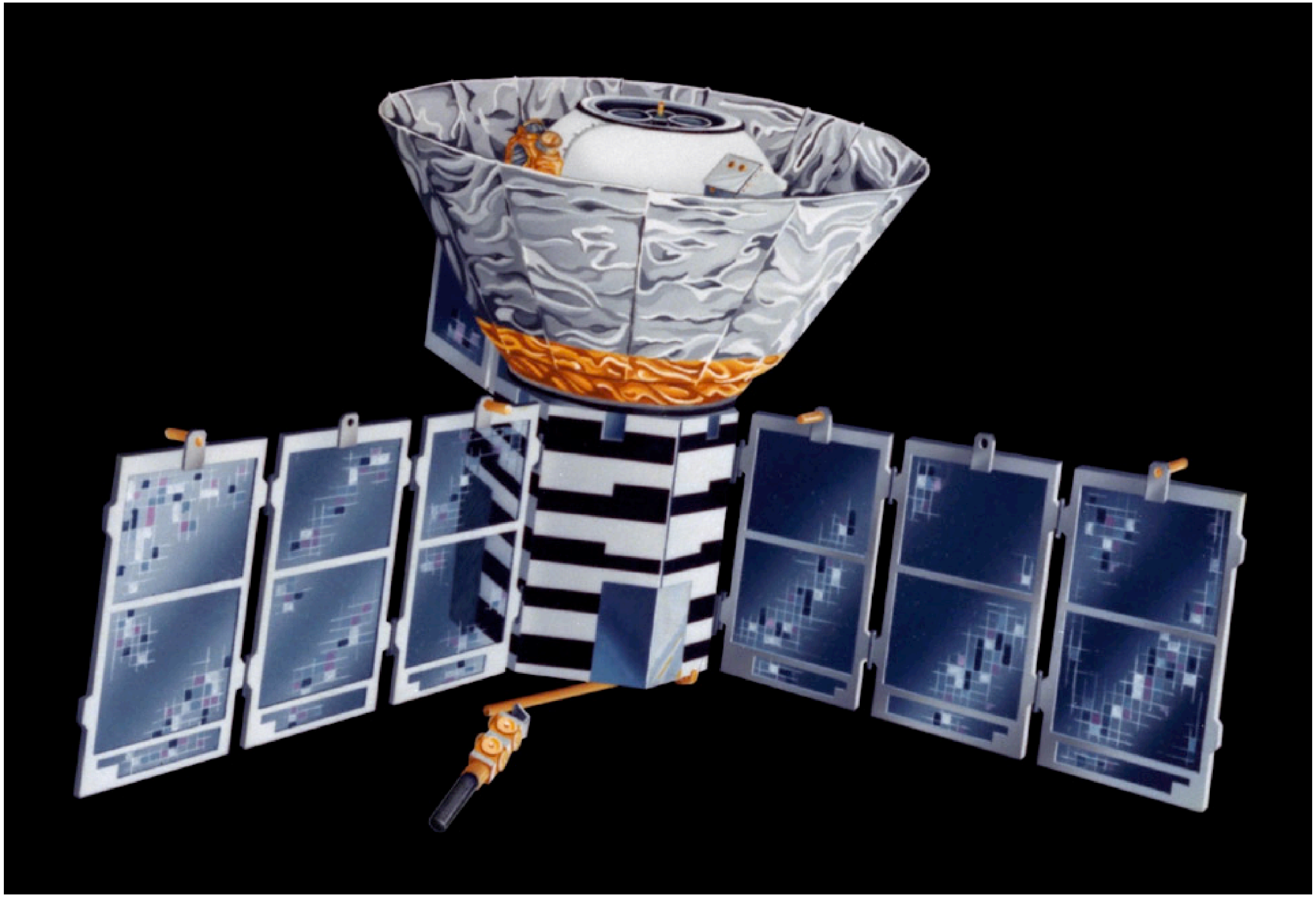
e - scattering optical depth

$$\tau(z) \approx \left(\frac{z}{1080} \right)^{13}$$

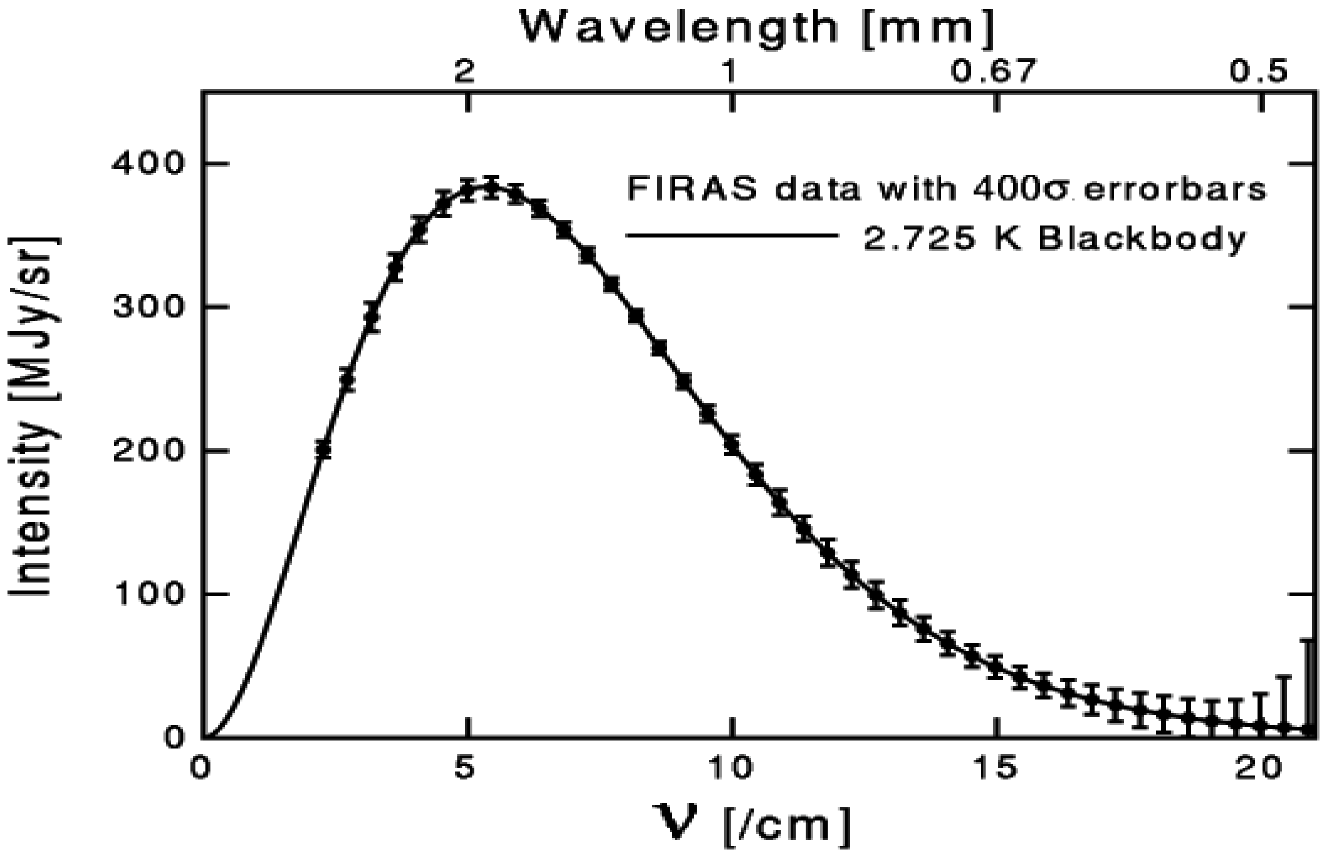
thin surface of last scattering


1965
Penzias and Wilson
1992
COBE
2003
WMAP

NASA 1992 - COBE
COsmic Background Explorer



COBE spectrum of CMB

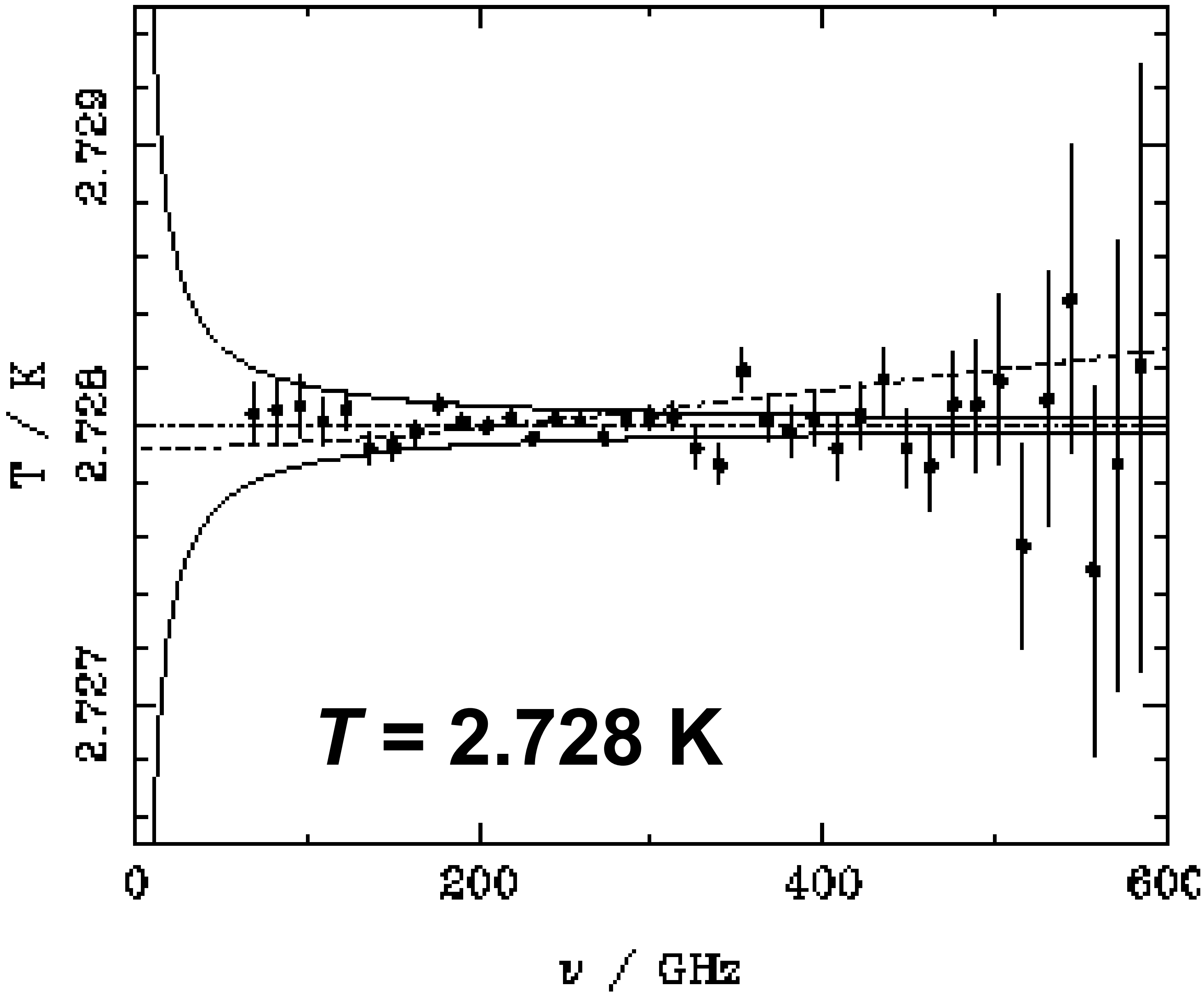


A perfect Blackbody !

No spectral lines -- strong test of Big Bang.
Expansion preserves the blackbody spectrum.

$$T(z) = T_0\,(1+z) \qquad T_0 \sim 3000\ K \quad z \sim 1100$$

COBE spectrum



T = 2.728 K

Radiation -> Matter -> Vacuum

$T = 2.728\ \mathrm{K}$

radiation energy density :

$$\rho_R = \frac{u(T)}{c^2} = \frac{4\,\sigma}{c^3} T^4$$

$$\Omega_R = 8.6\times10^{-5}\left(\frac{0.7}{h}\right)^2 \sim 0.01\%$$

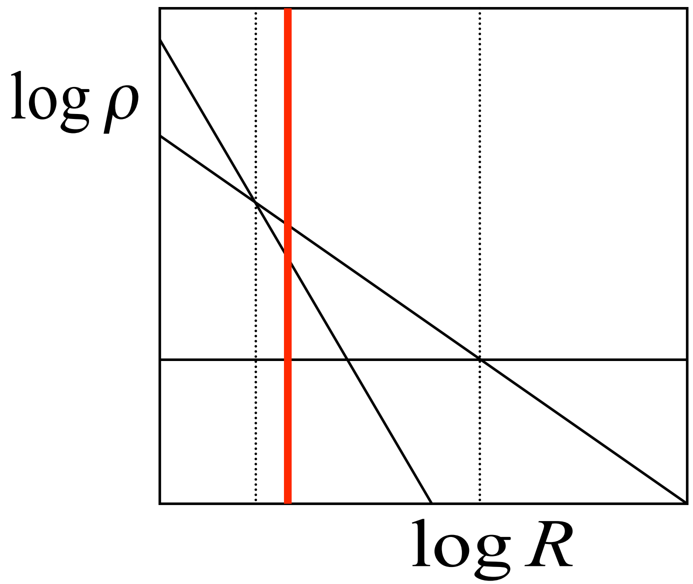
(*including neutrinos*)

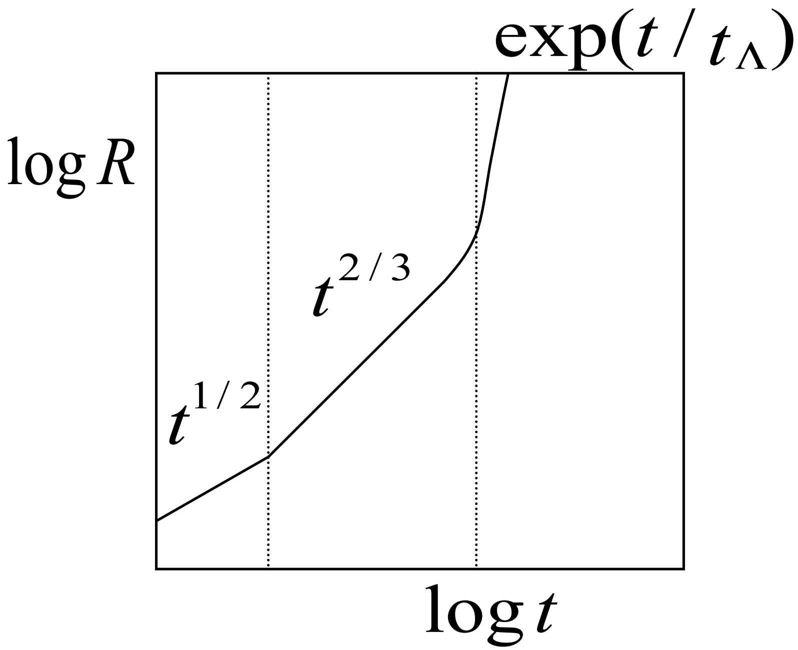
matter - radiation equality :

$$\frac{\Omega_R(z)}{\Omega_M(z)} = \frac{(1+z)^4\,\Omega_R}{(1+z)^3\,\Omega_M} = 1$$

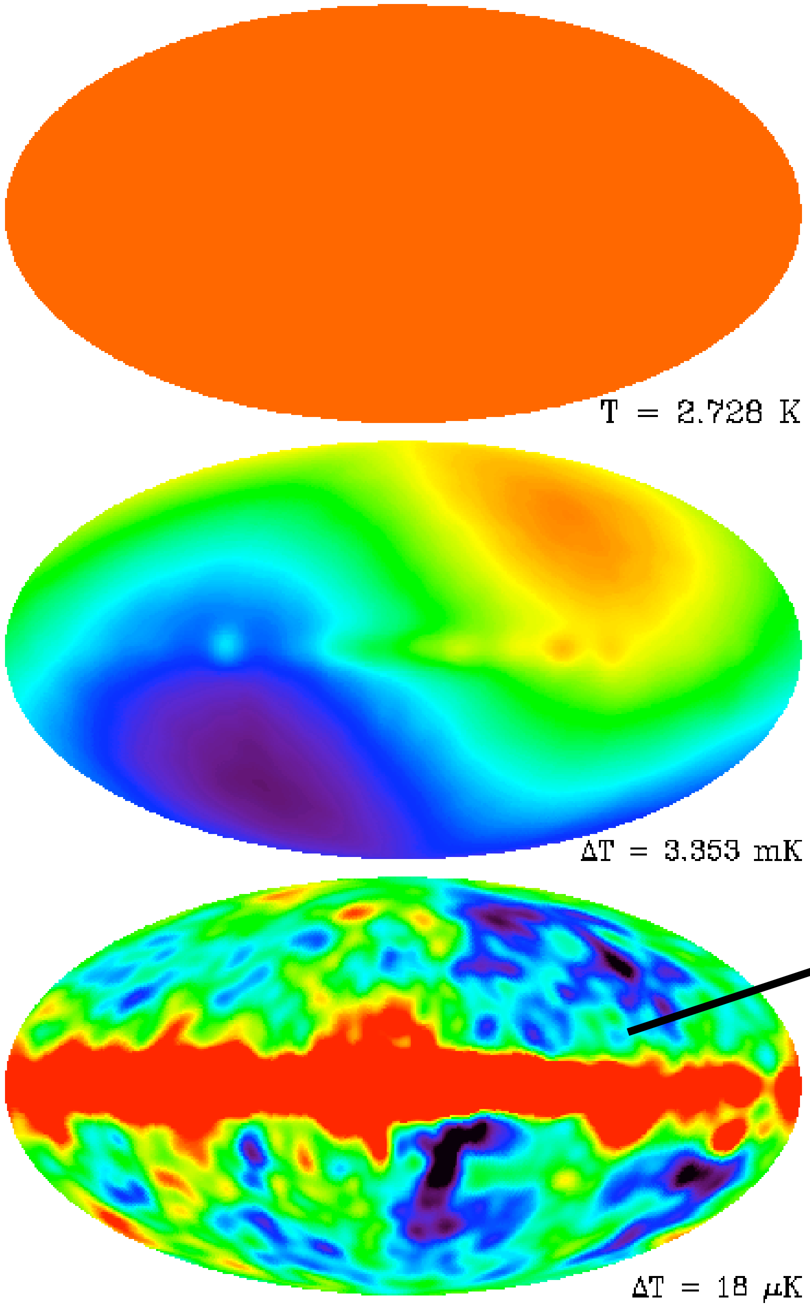
$$(1+z_{eq}) = \frac{\Omega_M}{\Omega_R} = 3500\left(\frac{\Omega_M}{0.3}\right)\left(\frac{2.73K}{T}\right)^4$$

