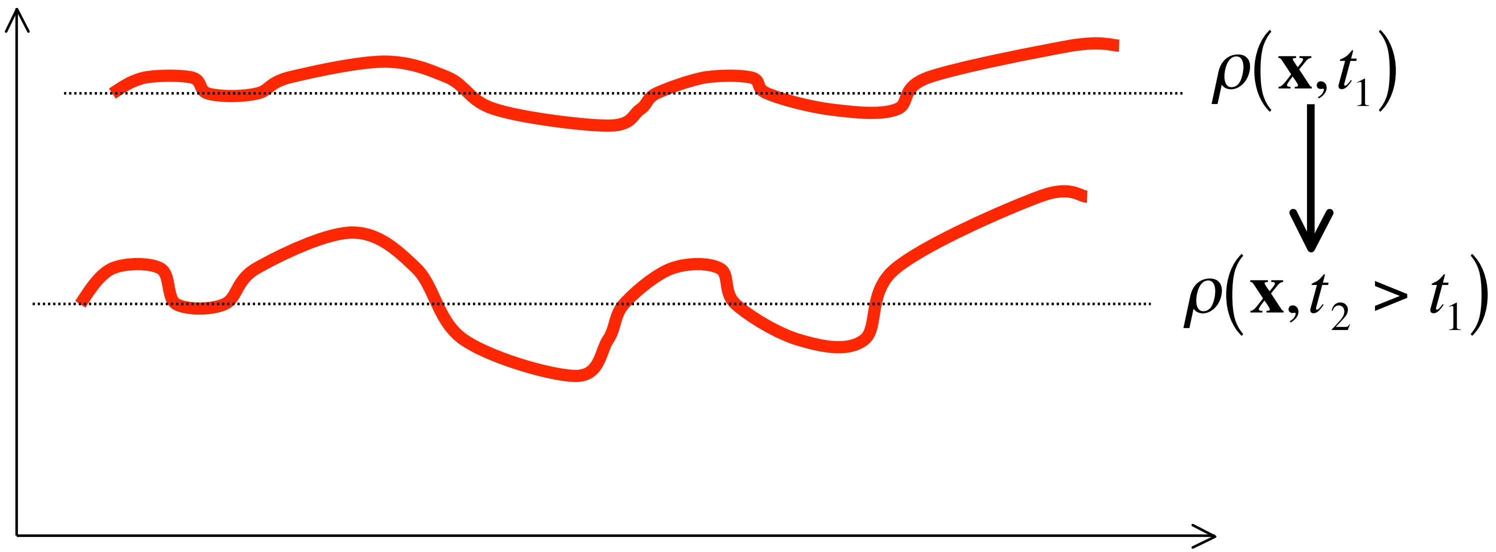
Large Scale Structure

- Galaxies are (biased) tracers of the mass
- "Bubbly" structure observed
  - Voids
  - Walls = edges of voids
  - Filaments = intersections of walls
  - Clusters = intersections of filaments
- Compare with supercomputer simulations.
  - initial density perturbations grow
  - stars / galaxies form when density high enough
- Determine Ω_M
  - High Ω_M => faster growth
  - => clusters form at earlier redshifts
  - => stronger clustering today
  - $\Omega_M \sim 0.3$ matches observed structure.

Density Perturbations Grow

$$\rho(\mathbf{x},t) = \bar{\rho}(t)\,\big(1+\delta(\mathbf{x},t)\big)$$



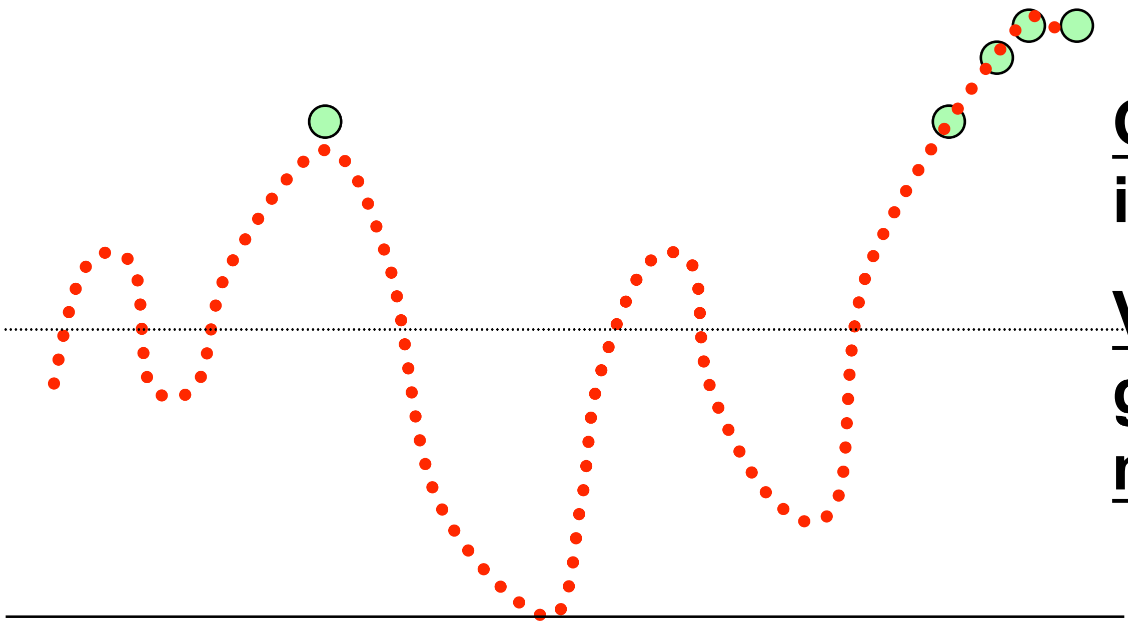
co-moving spatial coordinate : $\mathbf{x}$

Linear regime: $|\delta| < 1$ $\qquad \delta \equiv \dfrac{\rho - \bar{\rho}}{\bar{\rho}}$

$$\delta \propto R(t) \propto \frac{1}{1+z}$$

Biased Galaxy Formation

Non-Linear regime $\delta \geq 1$



<u>Galaxy clusters</u> form in density <u>maxima.</u>

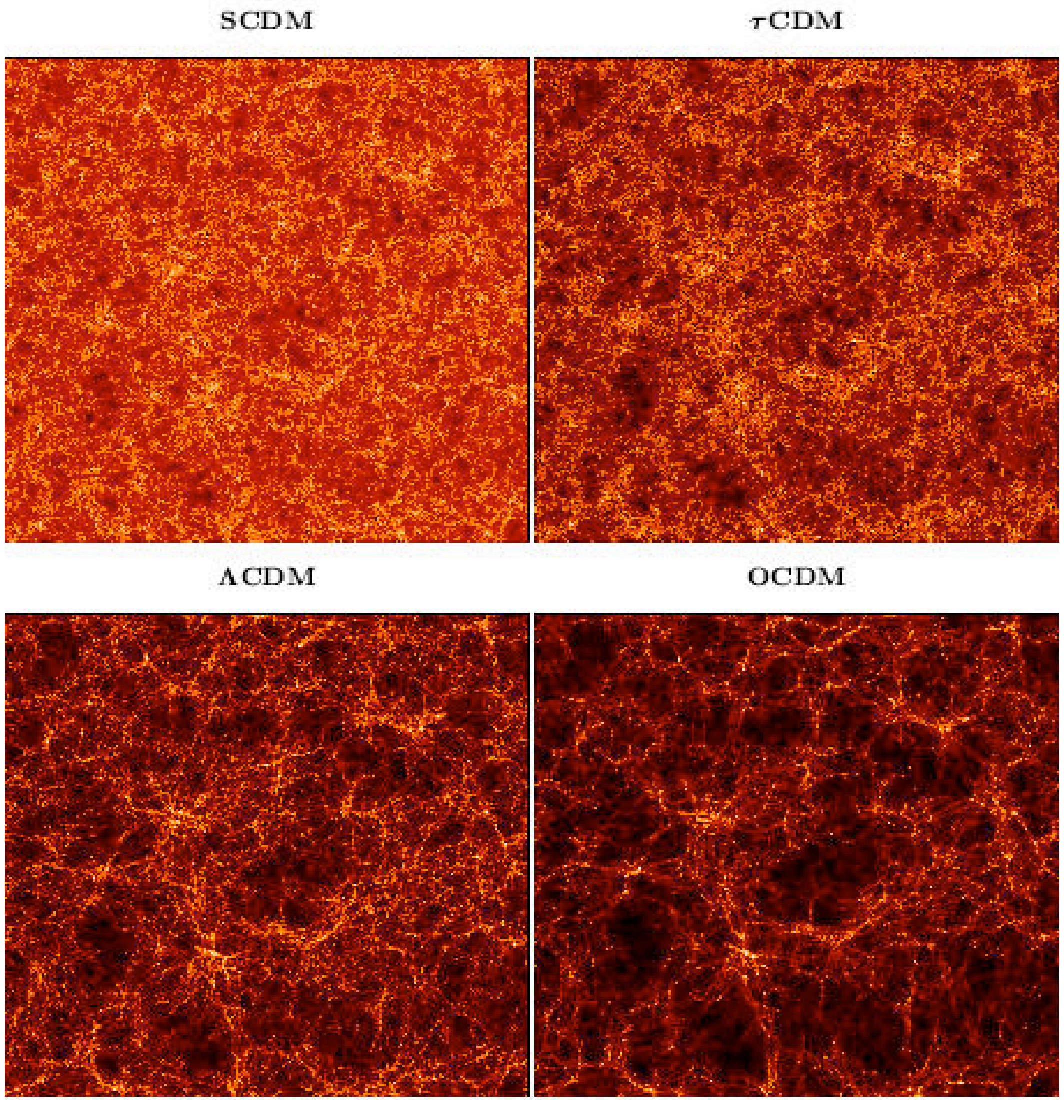
<u>Voids</u> (almost no galaxies) in density <u>minima.</u>

$$\delta_{galaxies} = b\,\delta_M$$

b = "bias parameter"