Matter-dominated at z=1100





Cosmic Microwave Background



Almost isotropic

$T = 2.728$ K

Dipole anisotropy

$$\frac{V}{c} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T} \approx 10^{-3}$$

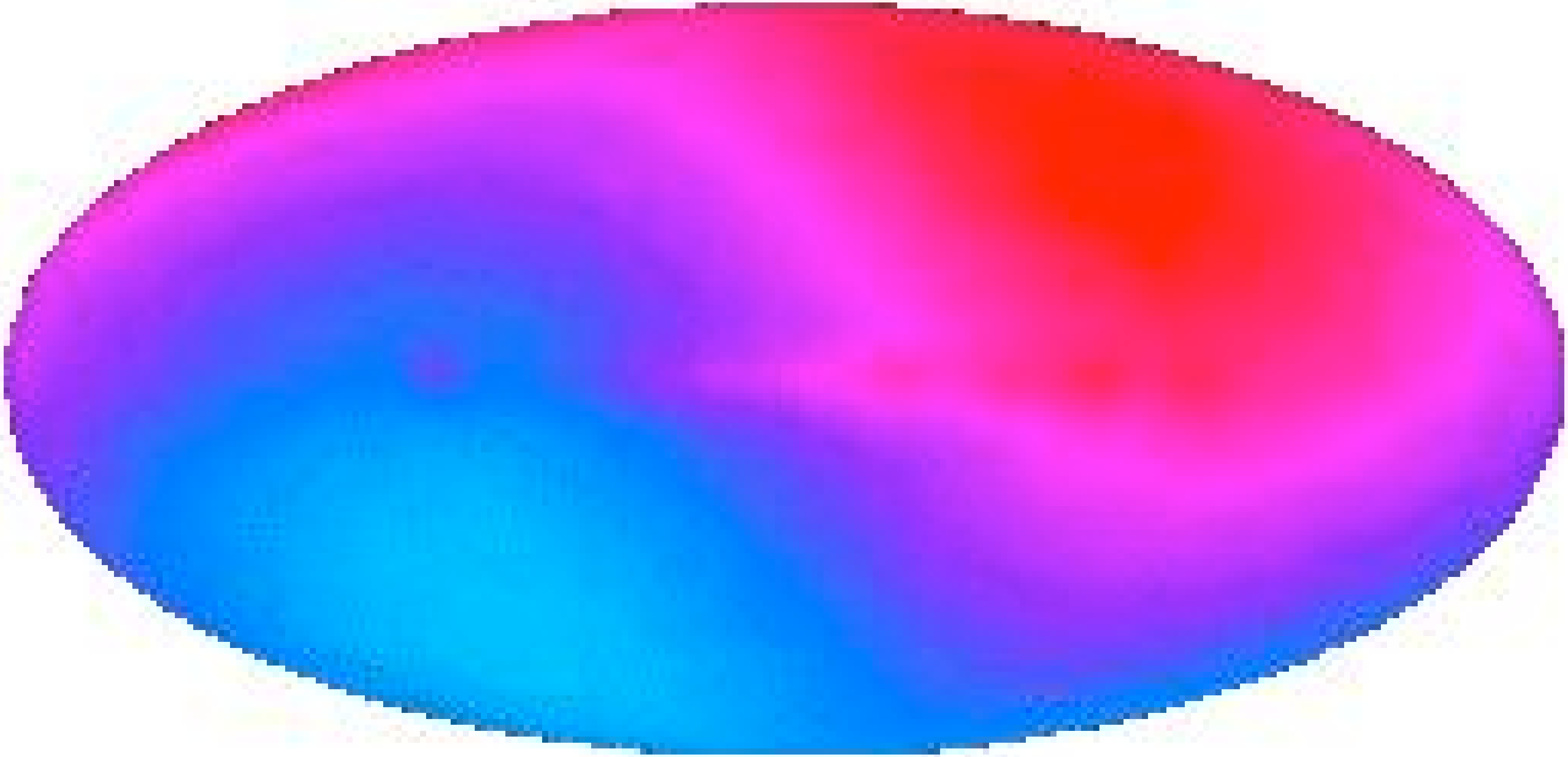
Our velocity:

$V \approx 600$ km/s

Milky Way sources

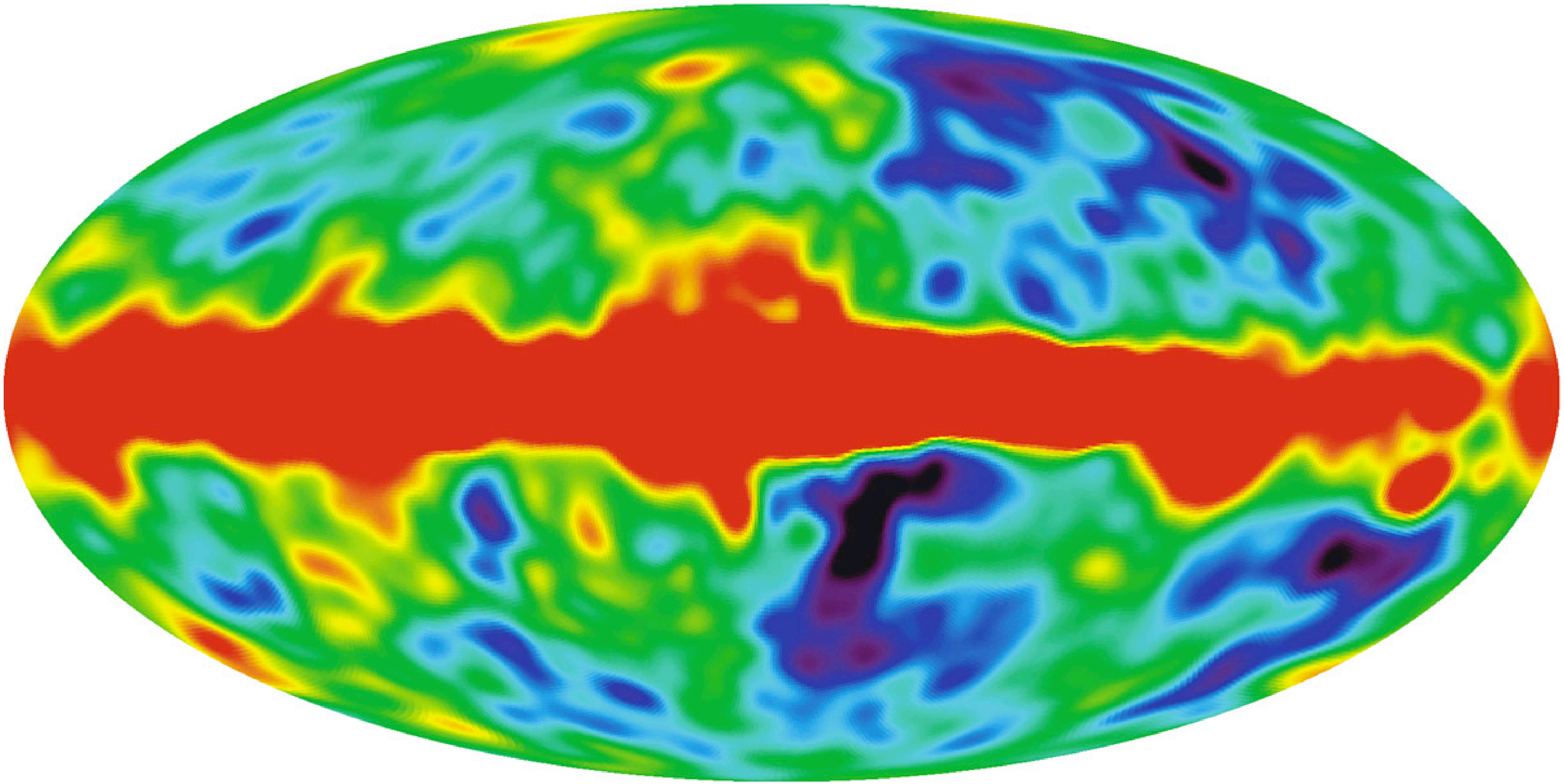
$+$ anisotropies $\dfrac{\Delta T}{T} \sim 10^{-5}$

0.1% CMB dipole anisotropy velocity relative to CMB photons



$$T(\theta) = T_0 \left(1 + \frac{V}{c}\cos\theta + \ldots\right) \rightarrow \begin{array}{c} V_{SUN} = 371 \pm 1\ \mathrm{km\ s^{-1}} \\ V_{MW} \approx 600\ \mathrm{km\ s^{-1}} \end{array}$$

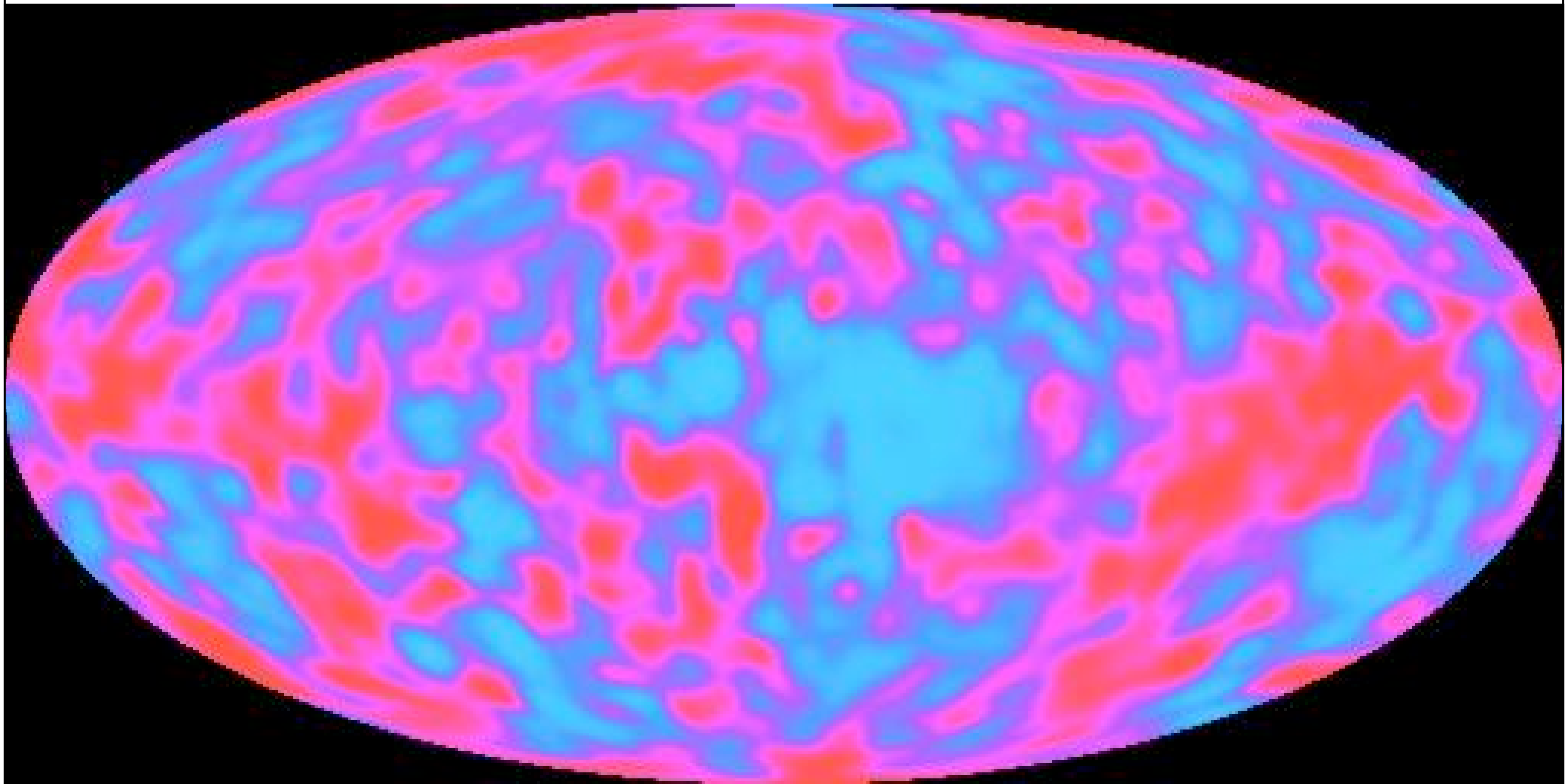
COBE 4-year map



Milky Way emission

subtract by using maps at several frequencies

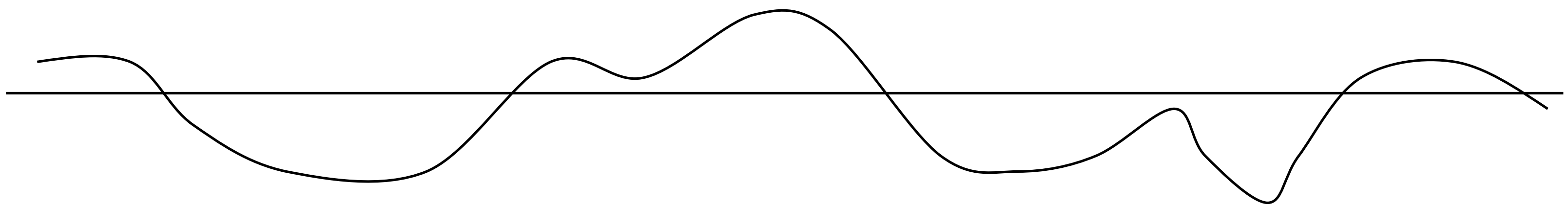
COBE - tiny ripples



$$\frac{\delta T}{T} \sim 10^{-5}$$

Resolution ~ 7°

Tiny Ripples at Redshift 1100



$$\frac{\Delta T}{T} \approx \frac{\Delta \rho}{4\rho} \sim 10^{-5} \text{ at } z = 1100$$

Ripples are :

relics of the Big Bang

initial quantum fluctuations expanded by early inflation

the seeds of later galaxy/cluster formation.

standard yardsticks for measuring curvature

(and other cosmology parameters)

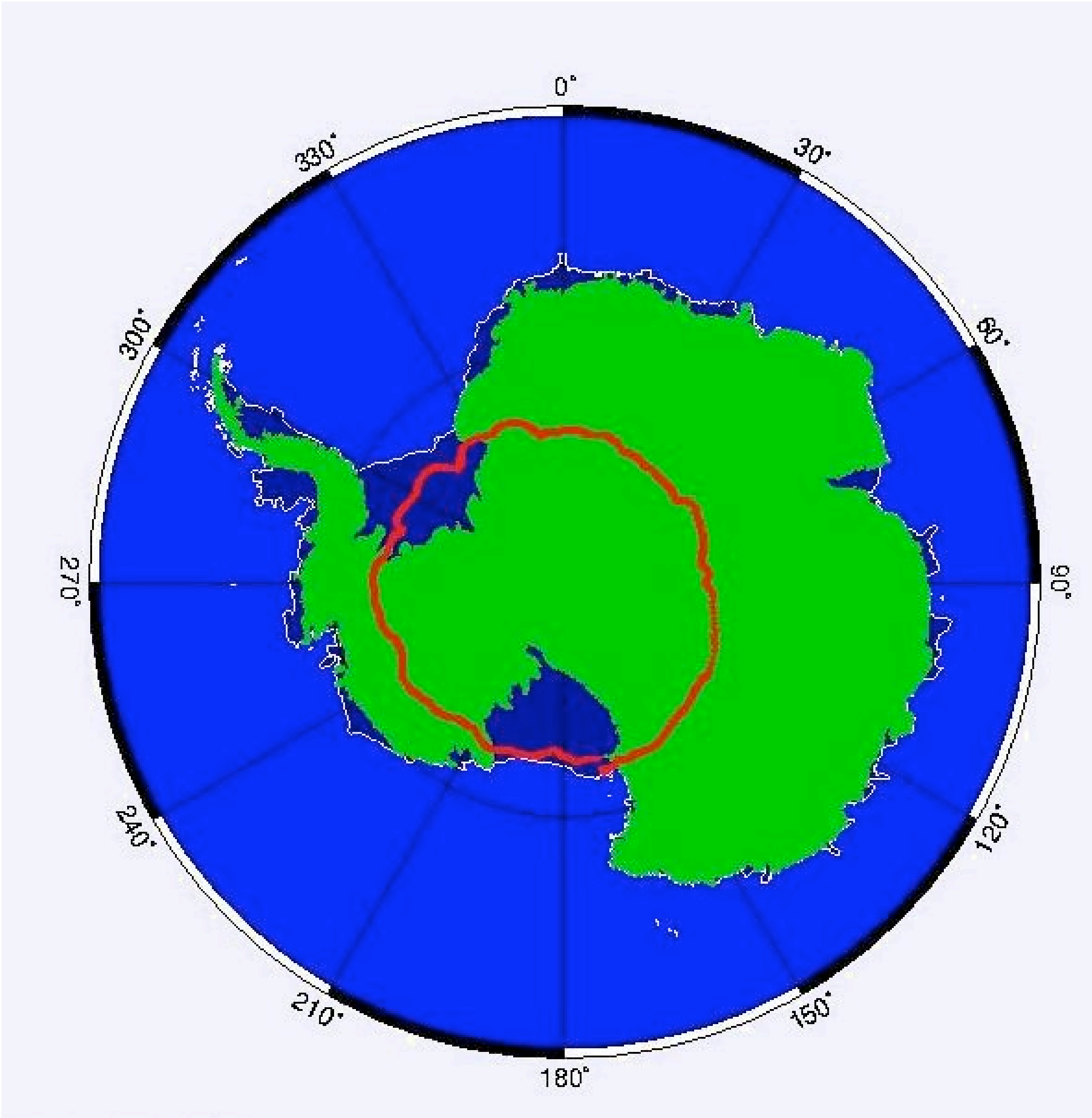
1999 - Boomerang in Antarctica

Baloon Observations Of Millimetric Extragalactic Radiation ANisotropy and Geophysics

AS 4022 Cosmology

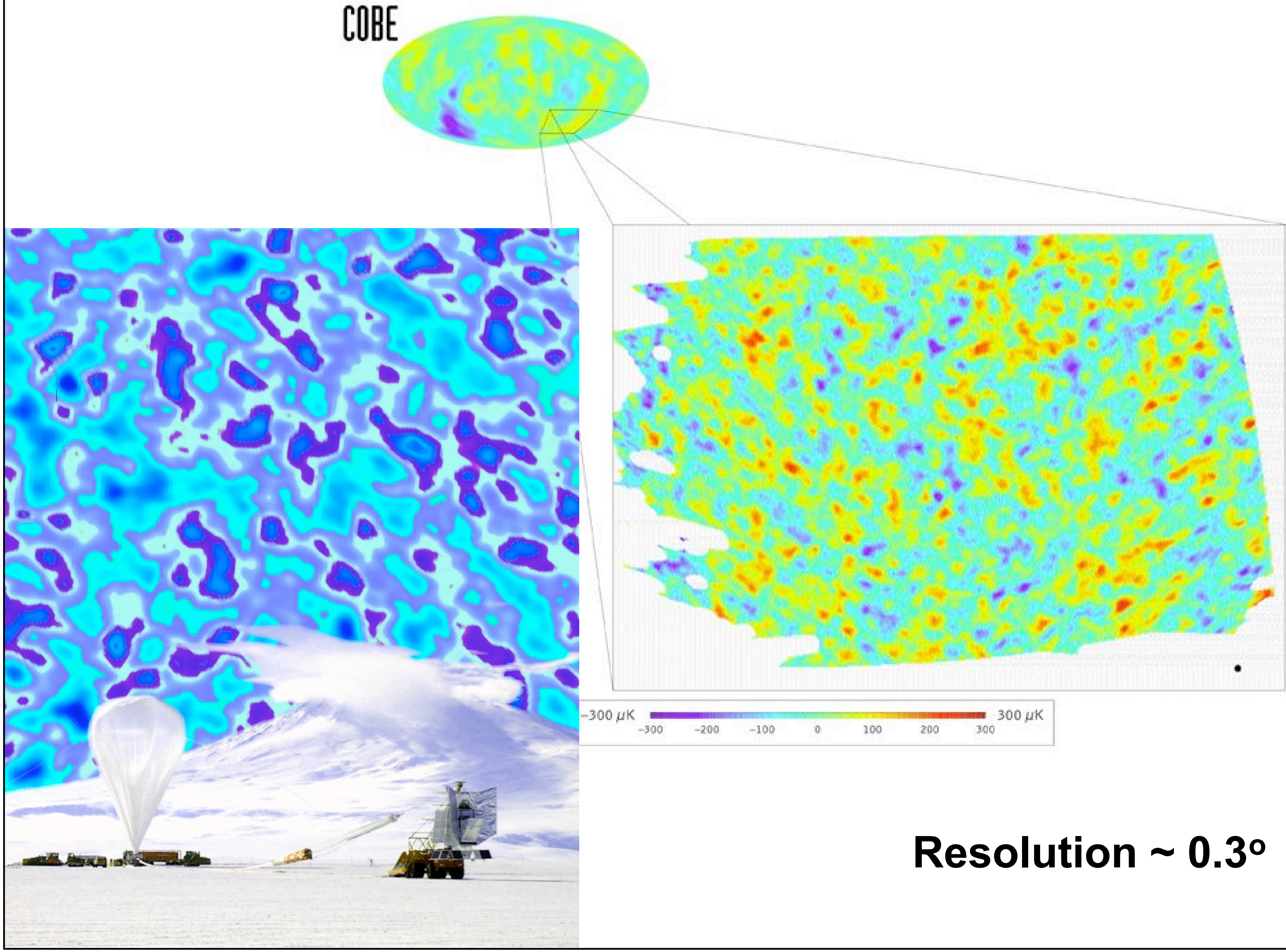
Boomerang's Baloon

Boomerang's Stratospheric Flight Track



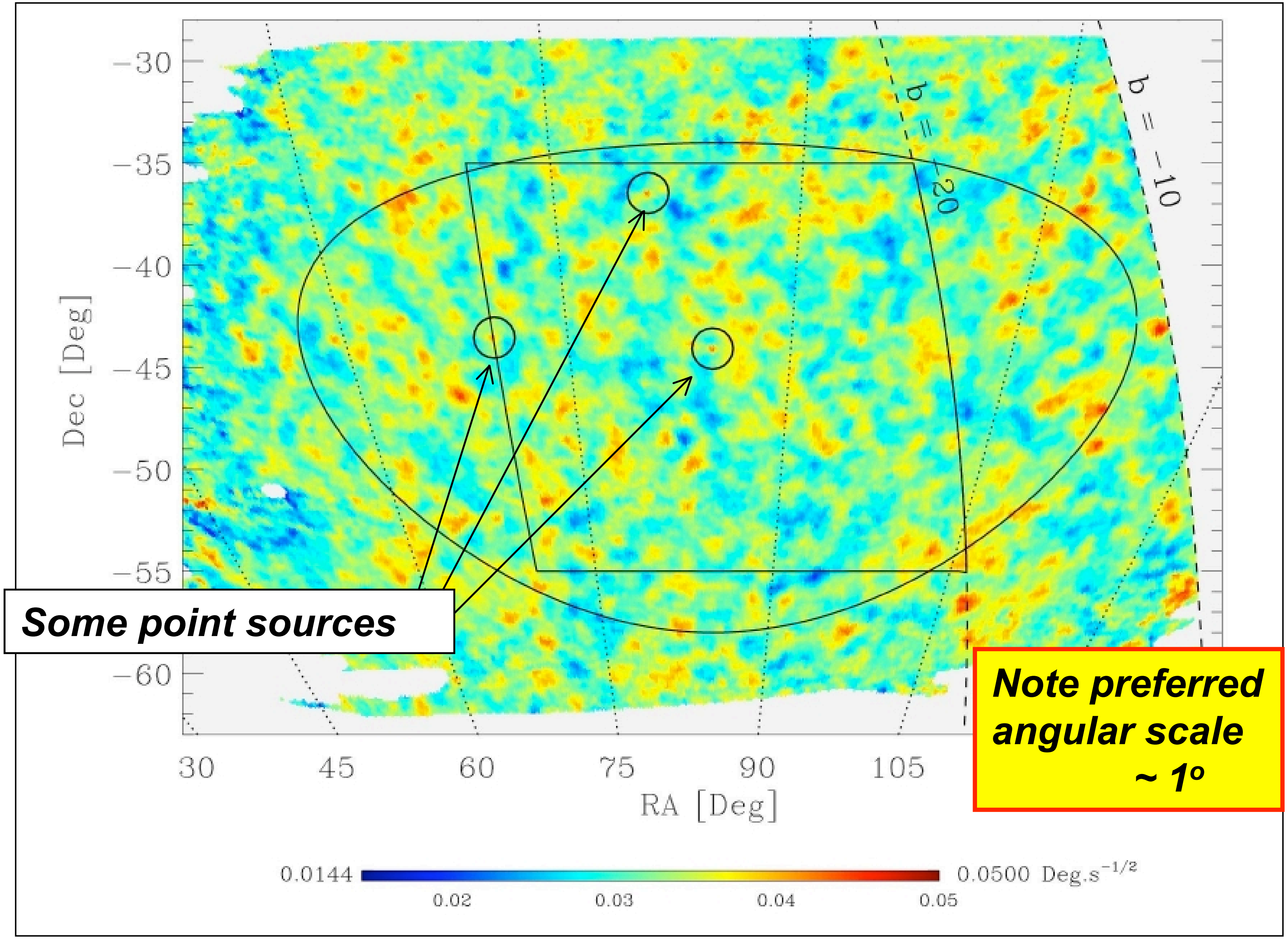
Altitude
37 km

10 days



Resolution ~ 0.3°

Boomerang Map



Spherical Harmonics

Fit temperature map with a series of

spherical harmonics

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi)$$

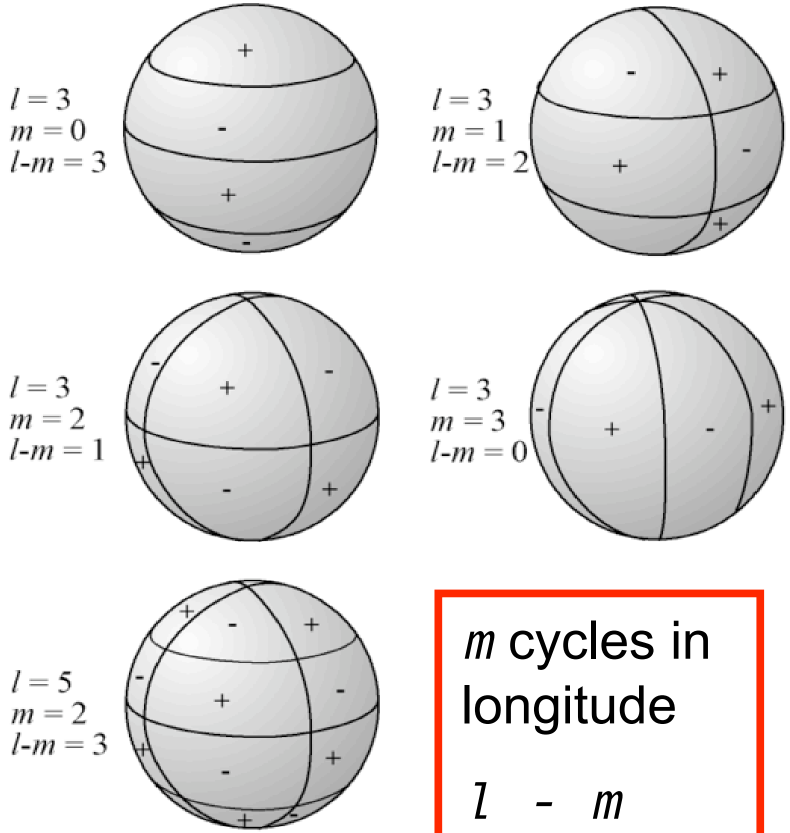
angular power spectrum

$$C_l = \left\langle \left| a_{lm} \right|^2 \right\rangle \quad \text{average} \; -l \le m \le l$$

dimensionless power spectrum

$$l(l+1) C_l \propto \frac{d\left\langle (\Delta T / T)^2 \right\rangle}{d \ln l}$$

$$\text{angular scale:} \quad \Delta\theta \approx \frac{\pi}{l} = \frac{180^\circ}{l}$$



m cycles in longitude

$l - m$ nodes in latitude

Spherical Harmonics

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{-i\varphi}$$

$$Y_1^{0}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\,\cos\theta$$

$$Y_1^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{i\varphi}$$

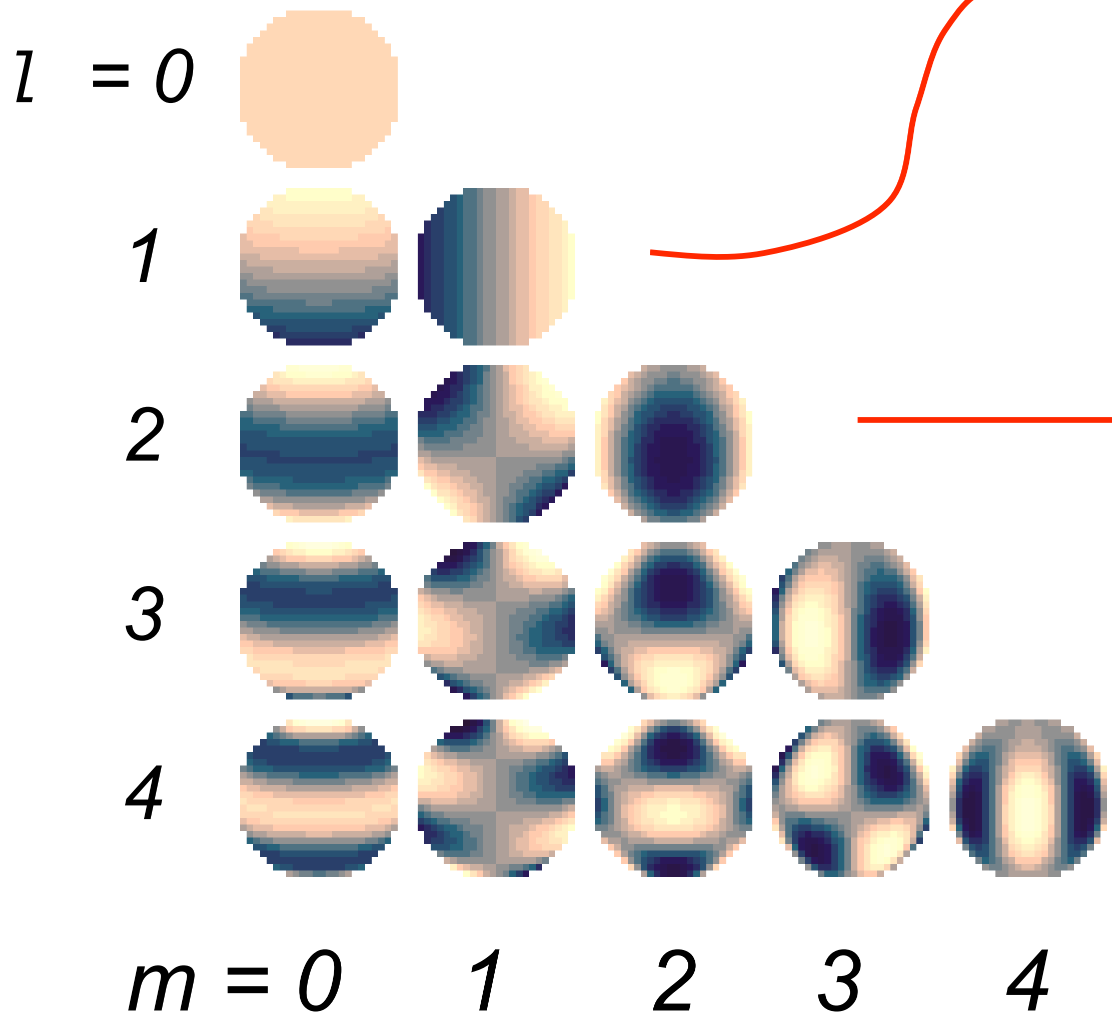
$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\, e^{-2i\varphi}$$

$$Y_2^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{15}{2\pi}}\,\sin\theta\,\cos\theta\, e^{-i\varphi}$$

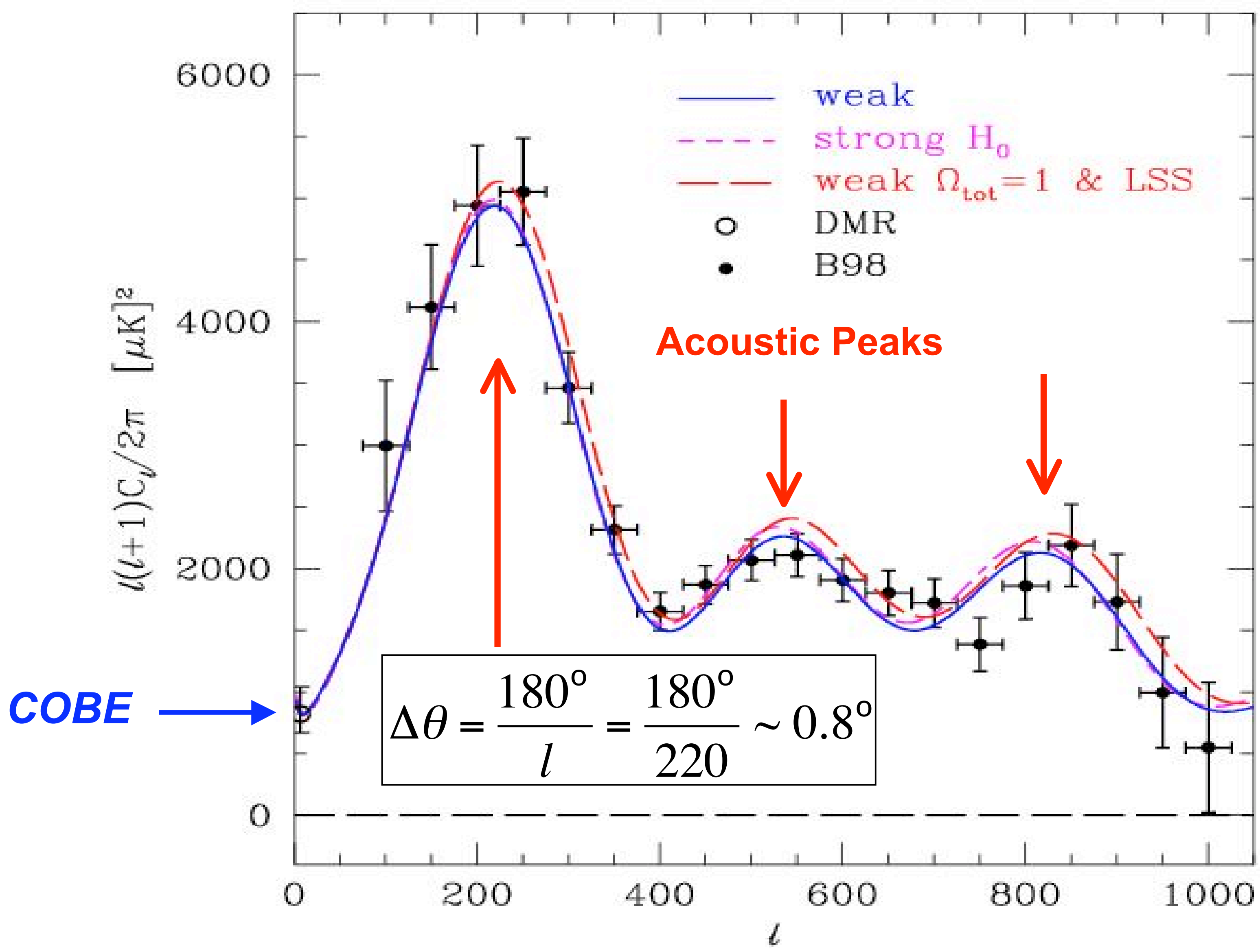
$$Y_2^{0}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}\,(3\cos^2\theta - 1)$$

$$Y_2^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{15}{2\pi}}\,\sin\theta\,\cos\theta\, e^{i\varphi}$$