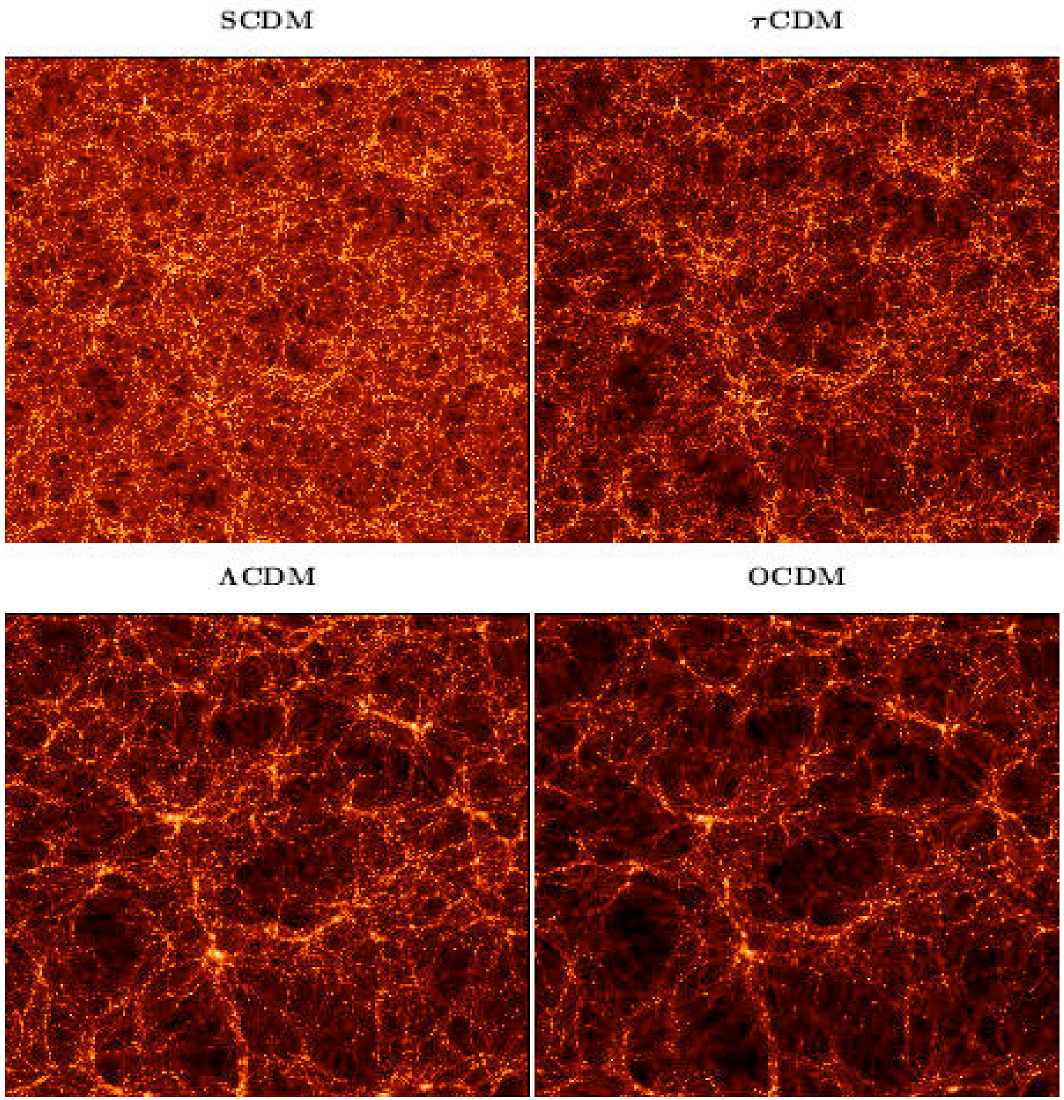
b > 1 --> Galaxies more strongly clustered than matter

Simulations: z = 3



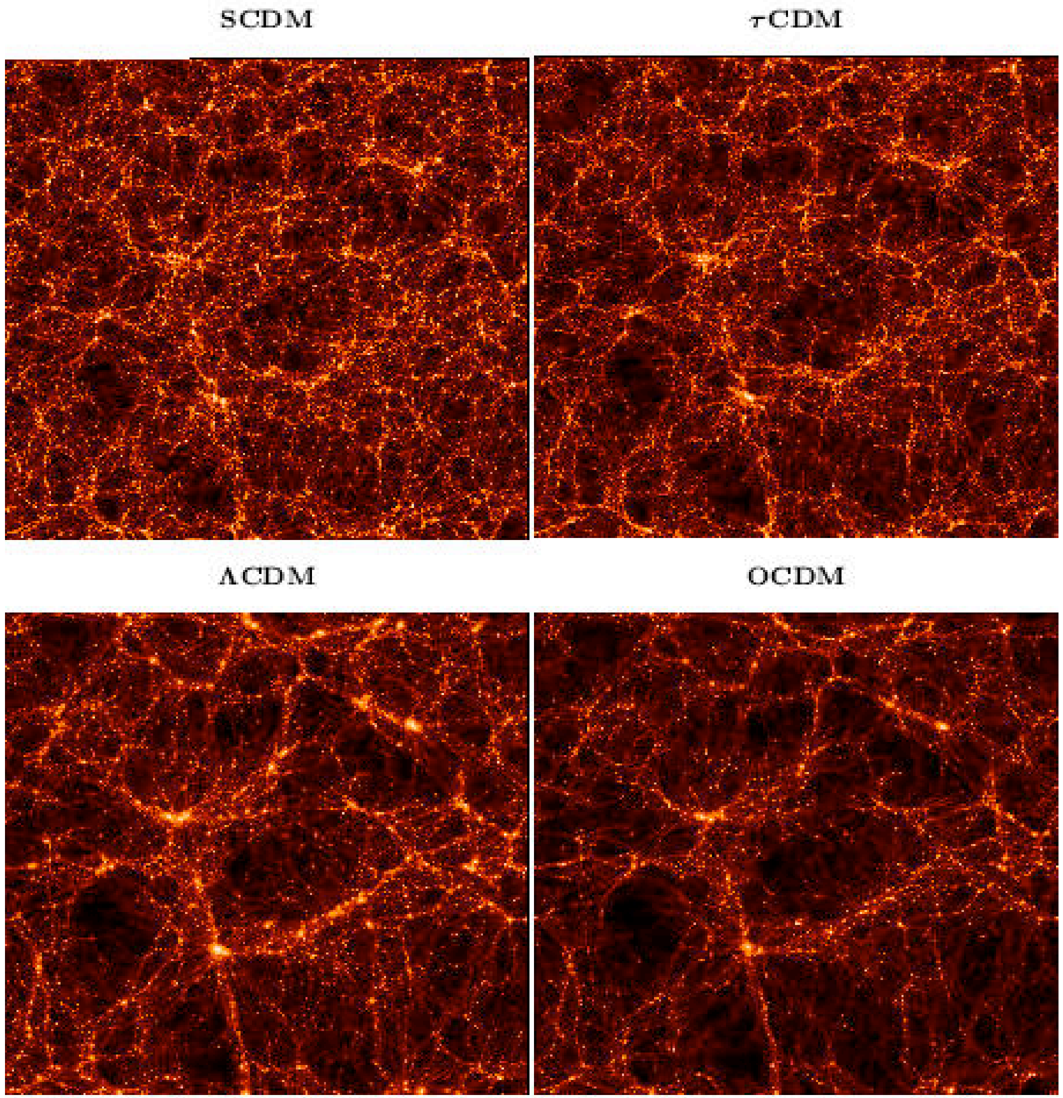
The VIRGO Collaboration 1996

Simulations: z = 1



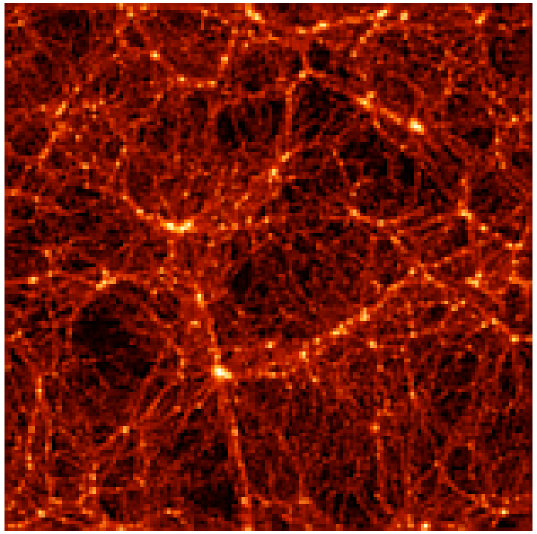
The VIRGO Collaboration 1996

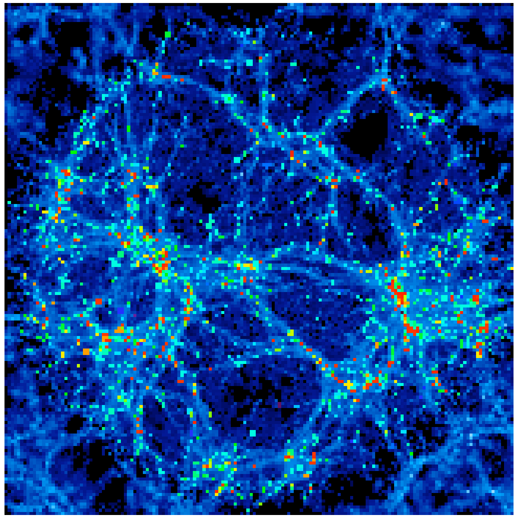
Simulations: z = 0

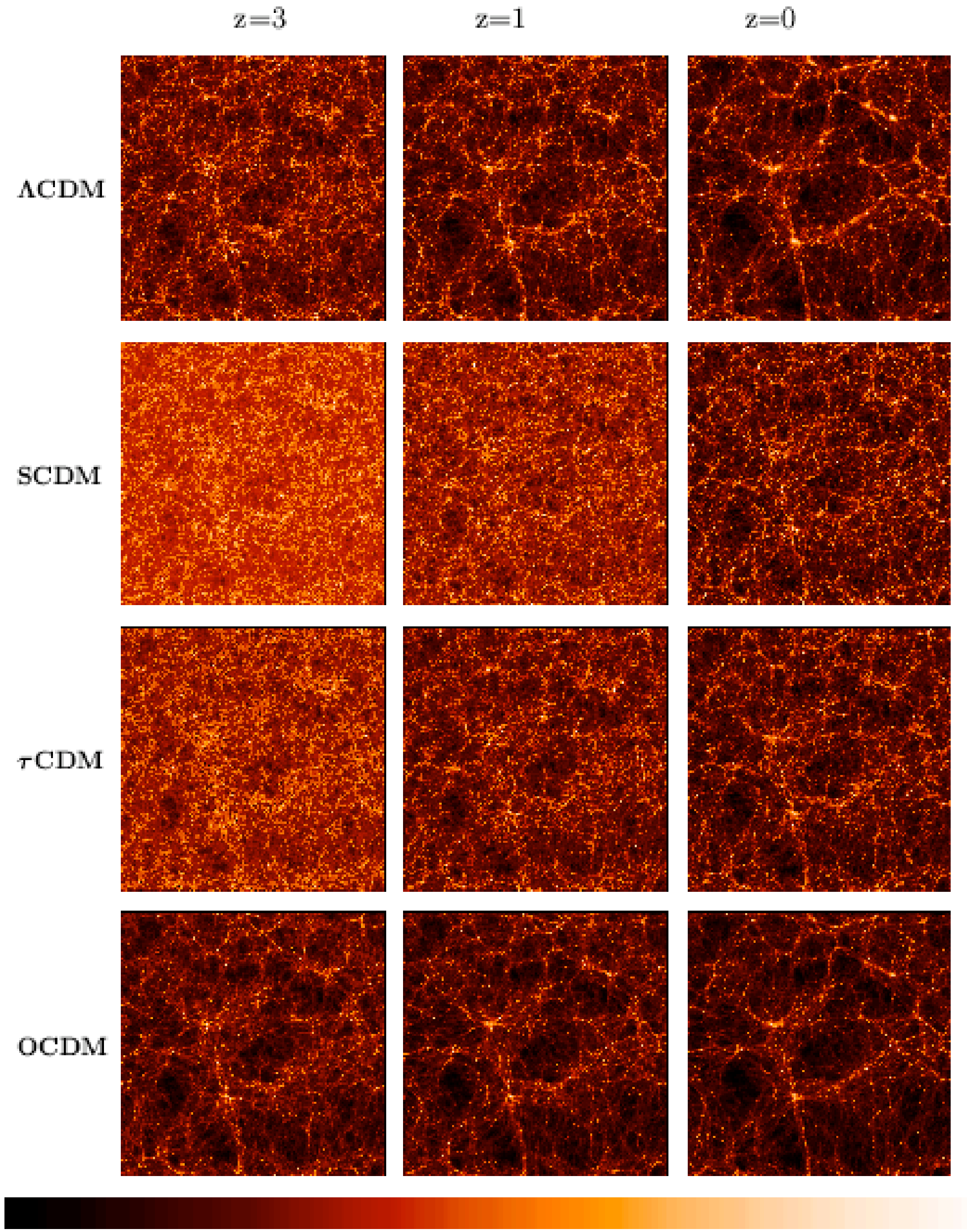


The VIRGO Collaboration 1996

Supercomputer Simulations



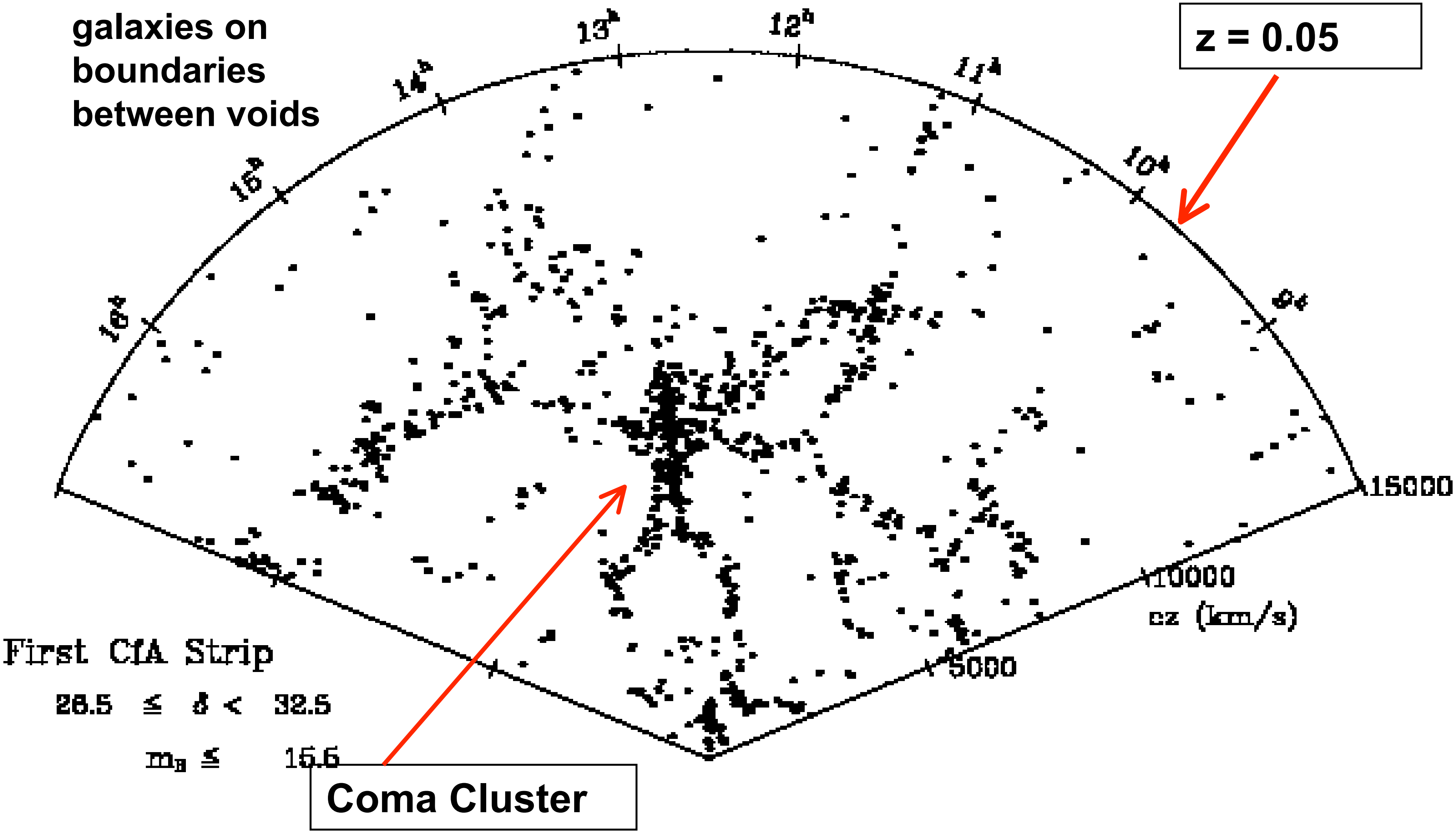




~1985 CfA Redshift Survey

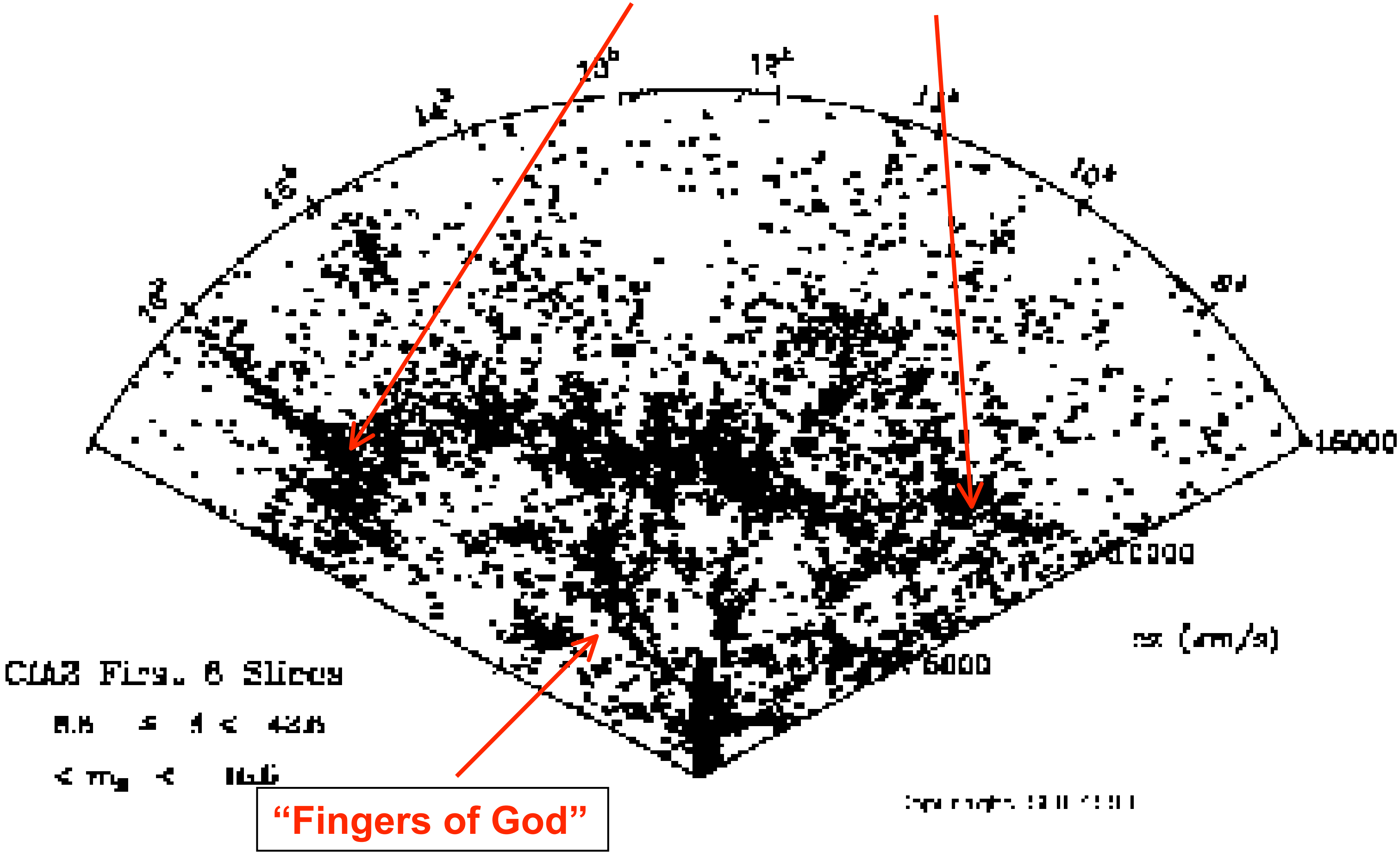
"Bubbly"