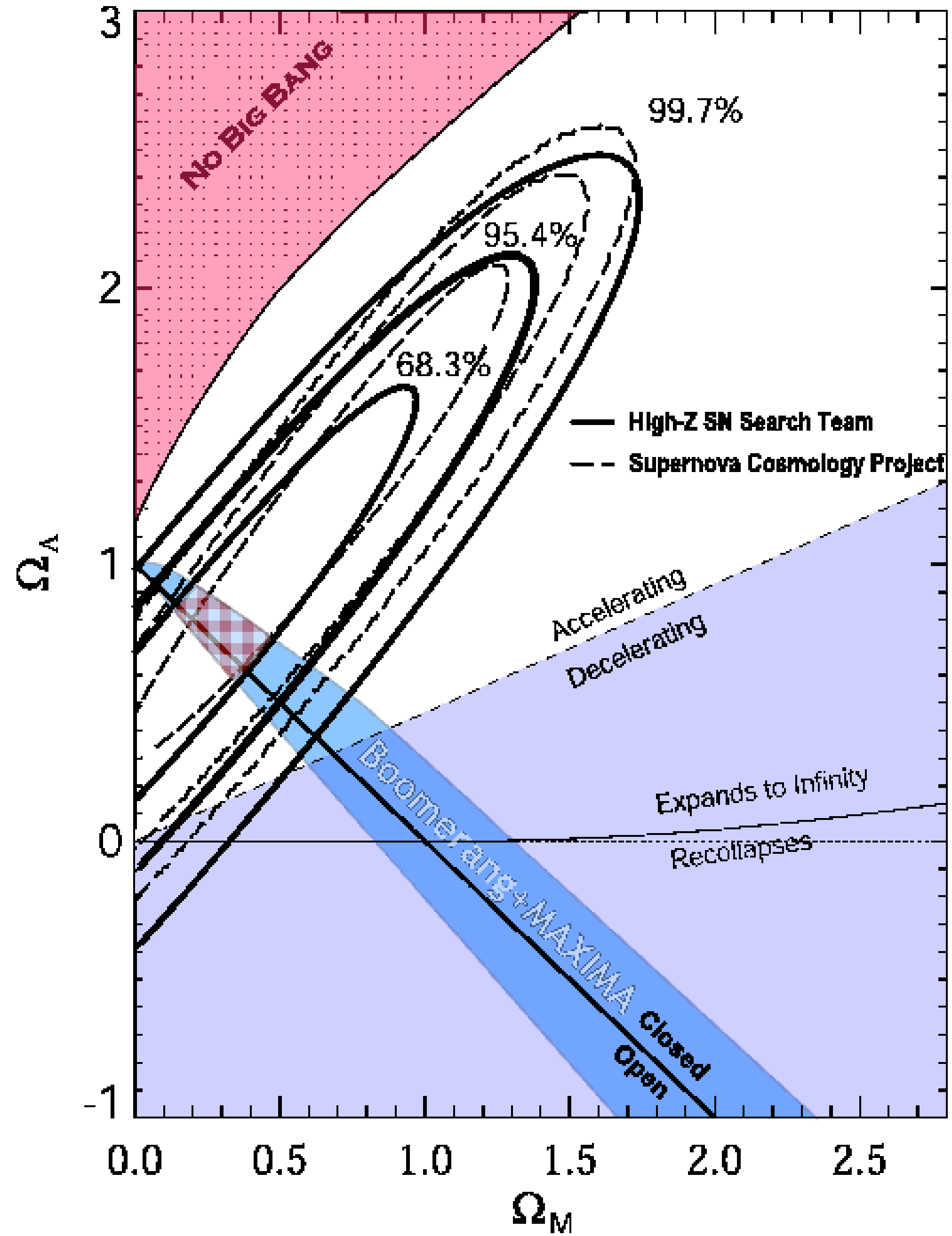
$$Y_2^{2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\, e^{2i\varphi}$$



Boomerang Power Spectrum



Supernovae
+
CMB ripples



Pre-WMAP constraints

From BOOMERANG

and MAXIMA

circa 2002

WMAP

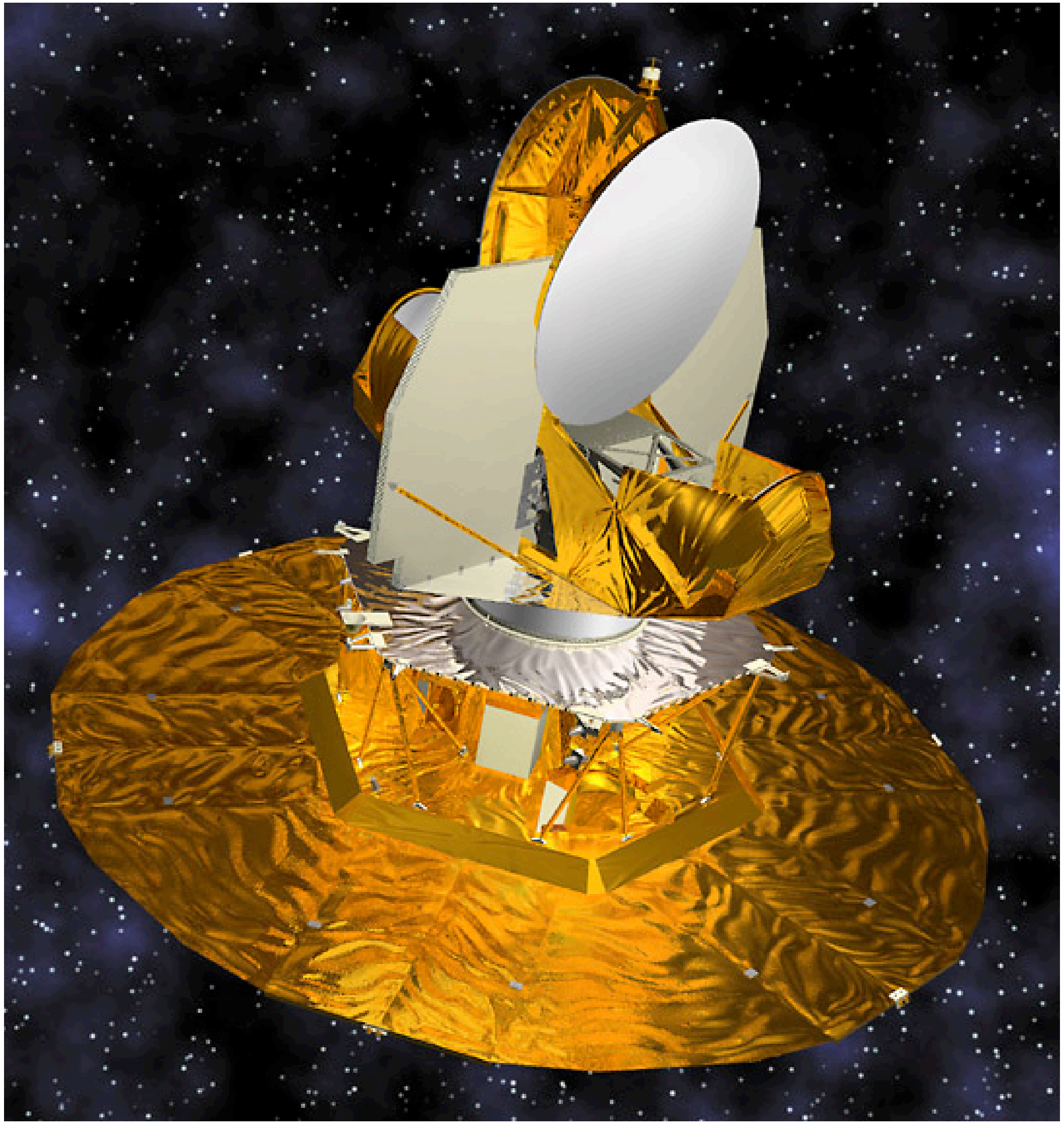
NASA 2001...