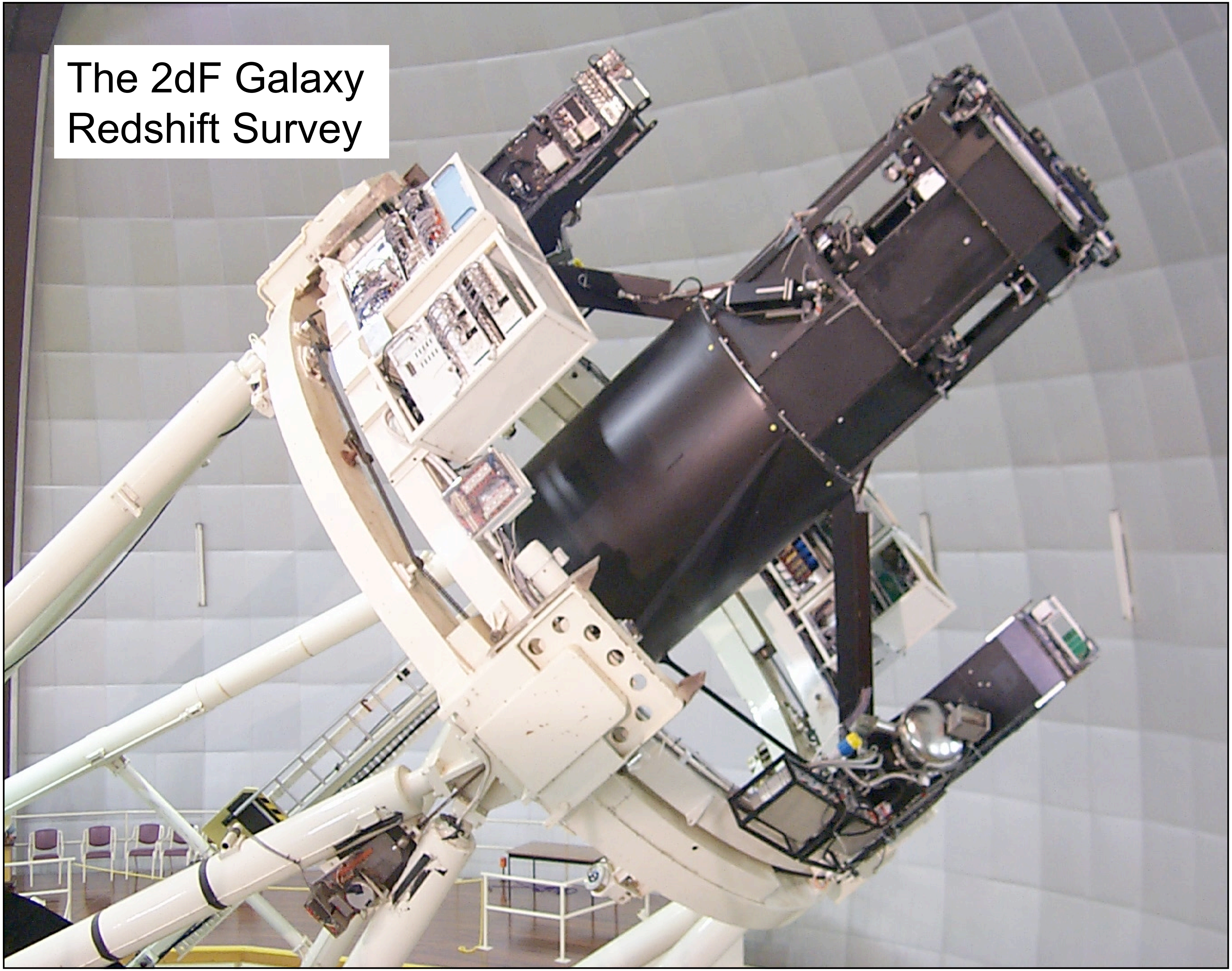
galaxies on boundaries between voids



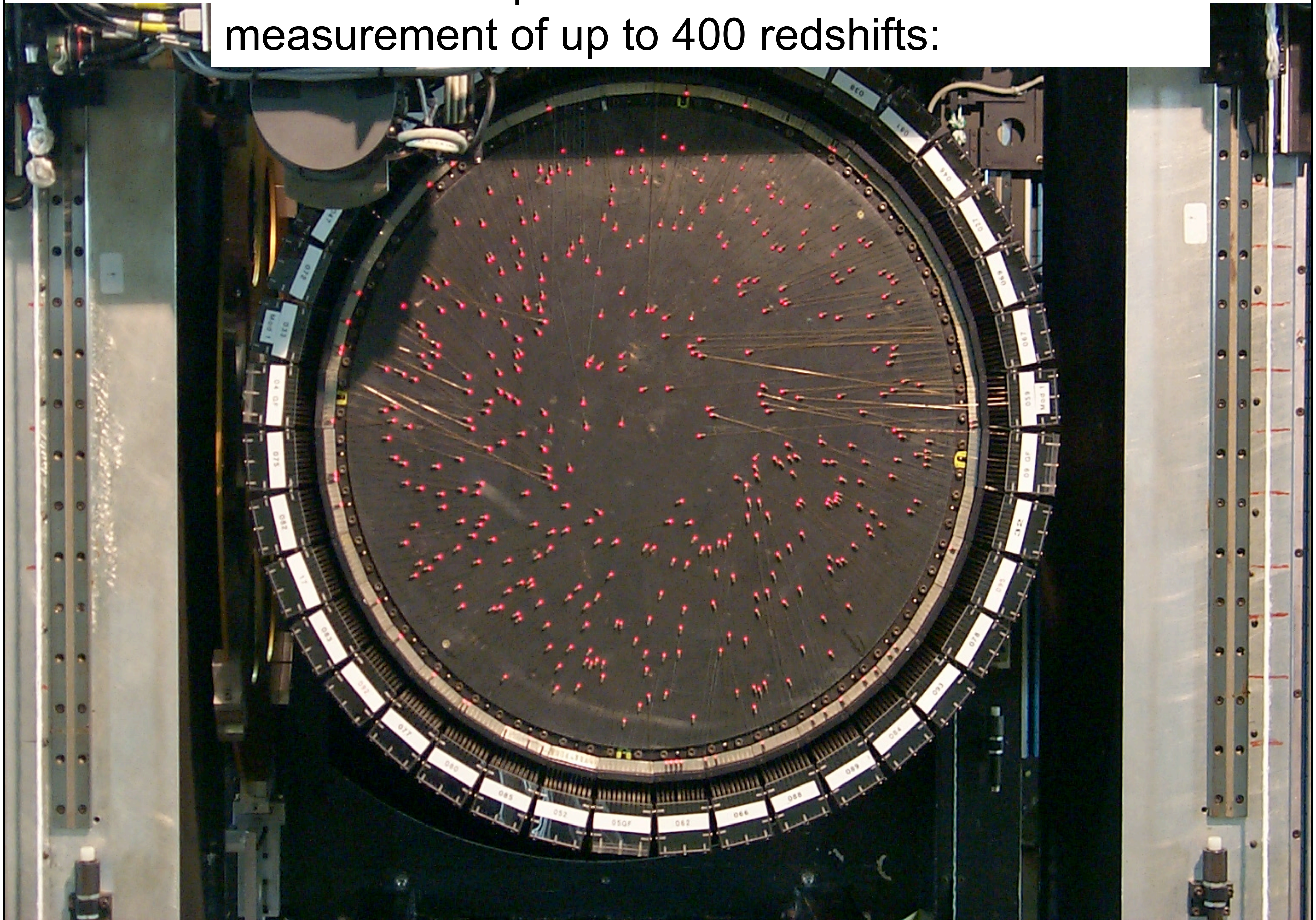
Huchra, Geller, de Laparet

“The Great Wall”

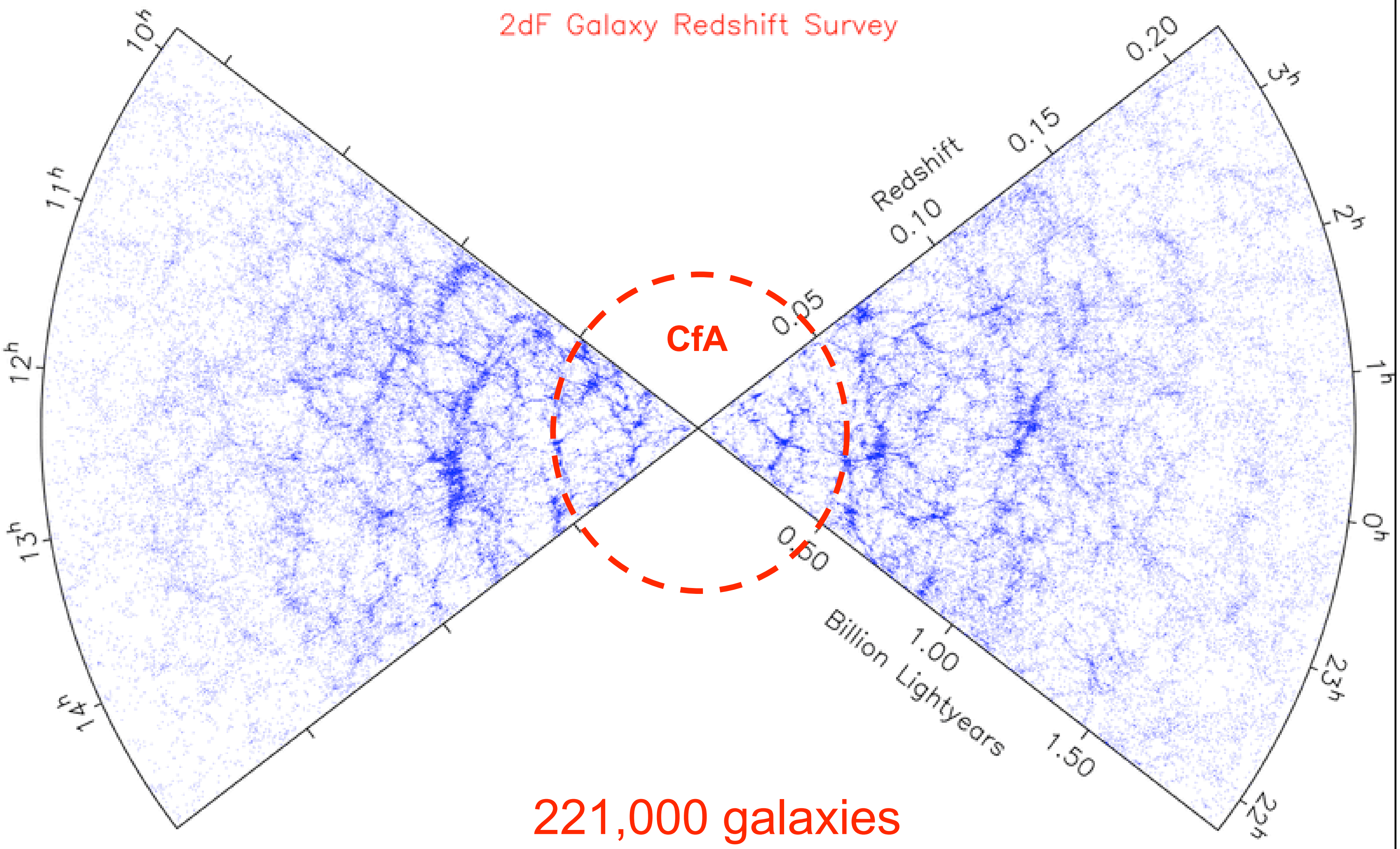



The 2dF Galaxy
Redshift Survey

AAT 2dF fibre positioner allows for simultaneous measurement of up to 400 redshifts:



2dF = 2 degree Field



Galaxy Redshift Surveys

Large Scale Structure:

Empty **voids**

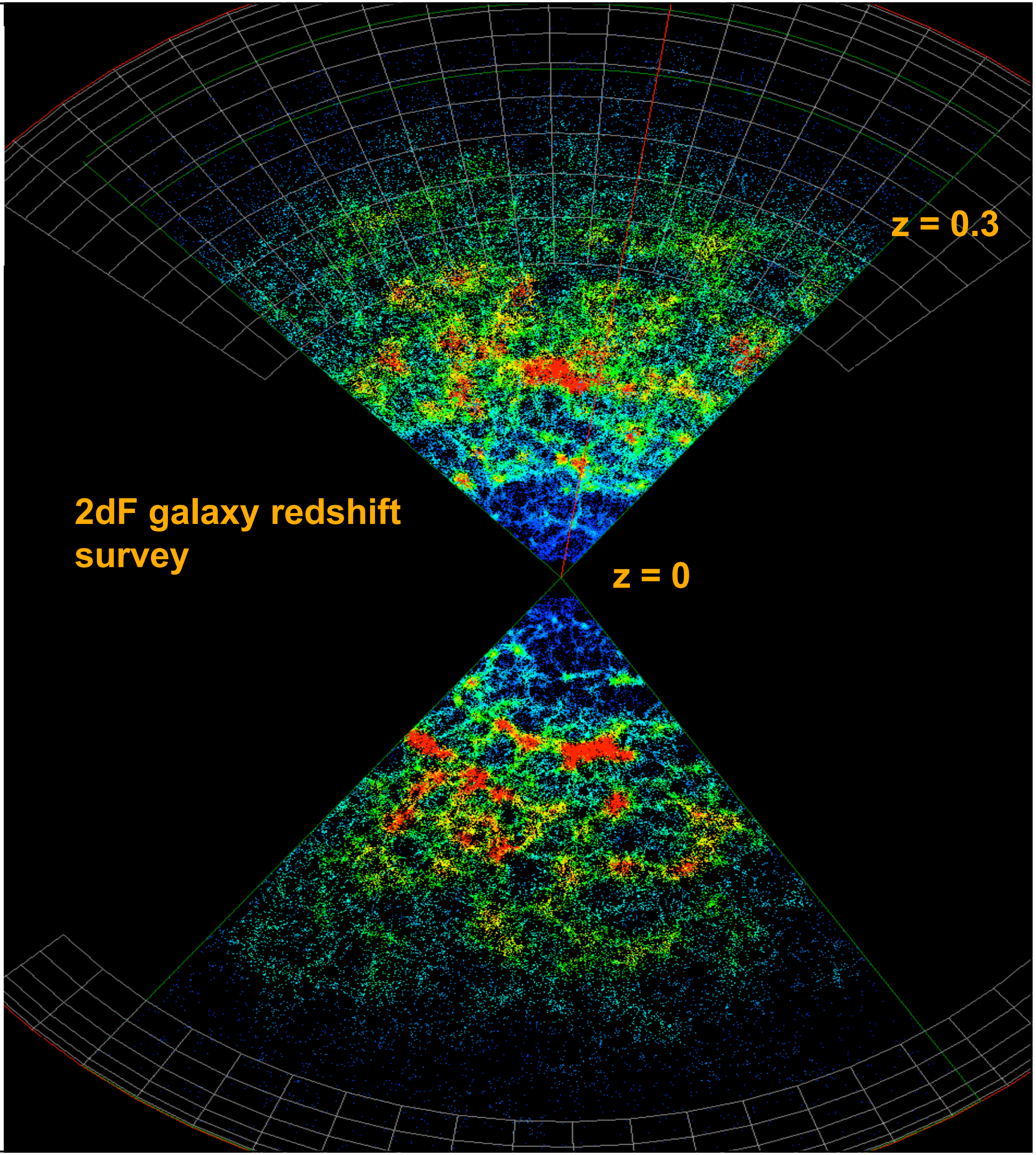
~50Mpc.

Galaxies are in
1. **Walls** between voids.

2. **Filaments** where walls intersect.

3. **Clusters** where filaments intersect.

Like Soap Bubbles !



Theory vs Observations

- **Can't directly compare simulations and observations**
  - details (exactly where density is high/low) don't matter.
- **Amplitude of structure vs size of structure is what matters. Quantify this using:**
  - Power Spectrum: $P(k)$ wavenumber $k = 2\pi / \lambda$
  - 2-point Correlation Function : $\xi(r)$
- **Biased galaxy formation:**
  - bias parameter b.
- **Initial conditions:**
  - Power-law power spectrum for initial amplitude vs scale.
  - Amplitude A, slope n $P_0(k) \sim A^2 k^n$

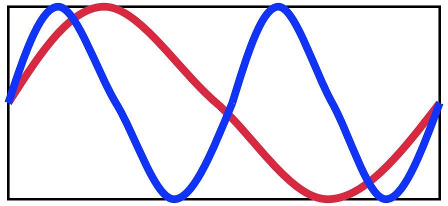
Fourier Analysis (Parceval's Theorem)

density perturbations

$$\delta(\mathbf{x}) \equiv \frac{\rho(\mathbf{x}) - \langle \rho \rangle}{\langle \rho \rangle} = \frac{\delta\rho}{\bar{\rho}}$$

fourier amplitudes $\delta_{\mathbf{k}}$

$$\delta(\mathbf{x}) = \sum_{\mathbf{k}} \delta_{\mathbf{k}} \exp(-i\,\mathbf{k}\cdot\mathbf{x})$$



mean

$$\langle \delta \rangle = 0$$

variance (average over volume V)

$$\langle \delta^2 \rangle = \frac{1}{V}\int \delta^2(\mathbf{x})\, d^3\mathbf{x}$$

$$= \frac{1}{V}\int \left| \sum_{\mathbf{k}} \delta_{\mathbf{k}} \exp(-i\,\mathbf{k}\cdot\mathbf{x}) \right|^2 d^3\mathbf{x}$$

$$= \frac{1}{V}\int \sum_{\mathbf{k}}\sum_{\mathbf{j}} \delta_{\mathbf{k}}\delta_{\mathbf{j}}^* \exp(-i\,(\mathbf{k}-\mathbf{j})\cdot\mathbf{x})\, d^3\mathbf{x}$$

$$= \sum_{\mathbf{k}}\sum_{\mathbf{j}} \delta_{\mathbf{k}}\delta_{\mathbf{j}}^* \underline{\int \exp(-i\,(\mathbf{k}-\mathbf{j})\cdot\mathbf{x})\, \frac{d^3\mathbf{x}}{V}}$$

1 if k = j

0 otherwise

$$\langle \delta^2 \rangle = \sum_{\mathbf{k}} |\delta_{\mathbf{k}}|^2$$

$$\approx \frac{V}{(2\pi)^3}\int |\delta_{\mathbf{k}}|^2\, d^3\mathbf{k}$$

mode spacing:

$$\lambda_x = L_x / n$$

$$k_x = \frac{2\pi\, n}{L_x} = n\,\Delta k_x$$

$$\Delta k_x = \frac{2\pi}{L_x}$$

$$d^3\mathbf{k} = \frac{(2\pi)^3}{L_x L_y L_z} = \frac{(2\pi)^3}{V}$$

= k - space volume per mode

Power Spectrum

For isotropic structure (consistent with observations) :

power spectrum (average over directions)

$$P(k) \equiv \left\langle \, \left|\delta_{\mathbf{k}}\right|^2 \, \right\rangle = \int\int \delta^2(k,\theta,\phi) \, \frac{\sin\theta \, d\theta \, d\phi}{4\pi} \qquad k = \left|\mathbf{k}\right| = \frac{2\pi}{\lambda}$$

variance of density fluctuations :

$$\left\langle \, \delta^2 \, \right\rangle = \sum_{\mathbf{k}} \left|\delta_{\mathbf{k}}\right|^2 \approx \frac{V}{(2\pi)^3} \int \left|\delta_{\mathbf{k}}\right|^2 d^3\mathbf{k}$$