Wilkinson

Microwave

Anisotropy

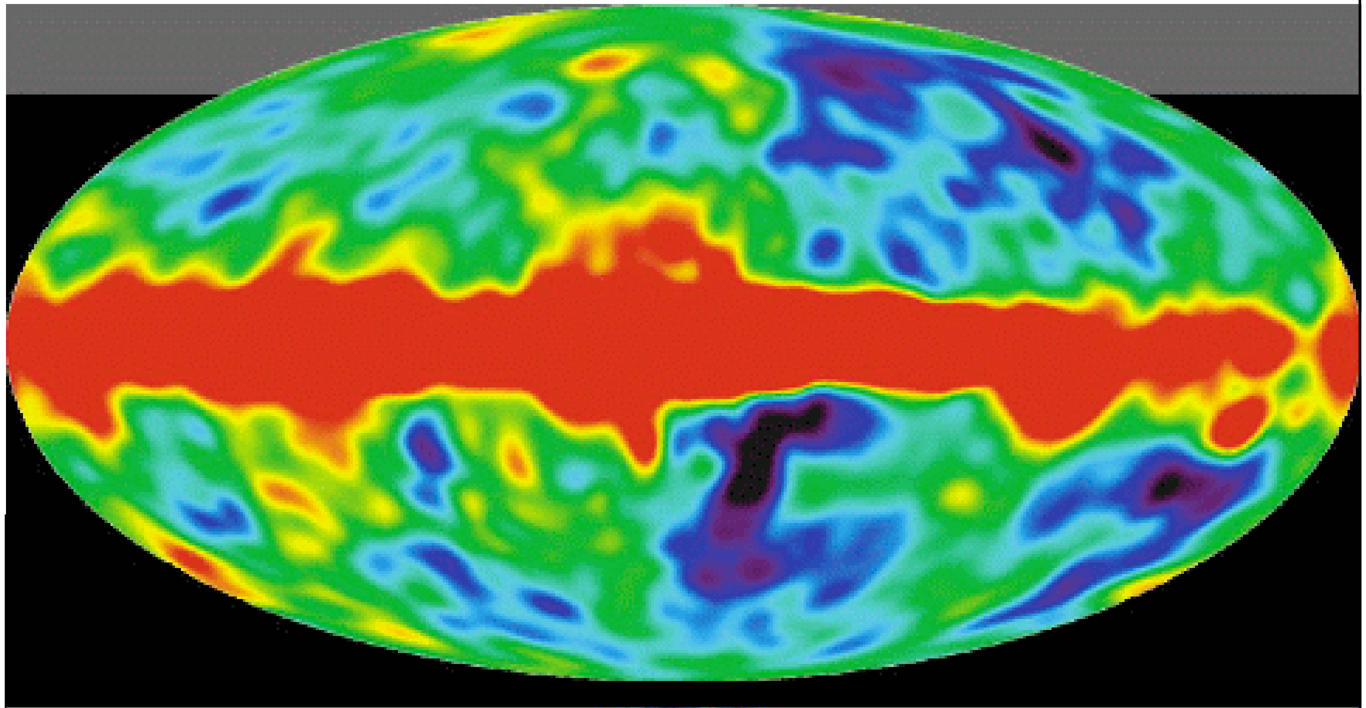
Probe



COBE

1992

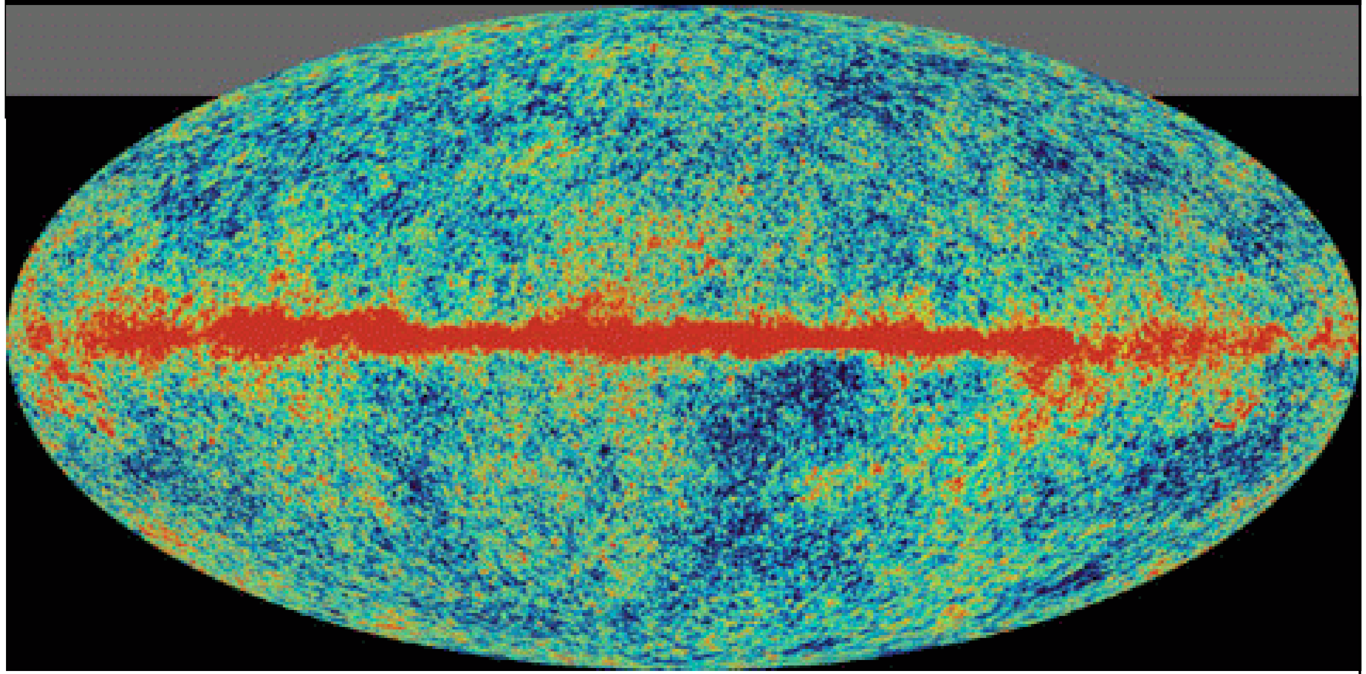
7 degree resolution



WMAP

2003

20 arcmin resolution



COBE

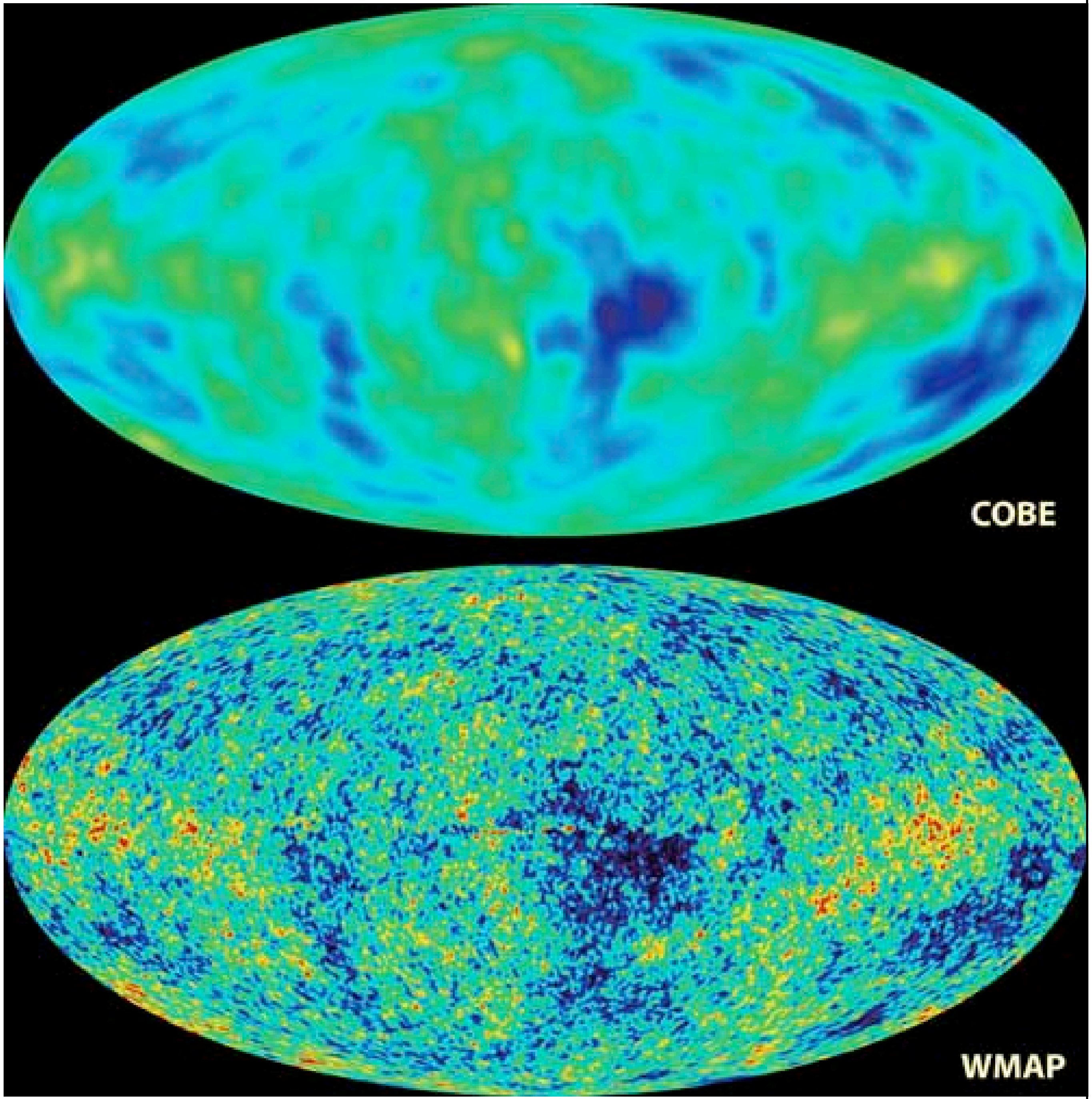
1992

7 degree resolution

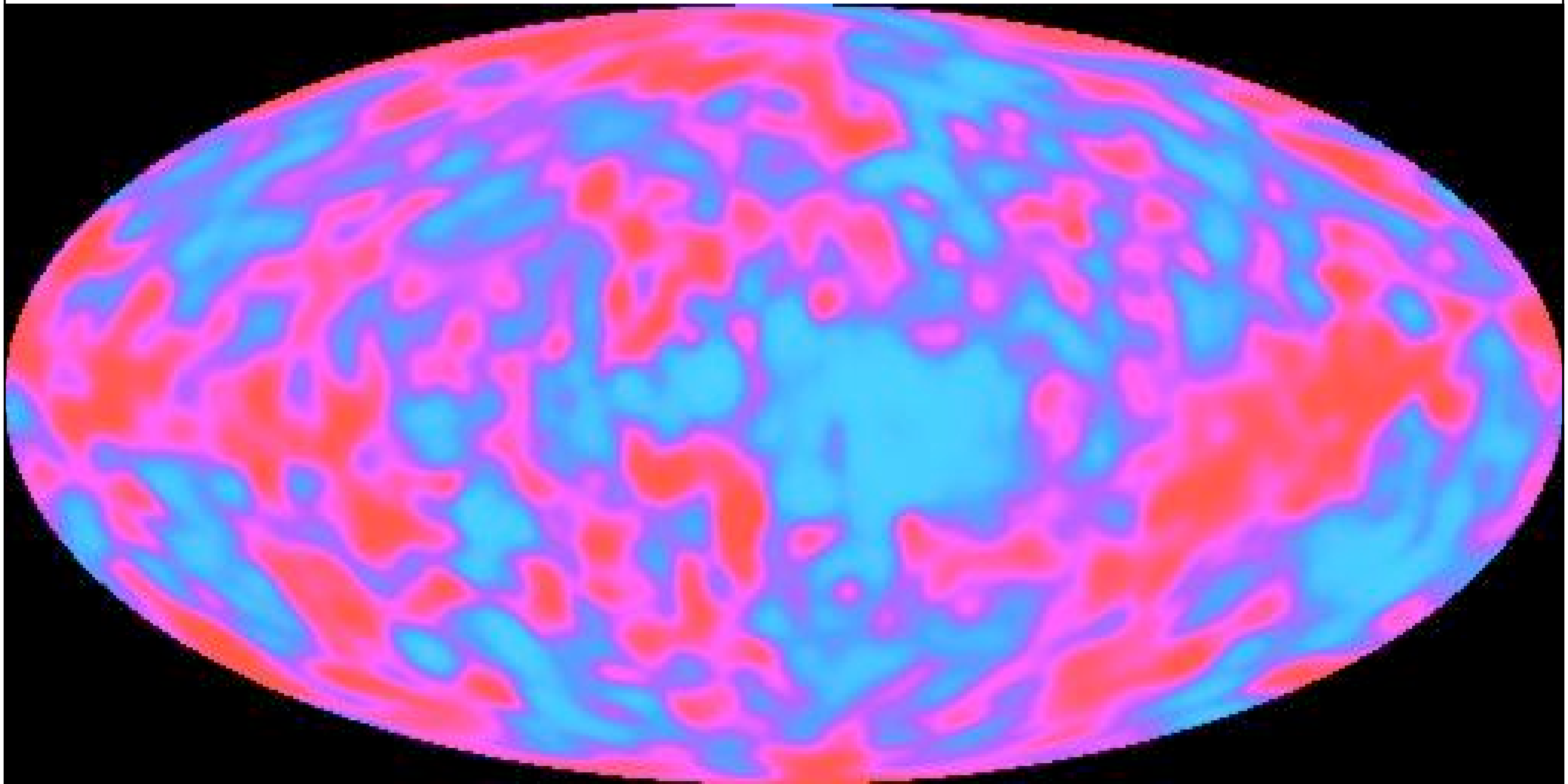
WMAP

2003

20 arcmin resolution



COBE - temperature ripples

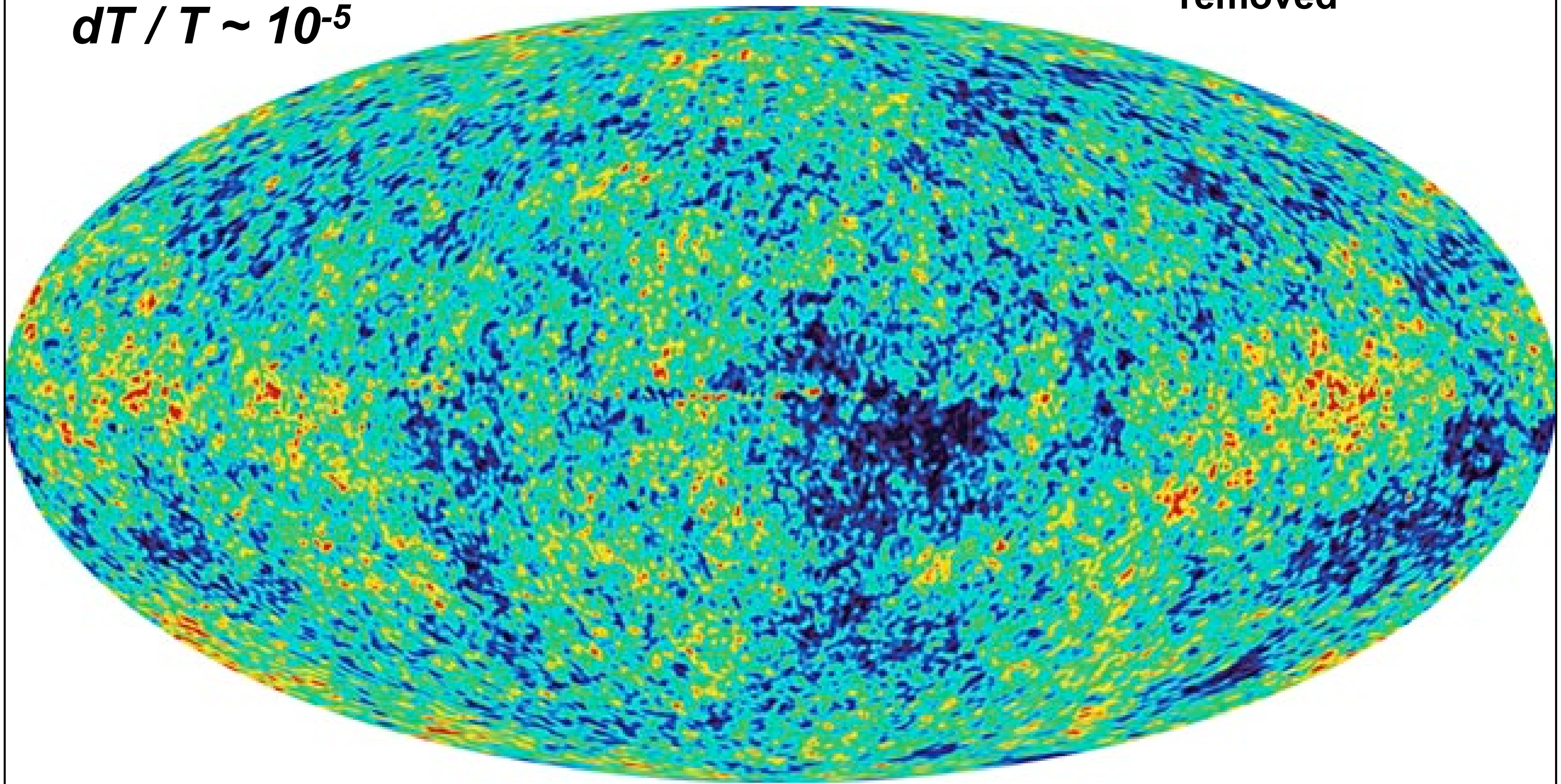


2003 WMAP all-sky

T = 2.73 K

dT / T ~ 10^{-5}

Dipole and foreground galaxy removed

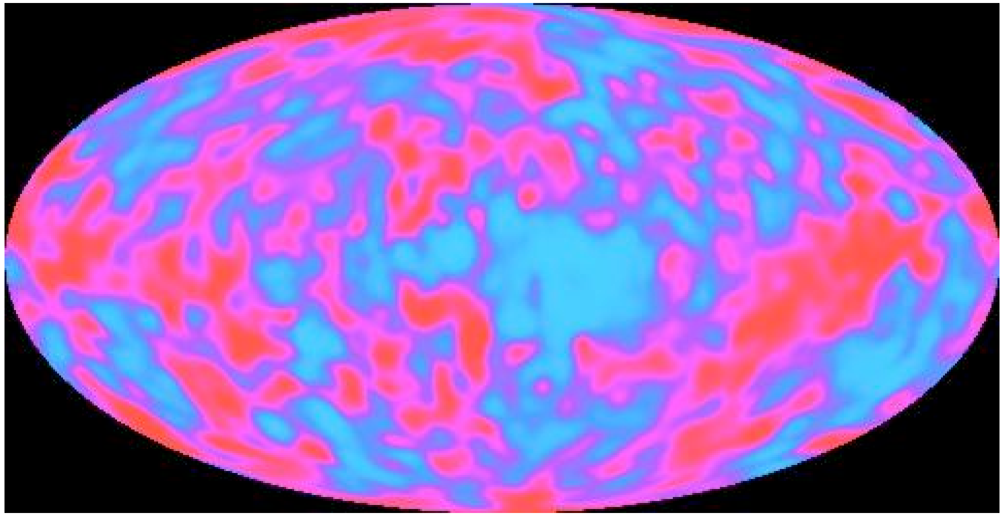


Snapshot at z=1100 of quantum fluctuations stretched by inflation.

Dark matter potential wells that seed later galaxy formation.

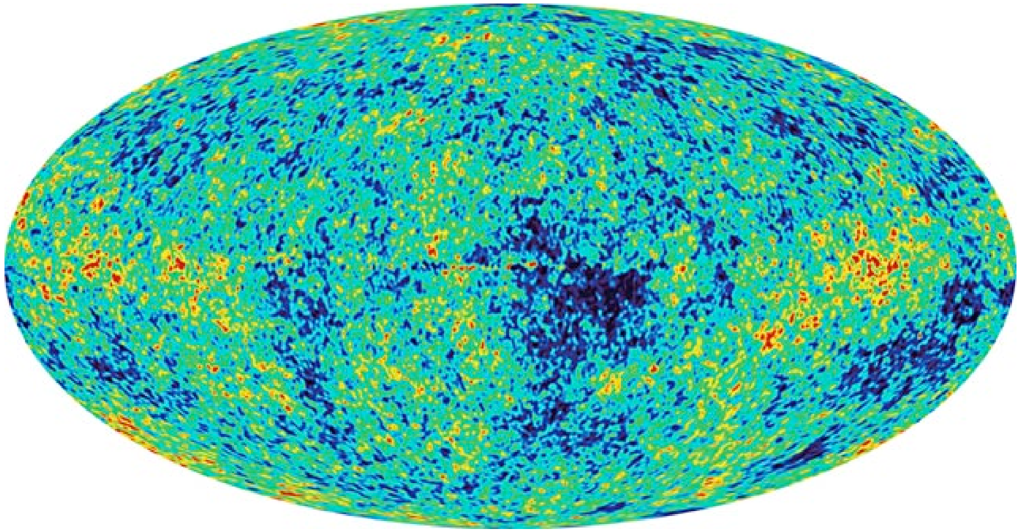
CMB Anisotropies

COBE
1994



$$\frac{\Delta T}{T} \sim 10^{-5}$$

WMAP
2004



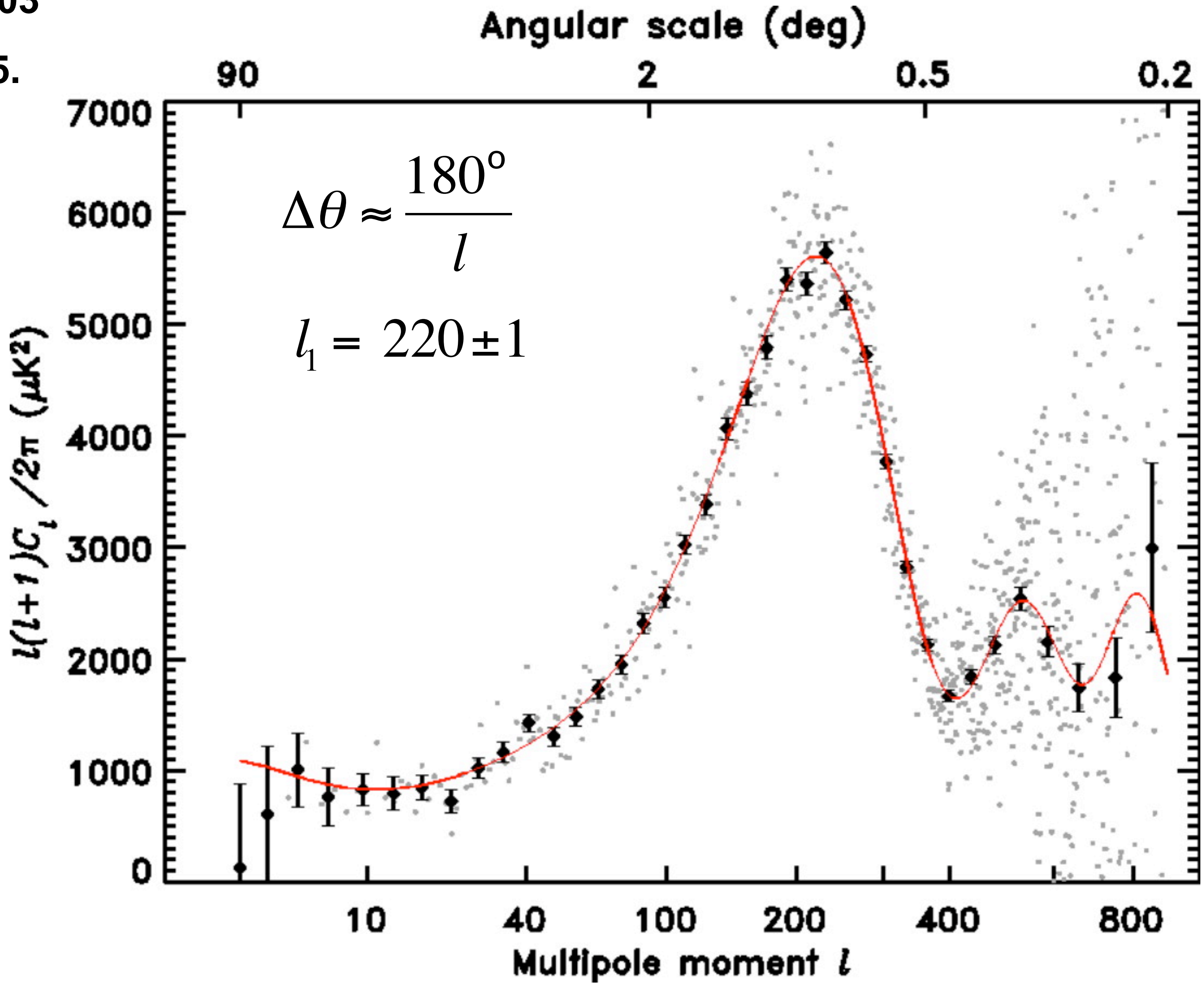
$$\Delta\theta \sim 1^{\circ}$$

Snapshot of Universe at z = 1100
Seeds that later form galaxies.

2003 -- WMAP Power Spectrum

Spergel et al. 2003

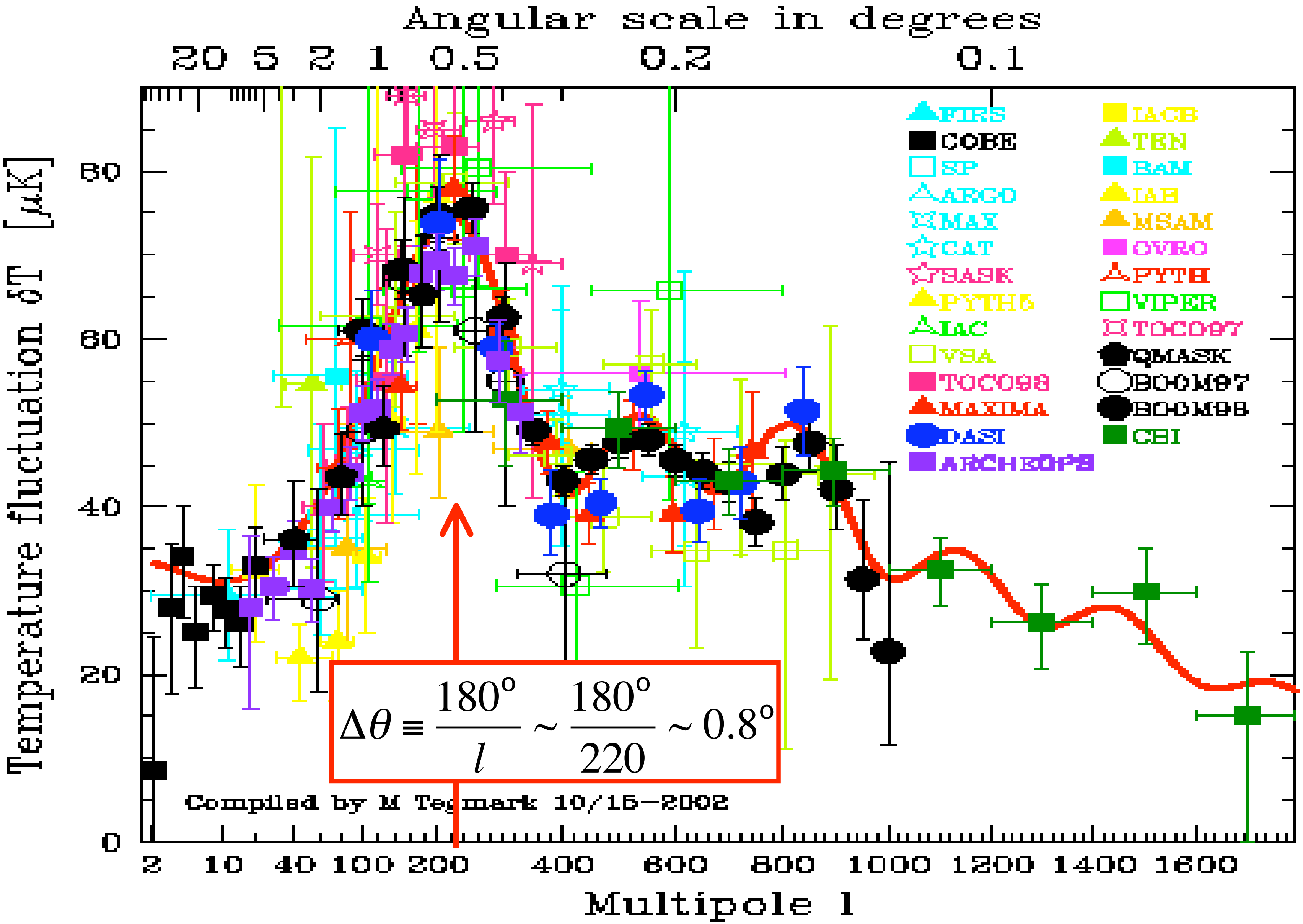
ApJSup 148,175.



$$\Delta\theta \approx \frac{180^{o}}{l}$$

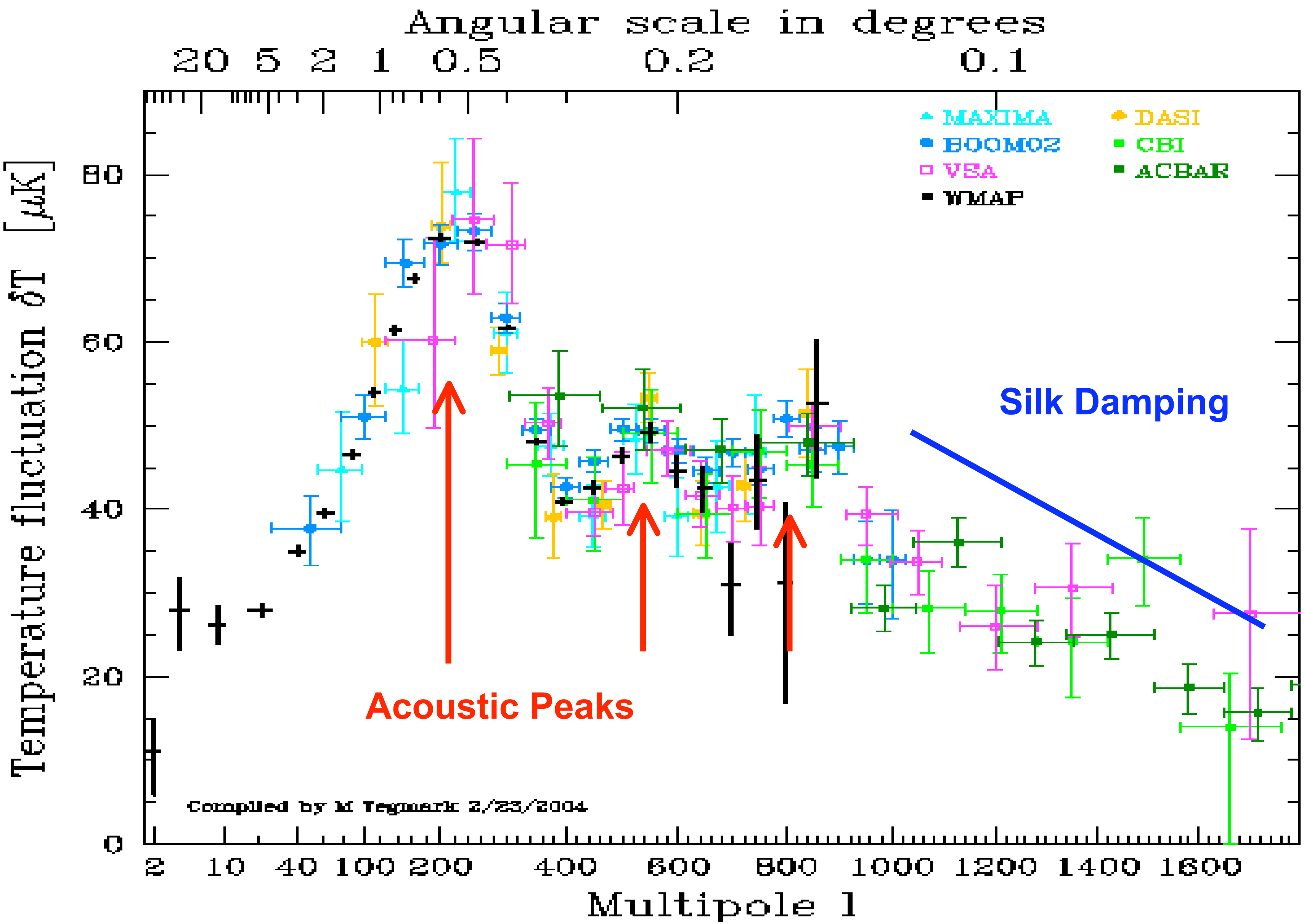
$$l_1 = 220 \pm 1$$

2002 - CMB Power Spectrum

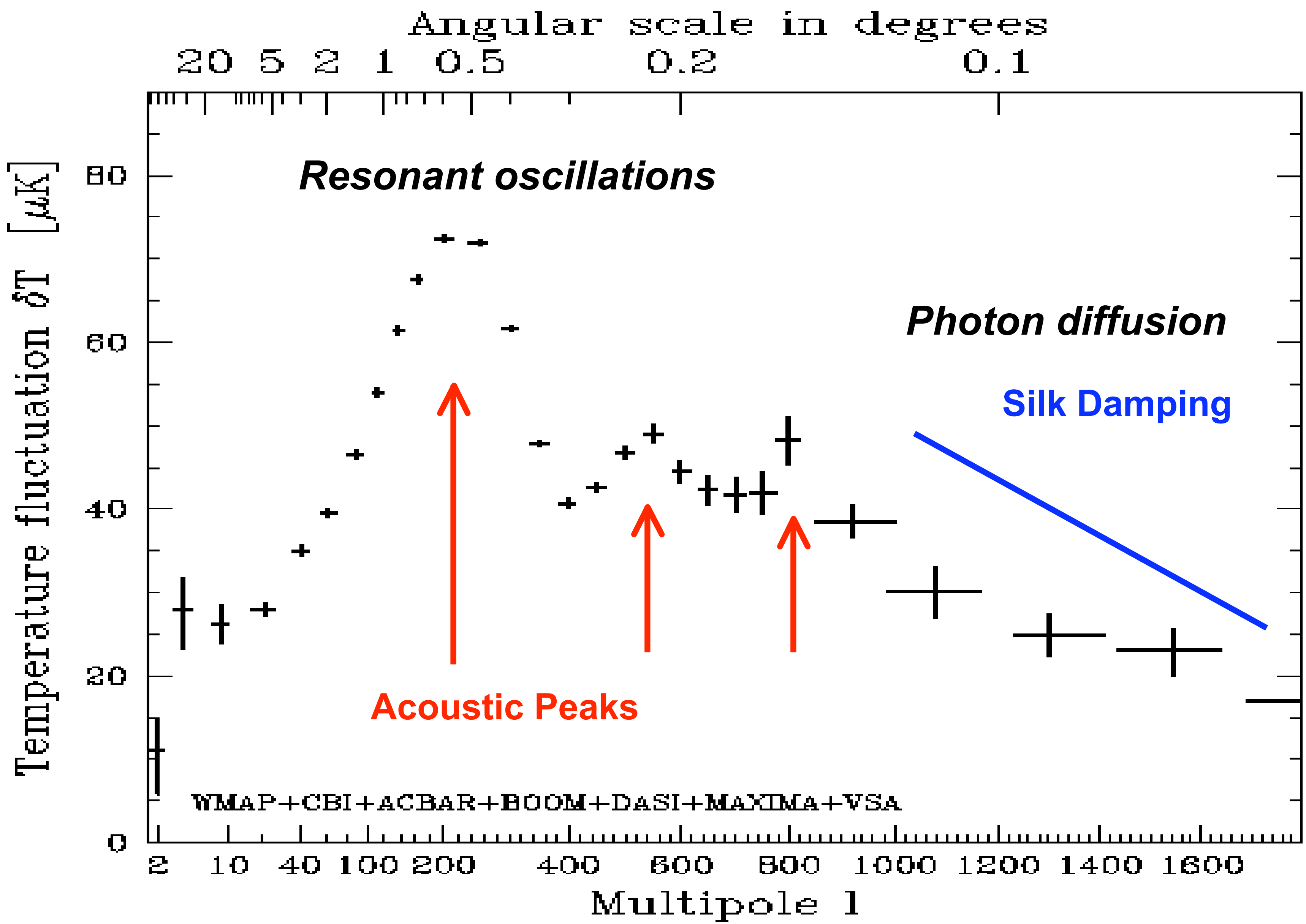


$$\Delta\theta \equiv \frac{180^{o}}{l} \sim \frac{180^{o}}{220} \sim 0.8^{o}$$

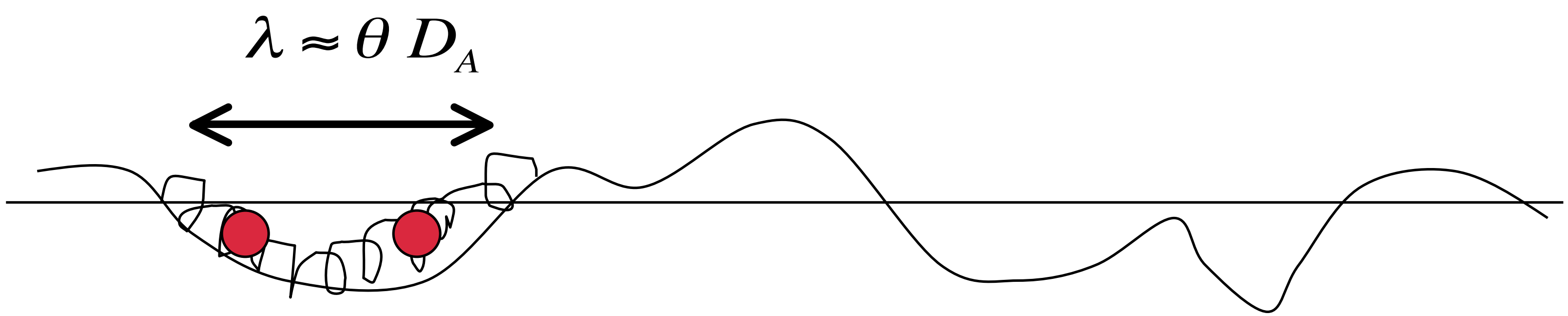
2004 - CMB Power Spectrum



2004 - CMB Binned



Acoustic Oscillations



Dark Matter potential wells - many sizes.

photon-electron-baryon fluid

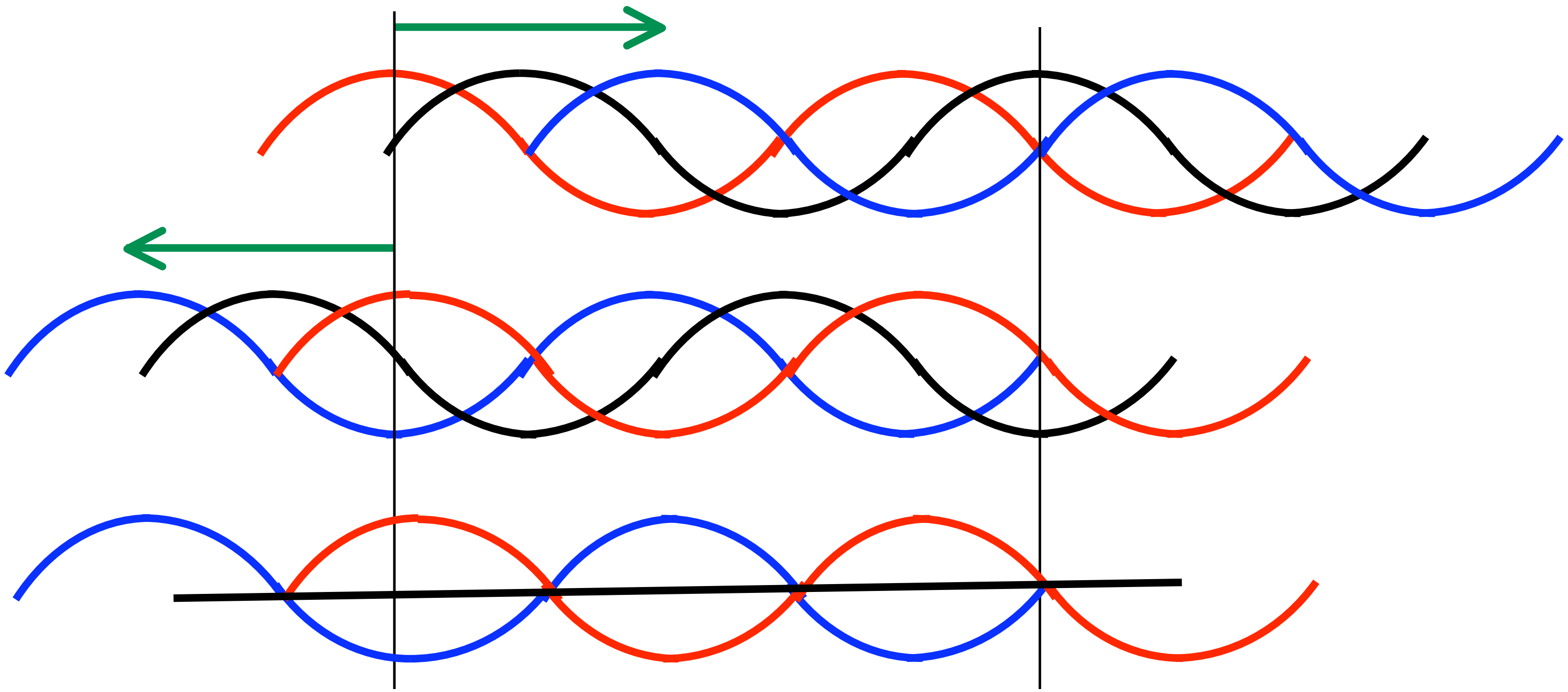
fluid falls into DM wells

photon pressure pushes it out again

oscillations starting at t = 0 (post-inflation)

stopping at z = 1100 (recombination)

Standing Sound Waves



temperature oscillations: $c_s = \frac{\lambda}{P} = \frac{\omega}{k} = \frac{c}{\sqrt{3}}$

$$\Delta T(x,t) = a\cos(\omega t)\cos(k x) \qquad \omega = \frac{2\pi}{P} \qquad k = |\mathbf{k}| = \frac{2\pi}{\lambda}$$

$$\Delta T(\mathbf{x},t) = \sum_{\mathbf{k}} a(\mathbf{k})\cos(\omega t)\cos(\mathbf{k}\bullet\mathbf{x}) \qquad \langle a(k)\rangle = a_0\left(\frac{k}{k_0}\right)^{n_S}$$

Resonant Oscillations

size of potential well λ

oscillation period $P \approx \dfrac{\lambda}{c_s}$

sound speed $c_s = \dfrac{c}{\sqrt{3}}$

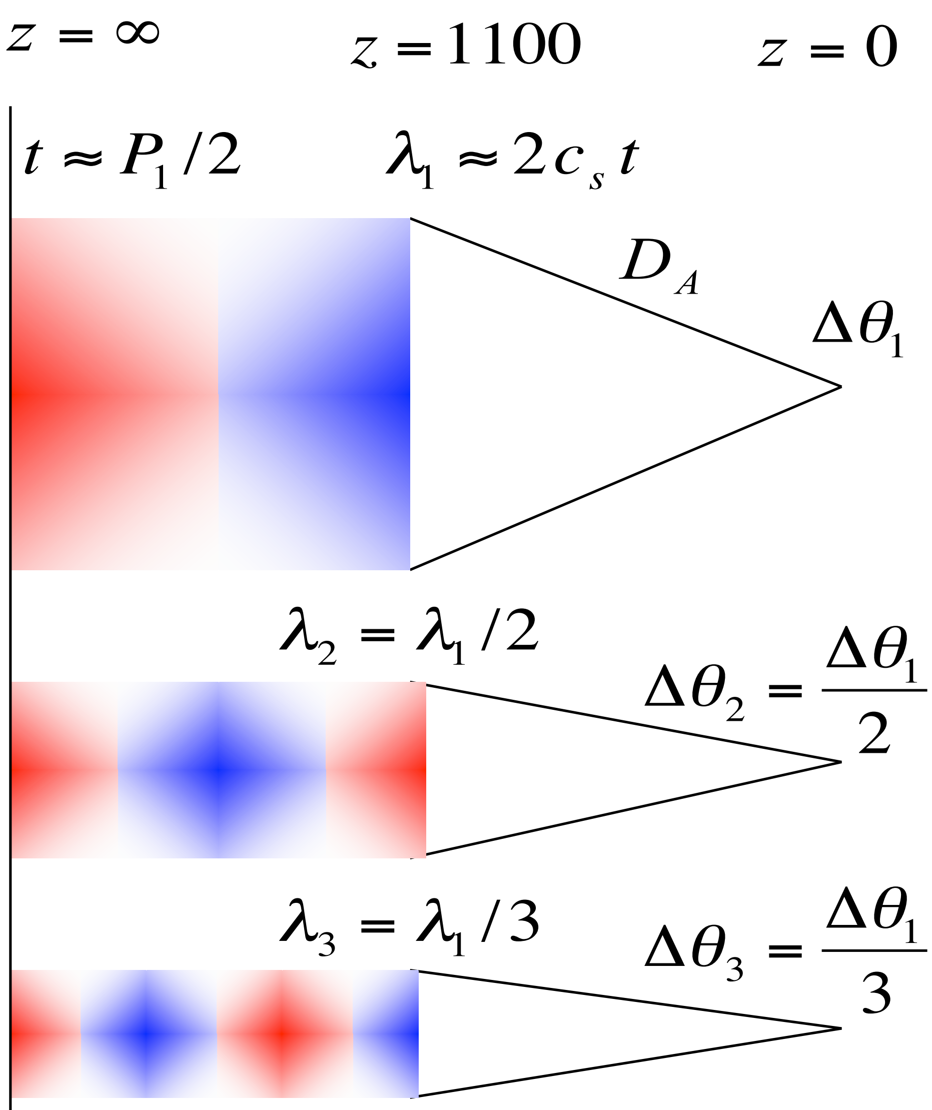
temperature oscillations

$$\Delta T(t) = \Delta T(0)\cos(2\pi\, t / P)$$

$$\max|\Delta T| \text{ at } t = \frac{n\,P}{2} \sim \frac{n\,\lambda}{2c_s}$$

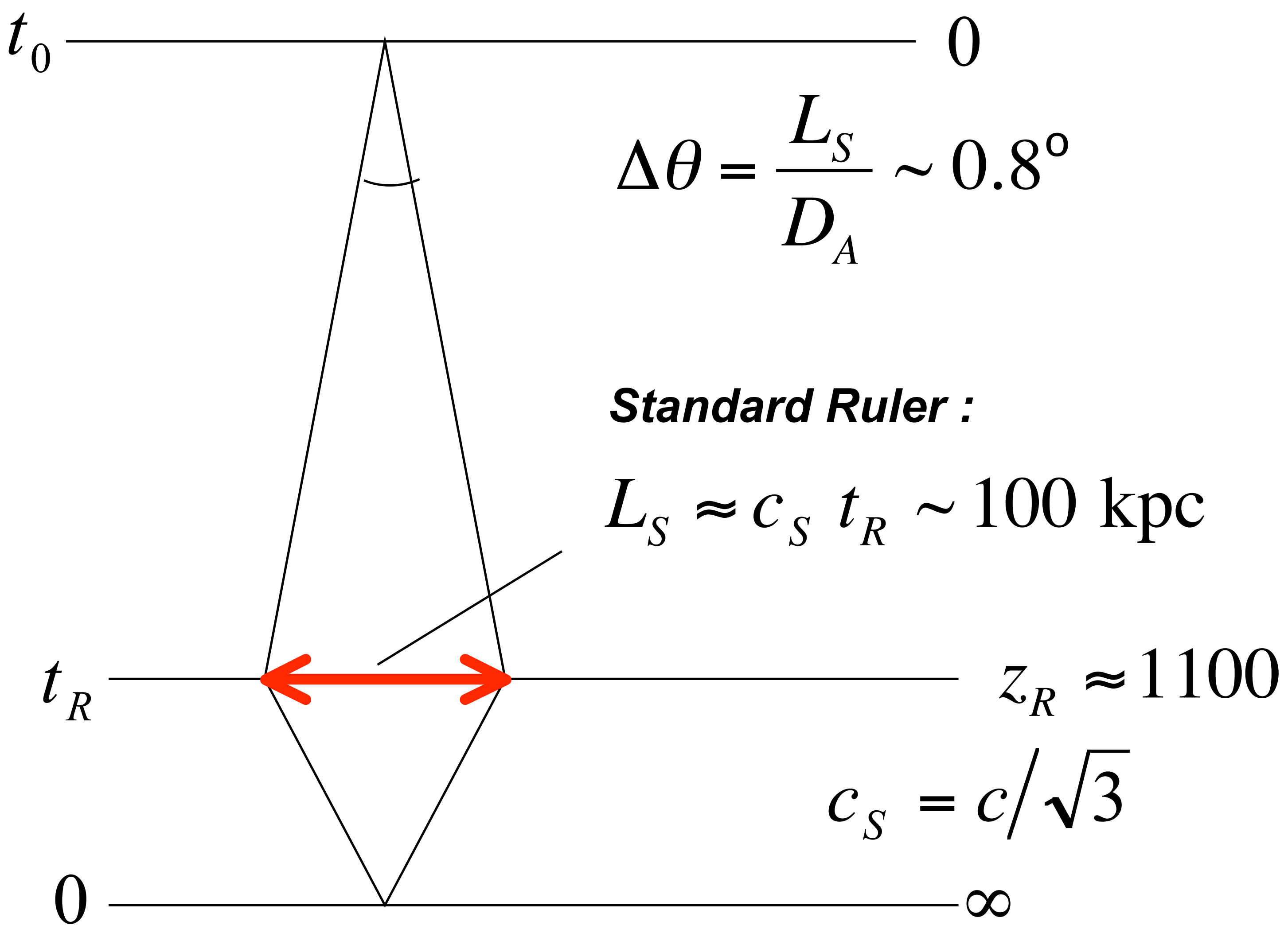
angular size

$$\Delta\theta_n = \frac{\lambda_n}{D_A} = \frac{\Delta\theta_1}{n} \qquad \Delta\theta_1 \approx \frac{2c_s\,t}{D_A} \sim 0.8^\circ$$



Smaller wells oscillate faster.