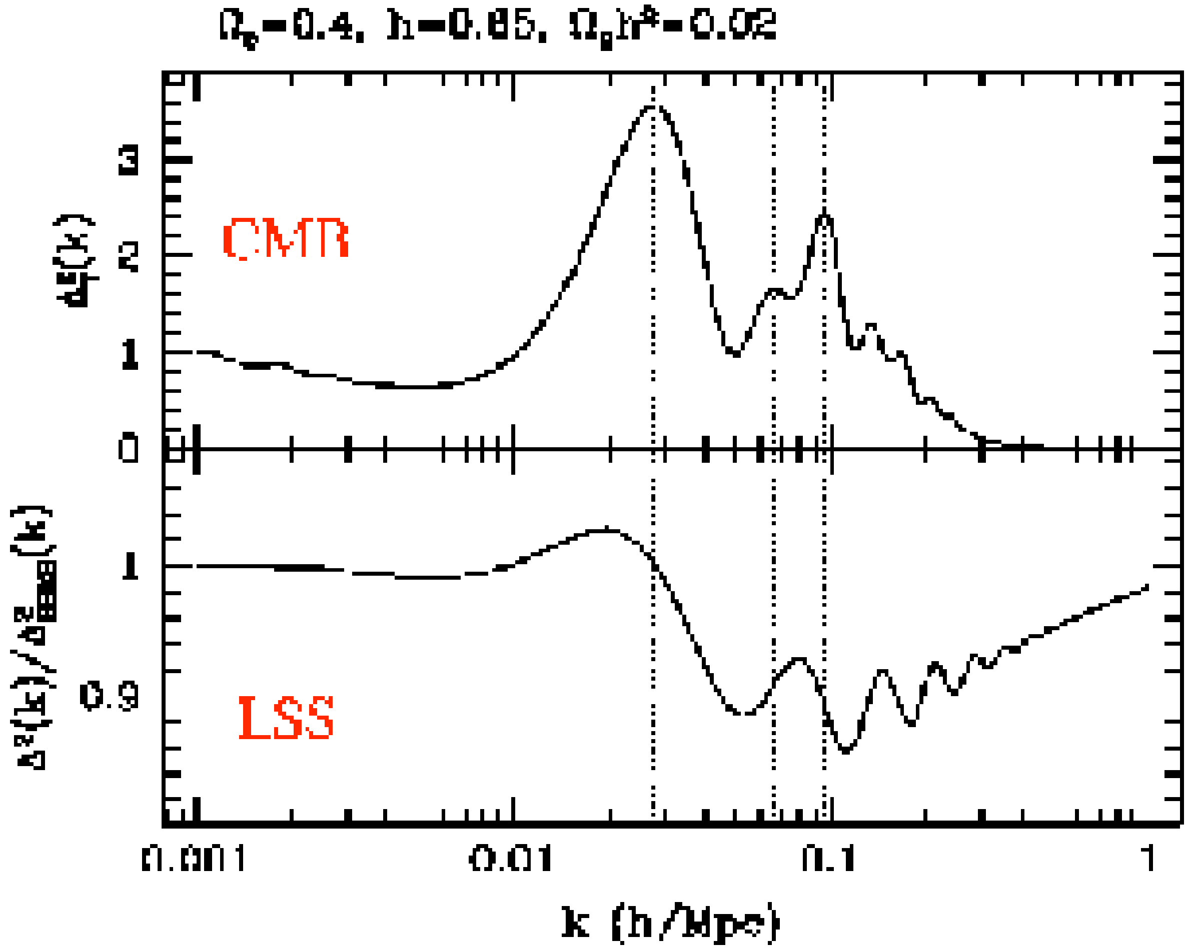
$$= \frac{V}{(2\pi)^3} \int P(k) \; 4\pi \, k^2 dk = \frac{V}{2\pi^2} \int P(k) \; k^2 dk$$

dimensionless power spectrum :

$$\Delta^2(k) \equiv \frac{d\left\langle \, \delta^2 \, \right\rangle}{d\ln k} \approx \frac{V}{2\pi^2} k \frac{d}{dk}\left(\int P(k) \, k^2 dk \right) = \frac{V}{2\pi^2} k^3 \; P(k)$$

Predicted Power Spectra

Independent constraints from CMB and Large-Scale Structure

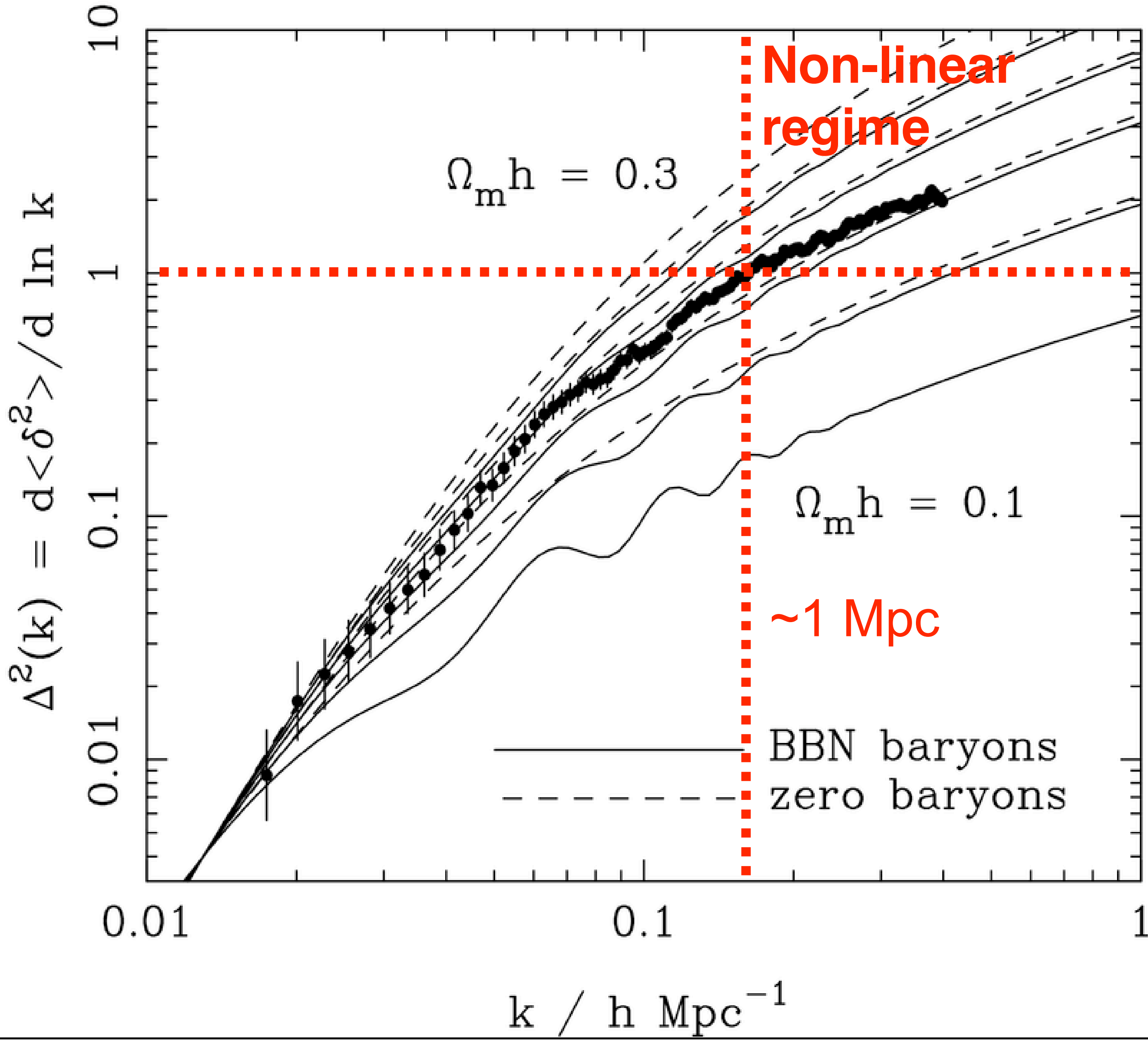


CMB and LSS out of phase:

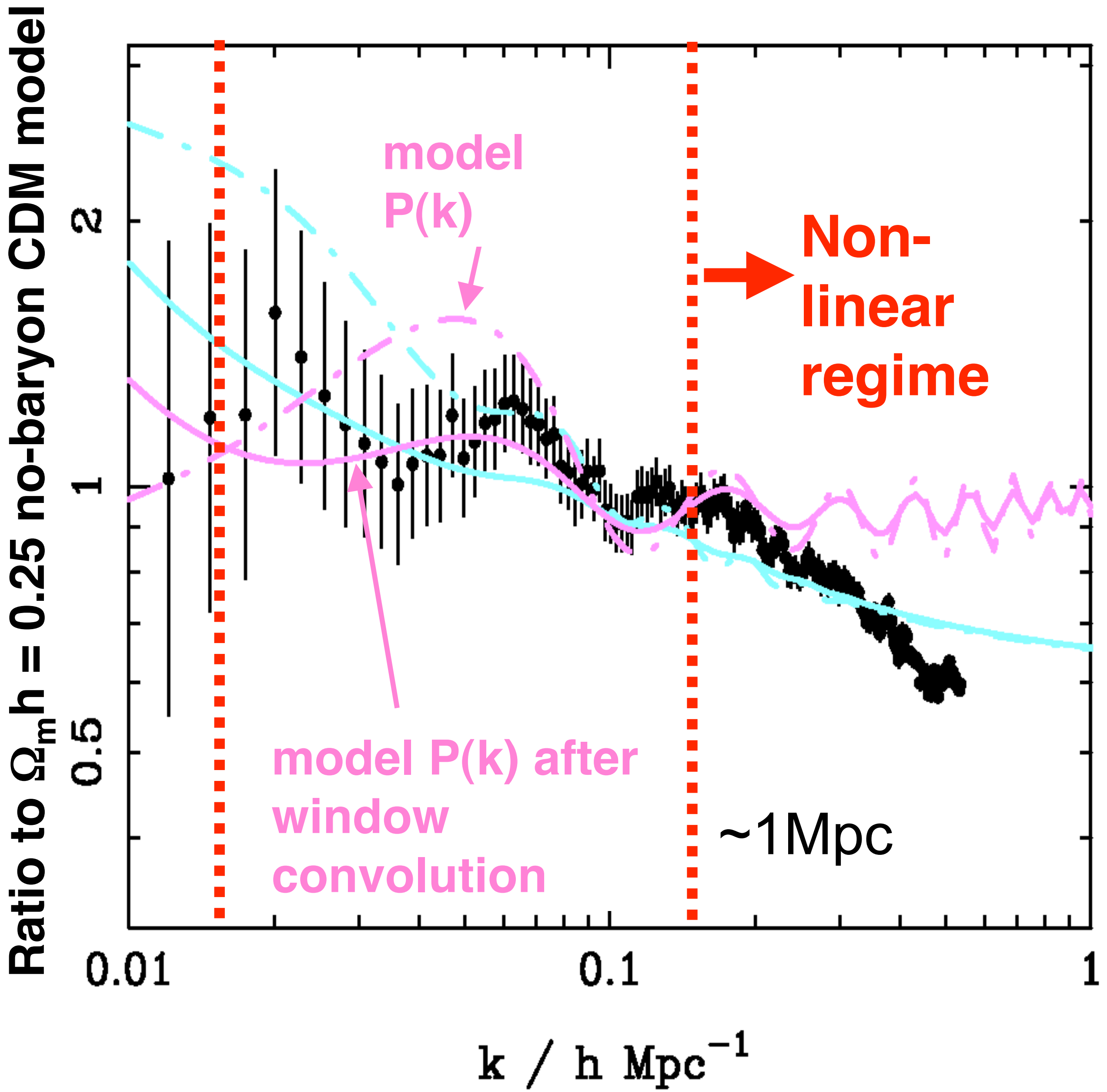
'velocity overshoot'

LSS amplitude smaller than CMB

CDM Model Fits to Galaxy Power Spectrum



CDM Model Fits to Galaxy Power Spectrum



Fit model CDM P(k) (with n=1) after convolution with survey window function.