Sound Horizon at z = 1100



$$\Delta\theta = \frac{L_S}{D_A} \sim 0.8^\circ$$

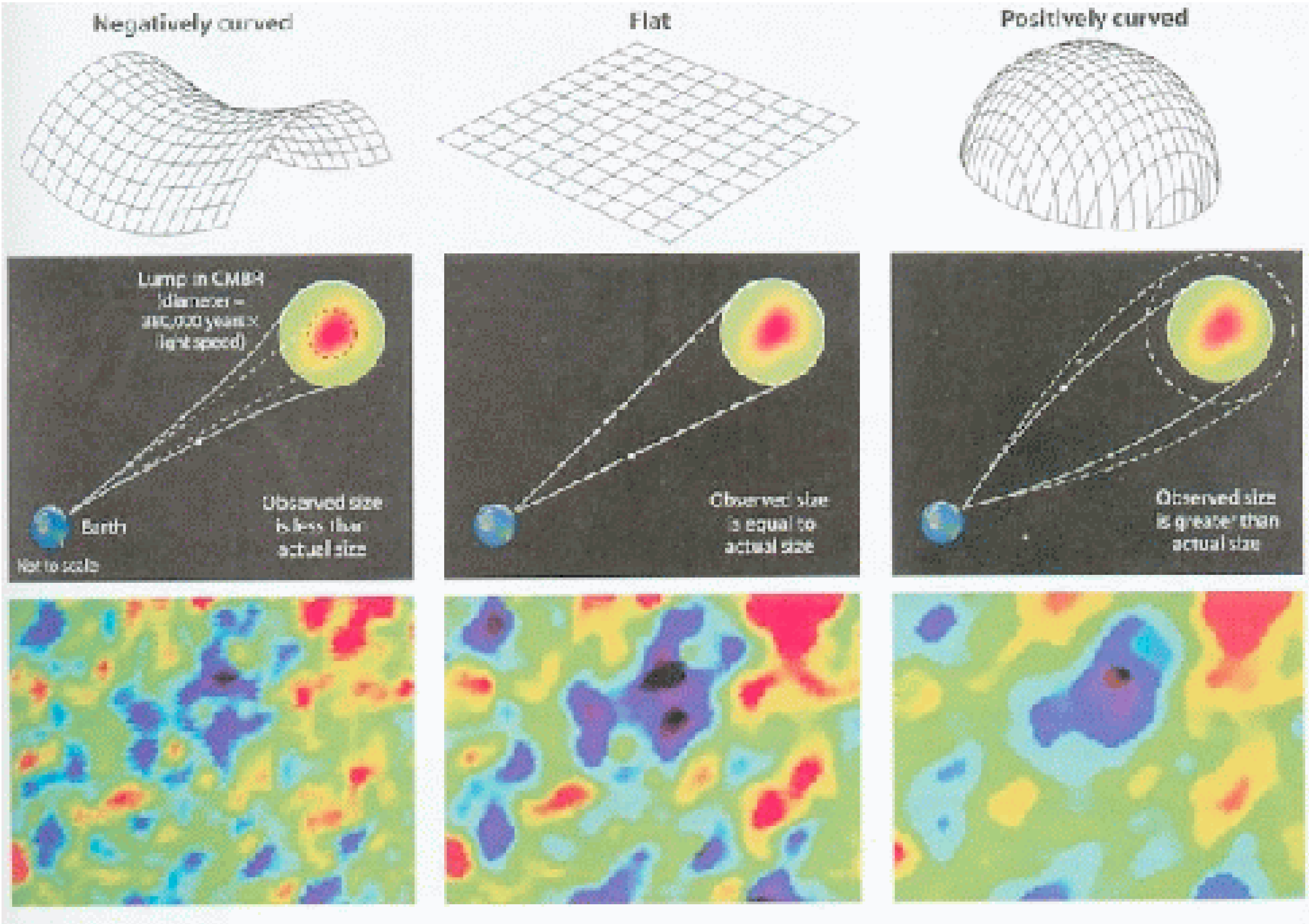
Standard Ruler :

$$L_S \approx c_S \, t_R \sim 100 \text{ kpc}$$

$$z_R \approx 1100$$

$$c_S = c/\sqrt{3}$$

Angular scale --> Geometry



Sound Horizon at z = 1100

distance travelled by a sound wave

$$c_S \, dt$$

recombination at ***z = 1100***

expand each step by factor $R(t_R)/R(t)$:

$$L_S(t_R) = R(t_R) \int_0^{t_R} \frac{c_S \, dt}{R(t)}$$

$$= \frac{R_0}{1+z} \int_{1+z}^{\infty} \frac{x}{R_0} \frac{c_S \, dx}{x \, H(x)}$$

dt = - dx / x H(x)
R(t) = R₀ / x

$$= \frac{c_S}{(1+z)} \int_{1+z}^{\infty} \frac{dx}{H(x)}$$

H(x) **from Friedmann Eqn.**

$$= \frac{c_S}{(1+z)\,H_0} \int_{1+z}^{\infty} \frac{dx}{\sqrt{x^4\,\Omega_R + x^3\,\Omega_M + \Omega_\Lambda + \left(1-\Omega_0\right)x^2}}$$

$$\approx \frac{c_S}{(1+z)\,H_0} \int_{1+z}^{\infty} \frac{dx}{\sqrt{x^4\,\Omega_R + x^3\,\Omega_M}}$$

keep 2 largest terms.

$$x \equiv 1+z = \frac{R_0}{R(t)}$$

$$dt = \frac{-dx}{x\,H(x)}$$

sound speed

$$c_S \approx \frac{c}{\sqrt{3}}$$

Sound Horizon at z = 1100

$$L_S(t_R) = \frac{c_S}{(1+z)} \int_{1+z}^{\infty} \frac{dx}{H(x)} \approx \frac{c_S}{(1+z)\,H_0} \int_{1+z}^{\infty} \frac{dx}{\sqrt{x^4\,\Omega_R + x^3\,\Omega_M}}$$

$$= \frac{c_S}{(1+z)\,H_0\sqrt{\Omega_R}} \int_{1+z}^{\infty} \frac{dx}{\sqrt{x^3(x+x_0)}}$$

$$x_0 \equiv \frac{\Omega_M}{\Omega_R} \approx 3500\left(\frac{\Omega_M}{0.3}\right)$$

$$= \frac{c_S}{(1+z)\,H_0\sqrt{\Omega_R}} \left(-\frac{2}{x_0}\sqrt{1+\frac{x_0}{x}} \right)_{1+z}^{\infty}$$

$$= \frac{2c_S}{(1+z)\,H_0\sqrt{\Omega_M x_0}} \left(\sqrt{1+\frac{x_0}{1+z}} - 1 \right)$$

$$c_S = \frac{c}{\sqrt{3}}$$

$$= \frac{c}{H_0} \frac{2\left(\sqrt{4.6}-1\right)}{1100\sqrt{3\times 0.3\times 3500}}$$

$$= 3.4\times 10^{-5}\,\frac{c}{H_0} \approx 110\left(\frac{0.7}{h}\right)\left(\frac{0.3}{\Omega_M}\right)^{1/2}\ \text{kpc}$$

Expands by factor
1 + z = 1100
to ~120 Mpc today.

Angular Scale measures Ω_0

sound horizon :

$$L_S(z) = \frac{1}{1+z}\int_{1+z}^{\infty}\frac{c_S\,dx}{H(x)}$$

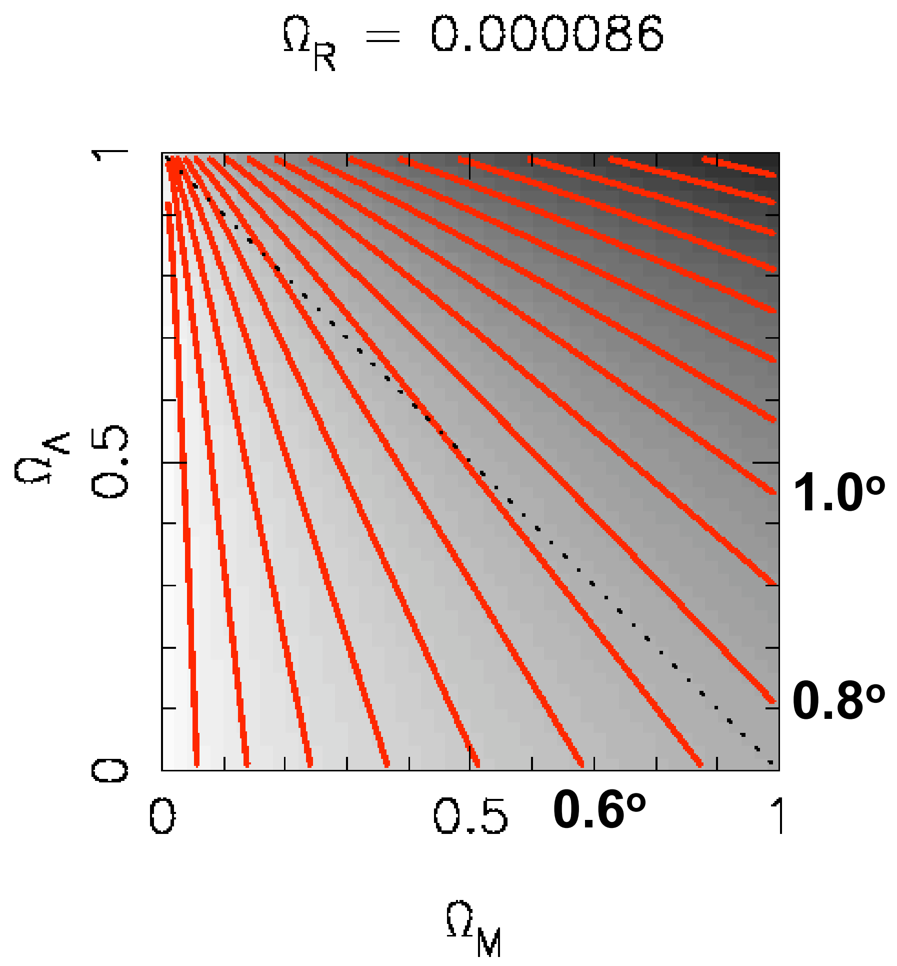
angular diameter distance :

$$D_A(z) = \frac{R_0\,S_K(\chi)}{1+z}$$

$$\chi = \int_t^{t_0}\frac{c\,dt}{R(t)} = \frac{c}{R_0}\int_1^{1+z}\frac{dx}{H(x)}$$

angular scale

$$\theta = \frac{L_S(z)}{D_A(z)} = \frac{\int_{1+z}^{\infty}\frac{c_S\,dx}{H(x)}}{R_0\,S_k\left(\frac{c}{R_0}\int_1^{1+z}\frac{dx}{H(x)}\right)}$$



Angular scale depends mainly on the curvature.

Gives θ ~ 0.8° for flat geometry,

$\Omega_0 = \Omega_M + \Omega_\Lambda = 1$

Finer Details: measure Ω_b and Ω_M

Sound speed not constant:

$$c_S(z) = \frac{c}{\sqrt{3\,(\,1 + R(z)\,)}}$$

$$R(z) \equiv \frac{3\,\rho_{\mathrm{b}}(z)}{4\,\rho_{\mathrm{R}}(z)} = \frac{3\,\Omega_b\,(1 + z)}{4\,\Omega_R}$$

Acoustic peaks not quite equally spaced:

$$l_n = l_A\left(\,n + \delta_n\,\right)$$

phase shifts

$$\delta_n \approx a_n \left(\frac{r}{0.3}\right)^{0.1} \qquad a_{1,2,3} \approx 0.267\,, 0.24, 0.35$$

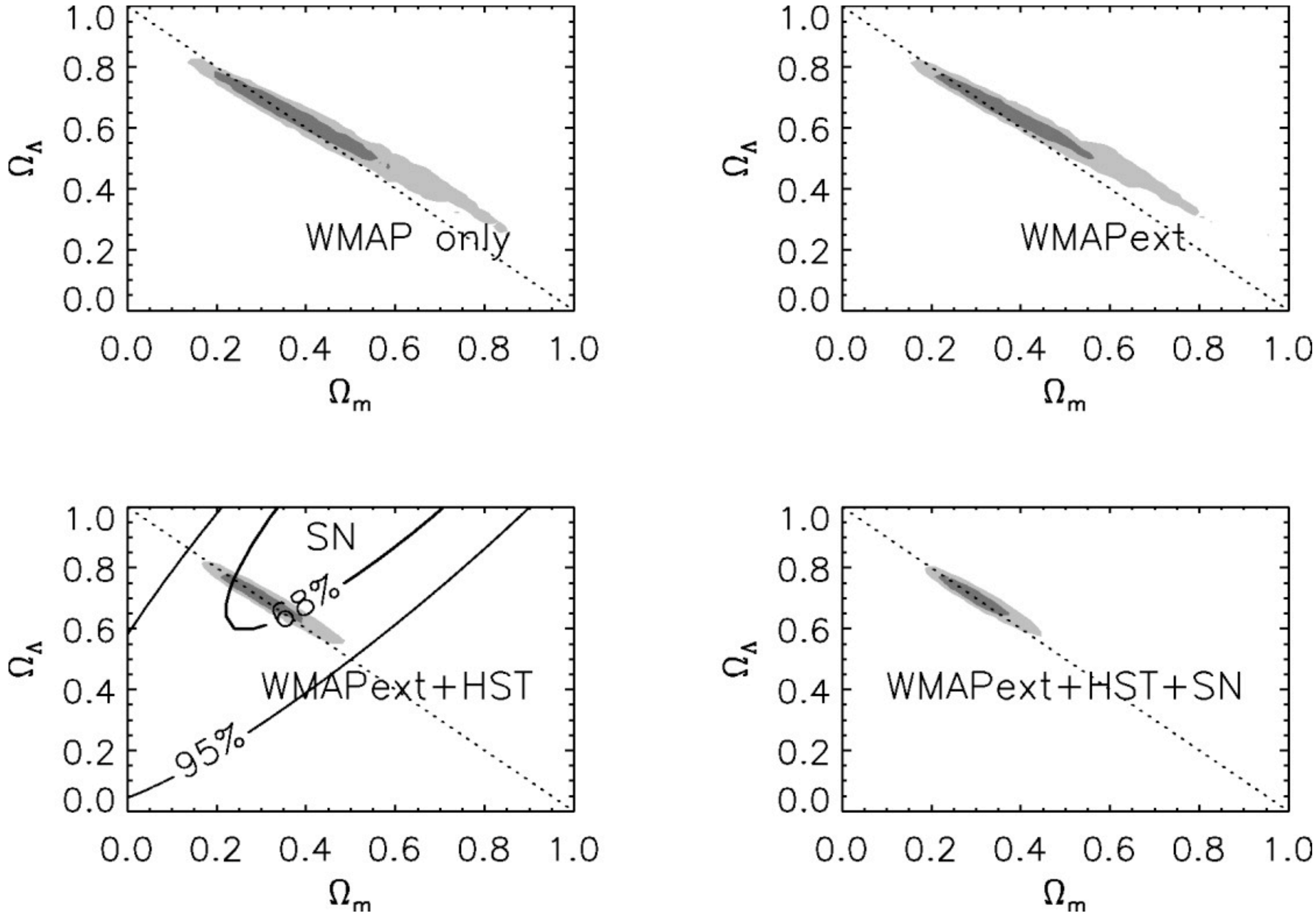
$$r \equiv \frac{\rho_{\mathrm{M}}(z)}{\rho_{\mathrm{R}}(z)} = \frac{\Omega_M\,(1 + z)}{\Omega_R}$$

Max Tegmark's CMB Movies

Shows how the **CMB power spectrum**
(and the **baryon power spectrum**)
depend on the cosmological parameters.

A link to these is available on the course web page.

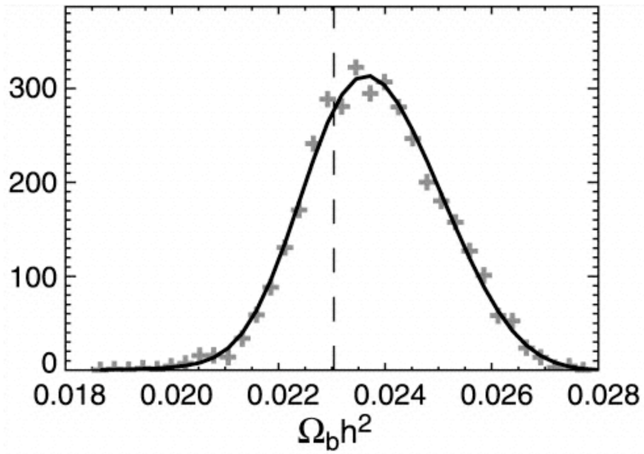
WMAP parameter constraints



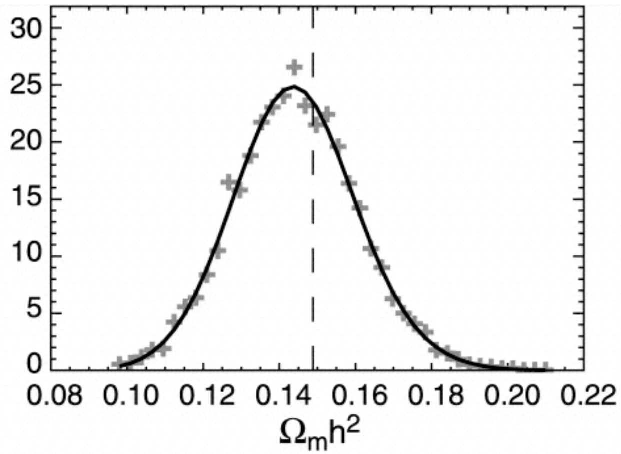
WMAP parameter constraints

Spergel et al. 2003   ApJSup 148,175.

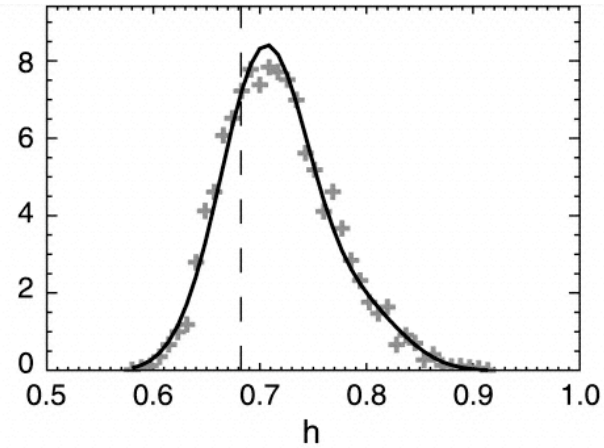
baryons



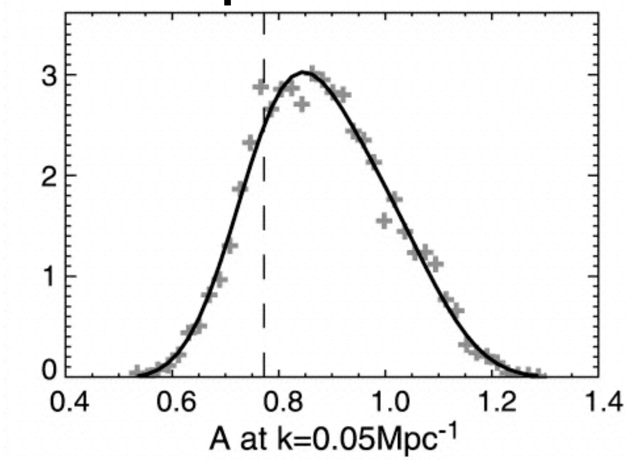
Dark Matter



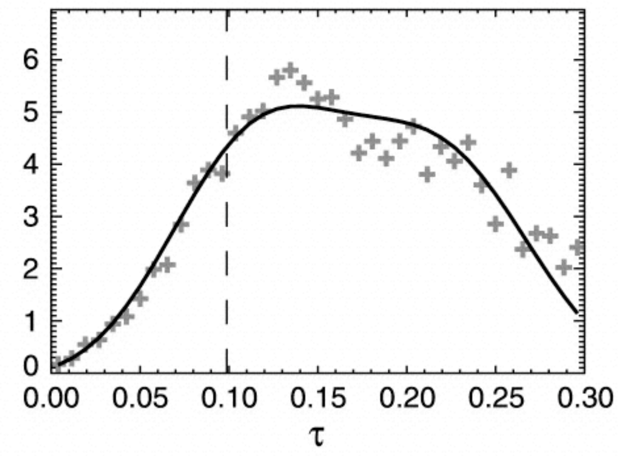
expansion



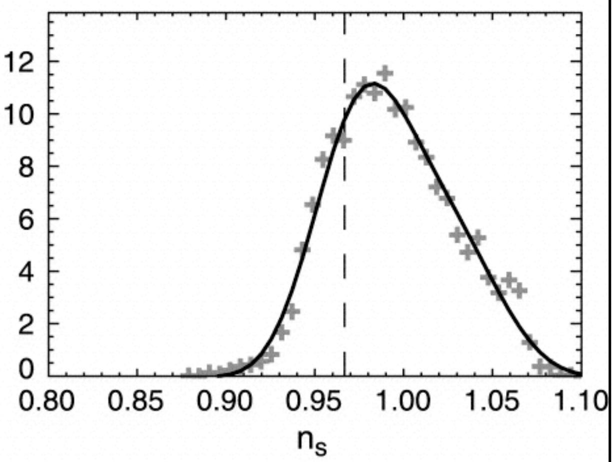
amplitude



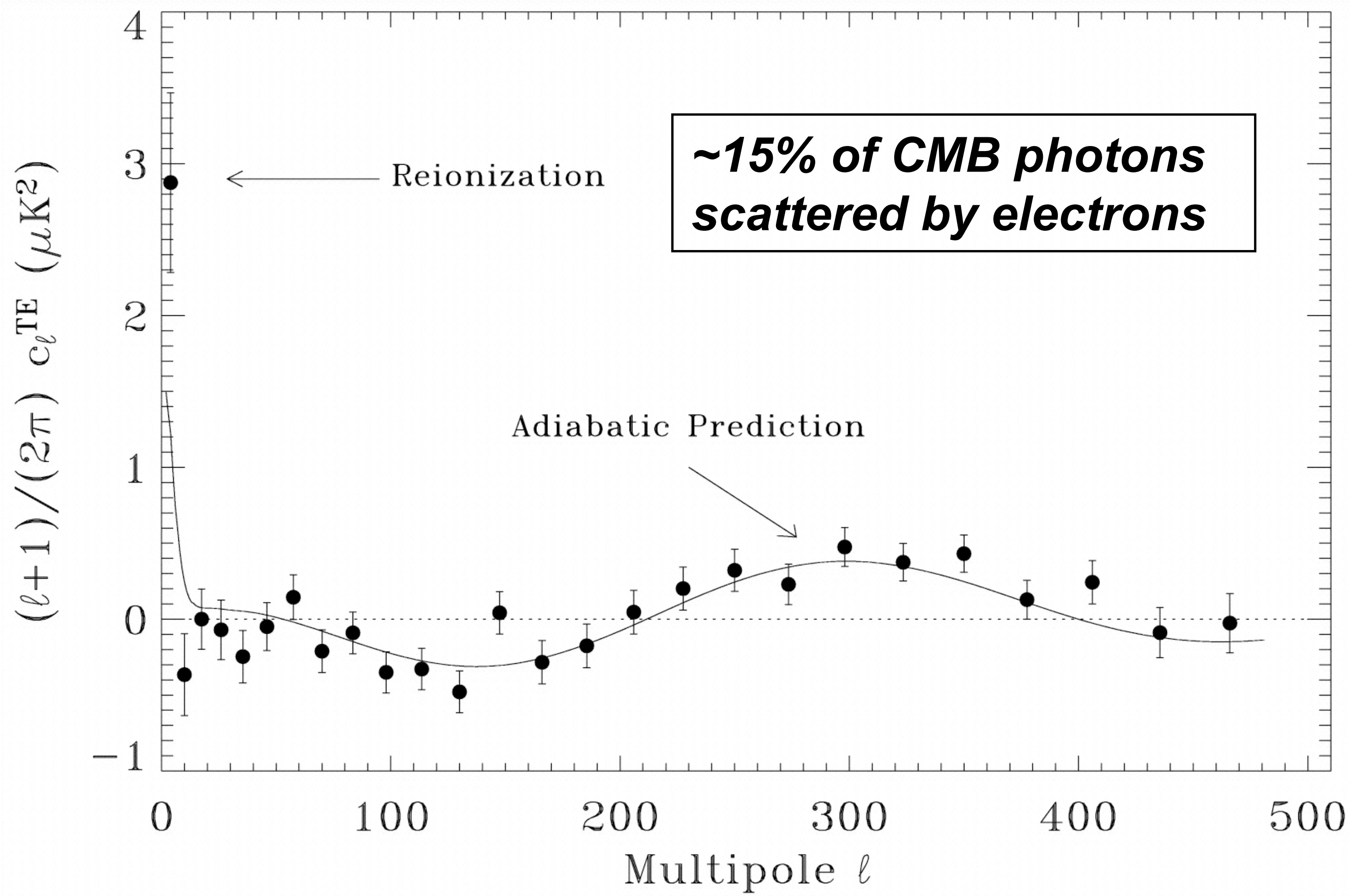
reionisation



tilt



WMAP Polarisation Power Spectrum



Spergel et al. 2003 ApJSup 148,175.

Epoch of Re-Ionisation

UV from first stars re-ionises gas.

Scatters ~15% of CMB photons yielding ~15% polarisation.

WMAP measured this !

$$dt = \frac{-dx}{x\,H(x)}$$

$$x \equiv 1 + z$$

Electron scattering optical depth:

$$d\tau = n\,\sigma_T\,dr$$

$$= n_0\,(1+z)^3\,\sigma_T\,c\,dt$$

$$\tau = n_0\,\sigma_T\,c \int_1^{1+z} \frac{x^3 dx}{x\,H(x)}$$

$$= \frac{n_0\,\sigma_T\,c}{H_0} \int_1^{1+z} \frac{x^2\,dx}{\sqrt{\Omega_M\,x^3 + \Omega_\Lambda + (1-\Omega_0)\,x^2}}$$

Thompson cross - section

σ_T

electron density today

$$n_0 = \frac{\Omega_b}{m_{\mathrm{H}}} \frac{3\,H_0^{\;2}}{8\,\pi\,G}\left(X + \frac{Y}{2}\right)$$

Gives ~15% optical depth at z ~ 20

Precision Cosmology

$h = 71 \pm 3$ expanding

$\Omega = 1.02 \pm 0.02$ flat

$\Omega_b = 0.044 \pm 0.004$ baryons

$\Omega_M = 0.27 \pm 0.04$ Dark Matter

$\Omega_\Lambda = 0.73 \pm 0.04$ Dark Energy

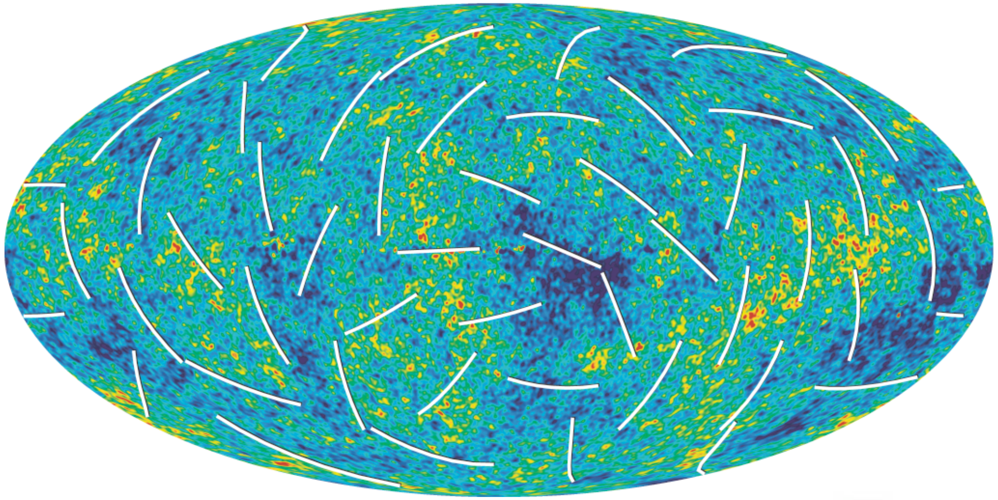
$t_0 = 13.7 \pm 0.2 \times 10^9$ yr now

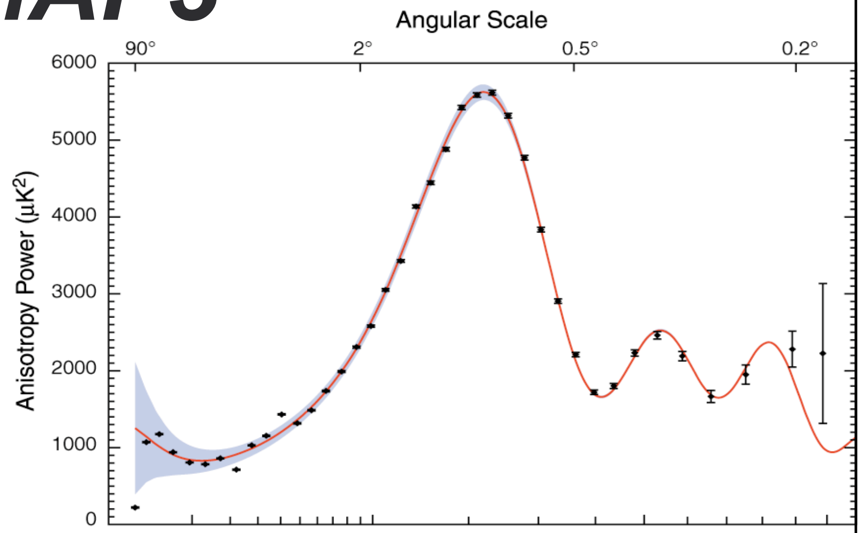
$t_* = 180^{+220}_{-80} \times 10^6$ yr $z_* = 20^{+10}_{-5}$ reionisation

$t_R = 379 \pm 1 \times 10^3$ yr $z_R = 1090 \pm 1$ recombination

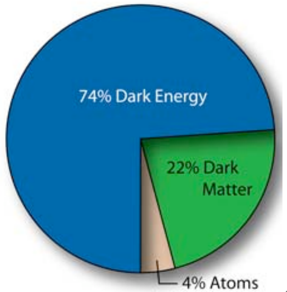
(From the WMAP 1-year data analysis)

2006 - WMAP3

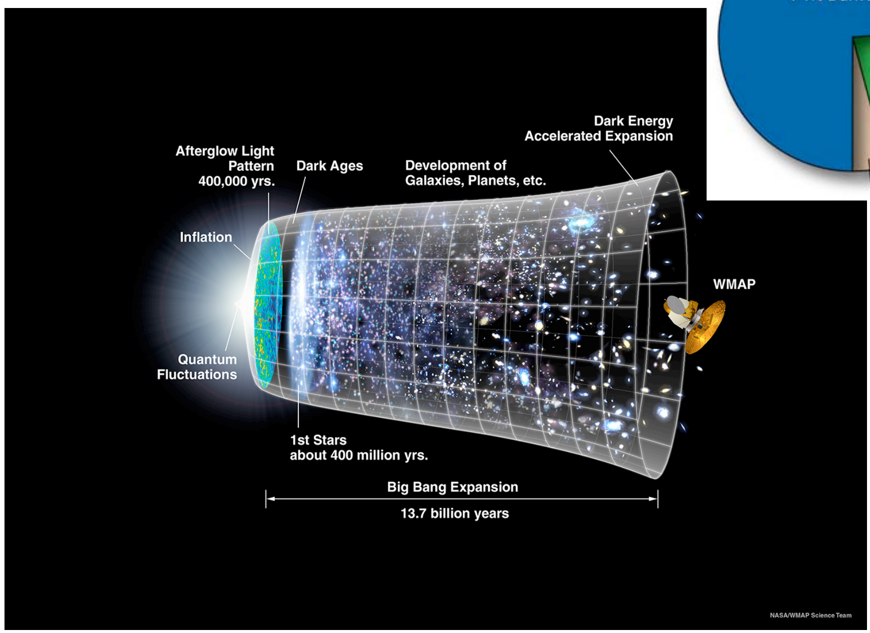




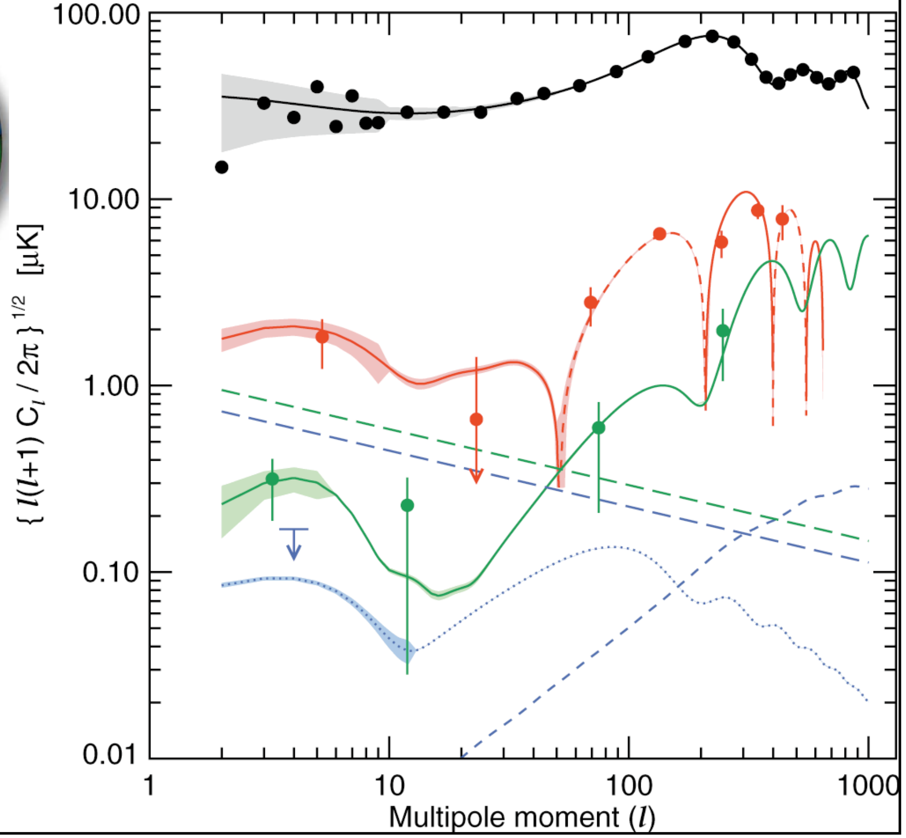
Angular Scale
90°
2°
0.5°
0.2°
Anisotropy Power (μK²)
6000
5000
4000
3000
2000
1000
0



74% Dark Energy
22% Dark Matter
4% Atoms



Dark Energy Accelerated Expansion
Afterglow Light Pattern 400,000 yrs.
Dark Ages
Development of Galaxies, Planets, etc.
Inflation
WMAP
Quantum Fluctuations
1st Stars about 400 million yrs.
Big Bang Expansion
13.7 billion years



100.00
10.00
1.00
0.10
0.01
{ l(l+1) C_l / 2π }^1/2 [μK]
1
10
100
1000
Multipole moment (l)

Planck -- 2009 launch