Parameters:

(1) Ω_m h

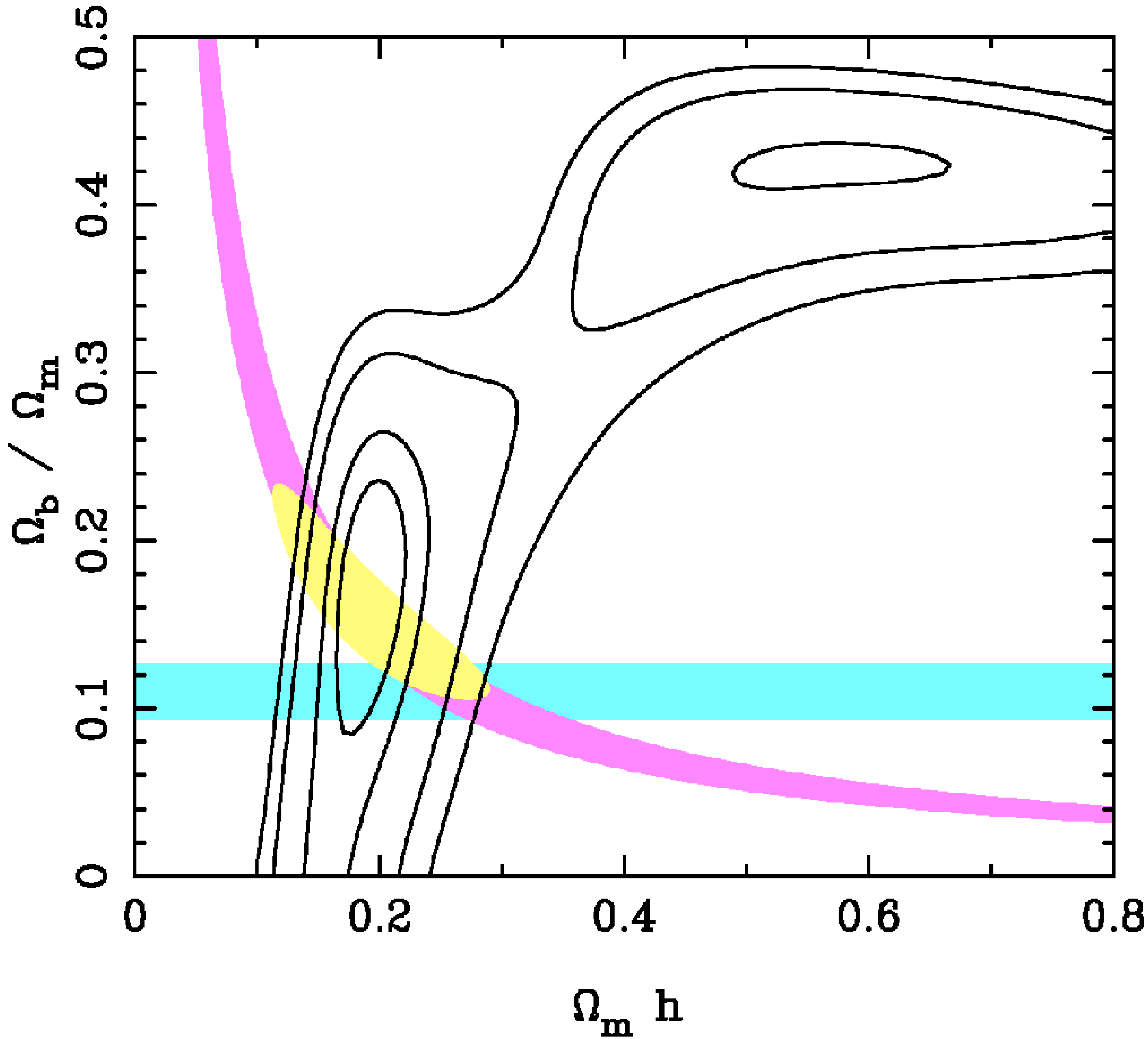
(2) Ω_b / Ω_m

(3) h (marginalise)

Window flattens P(k) and depresses baryon features.

Fits limited to $0.015 < k < 0.15$.

Consistency with Other Constraints



Cluster baryon fraction

Nucleo-synthesis

CMB anisotropies

Galaxy-Galaxy Correlation Function

correlation function = fourier transform of power spectrum

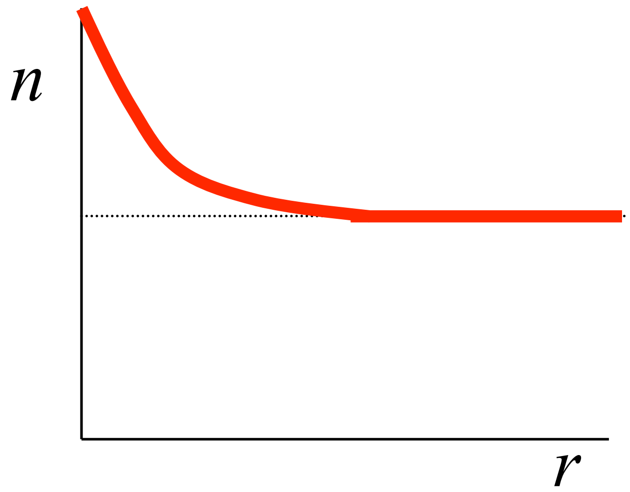
$$\xi(\mathbf{r}) \equiv \langle \, \delta(\mathbf{x})\, \delta(\mathbf{x}+\mathbf{r}) \, \rangle = \sum_{\mathbf{k}} \left|\delta_{\mathbf{k}}\right|^2 \exp(-i\, \mathbf{k}\cdot\mathbf{r}\,)$$

radial correlation function

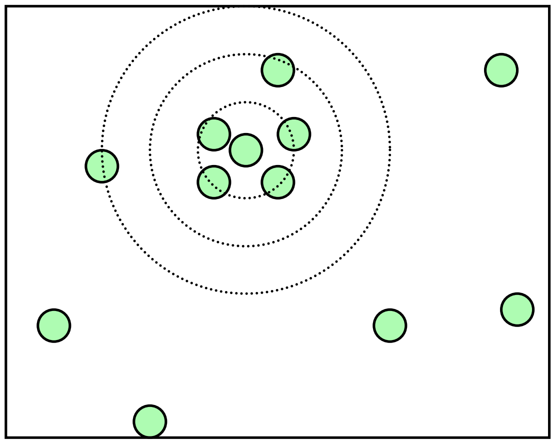
$$\xi(r) = \langle \, \xi(\mathbf{r}) \, \rangle \approx \left(\frac{r}{8\ \mathrm{Mpc}} \right)^{-1.8}$$

measures galaxy clustering

$$n(r) = n_0 \left(\, 1 + \xi(r) \, \right)$$



Galaxy counts at separation *r* <u>larger than expected</u> for random distribution.



Power-Law Models

power - law power spectrum

$$P(\mathbf{k}) = \left\langle \delta_{\mathbf{k}}^{\ 2} \right\rangle = P_0 \left(\frac{k}{k_0} \right)^n$$

$n =$ "tilt"

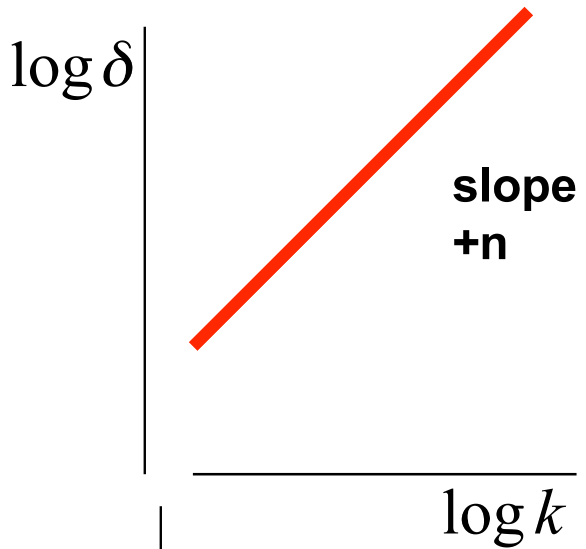
$n = +1 \Rightarrow$ "scale - invariant"

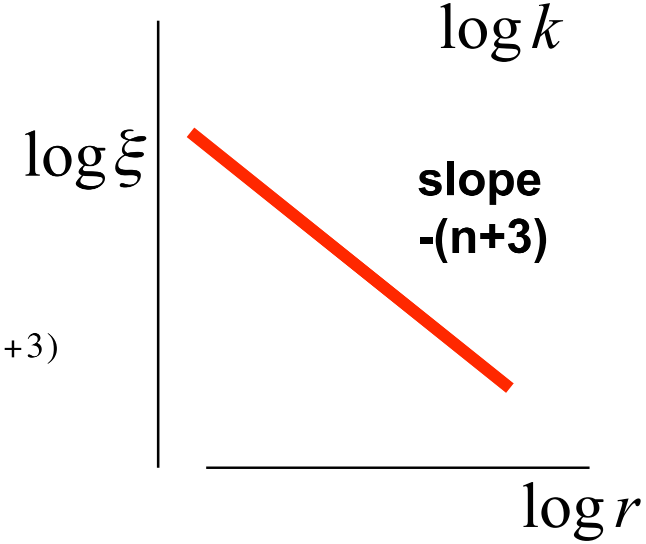
variance after smoothing on scale r

$= $ sum of power for $k = |\mathbf{k}| \leq k_{\max} = 2\pi / r$

$$\left\langle \delta^2 \right\rangle = \sum_{\mathbf{k}} P(\mathbf{k}) \propto \int_0^{k_{\max}} k^n \left(4\pi\, k^2 dk\right) \propto k_{\max}^{n+3} \propto r^{-(n+3)}$$

$$\xi(r) = \left(\frac{r}{r_0} \right)^{-\gamma} \qquad r_0 \approx 8 \text{ Mpc} \qquad \gamma = n + 3 \approx 1.8 \rightarrow n \approx -1.2$$





Easier to derive γ = 1.8 from 2-point correlation function.

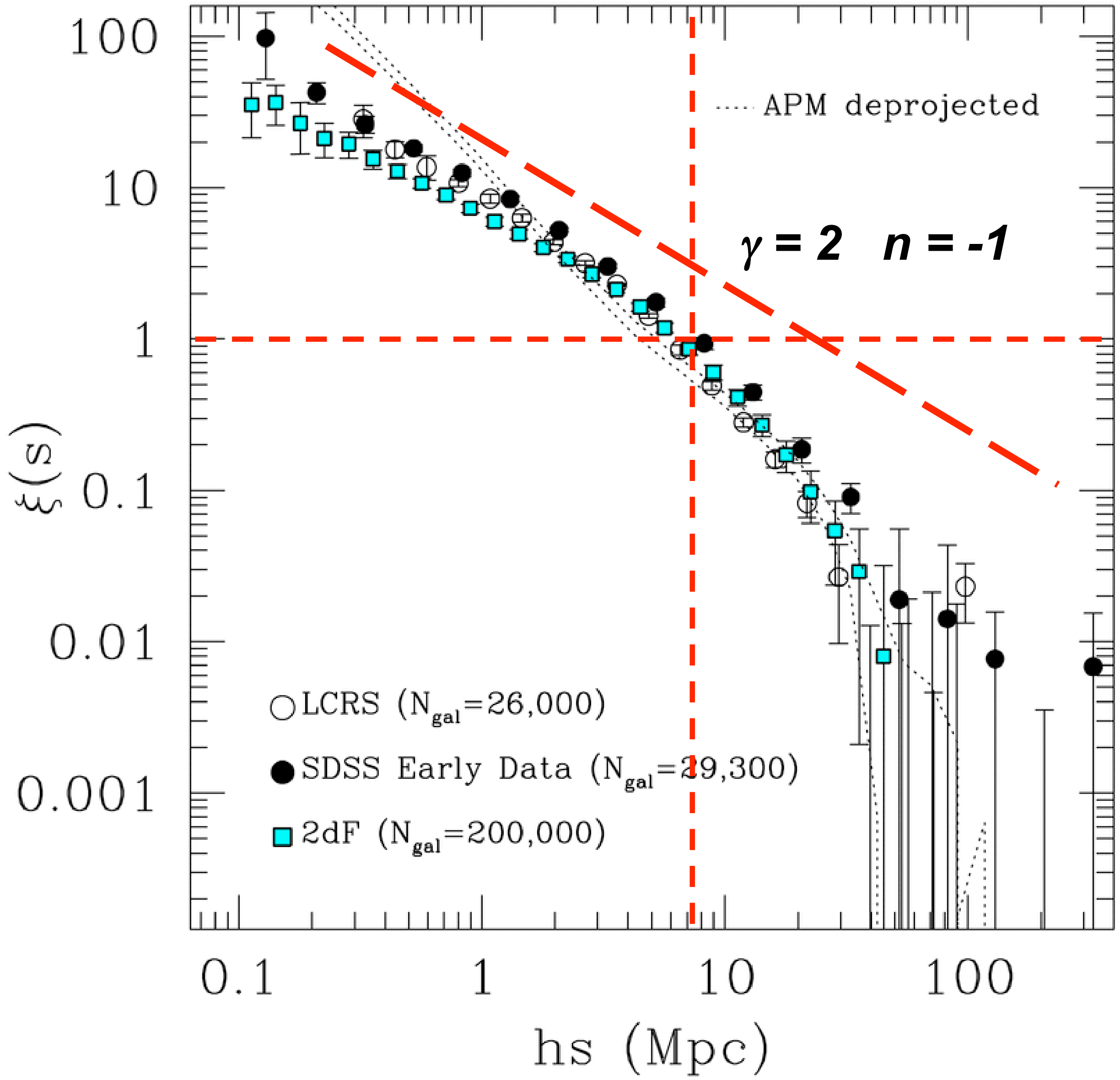
Galaxy-Galaxy Correlation Function

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

$$r_0 \approx 8 \text{ Mpc}$$

$$\gamma = n + 3 \approx 1.8$$

$$\rightarrow n \approx -1.2$$



2-D Correlation Function

angular separation

$$\sigma \equiv \frac{\Delta\theta}{D_A}$$

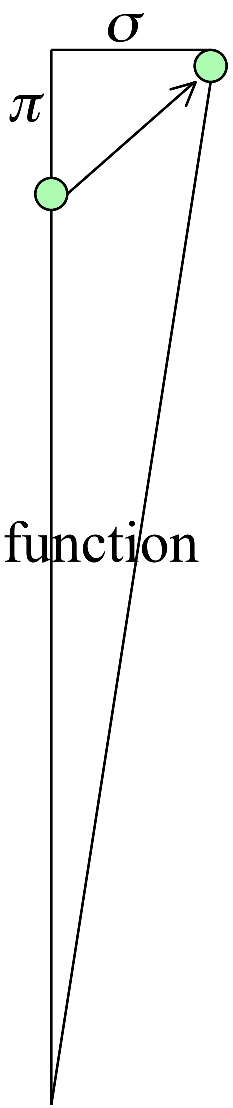
radial separation

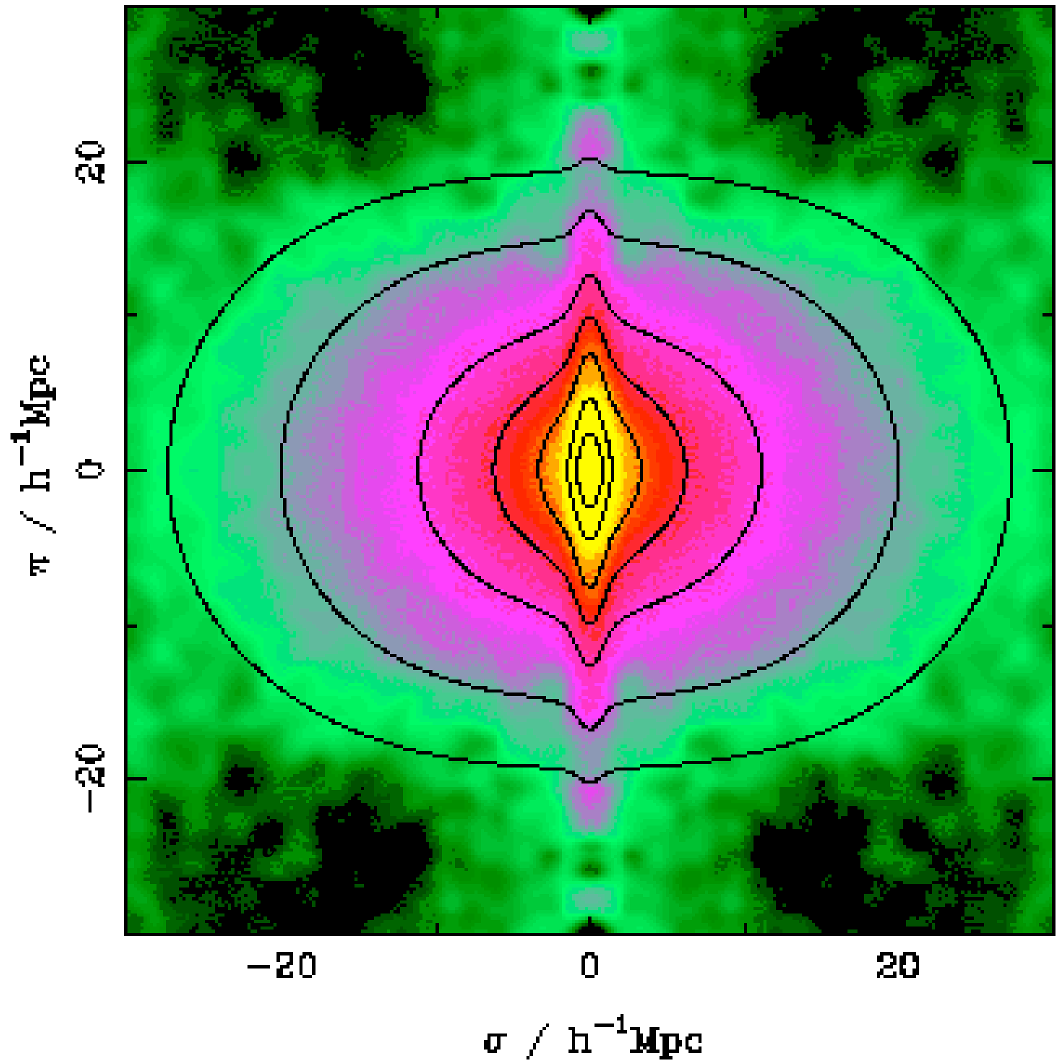
$$\pi \equiv \frac{c\,\Delta z}{H_0}$$

2 – point correlation function

$$\xi(\sigma, \pi)$$

Why flattened ?

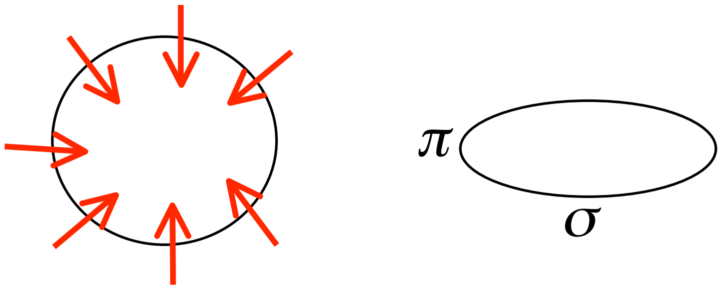




Hawkins et al. 2002

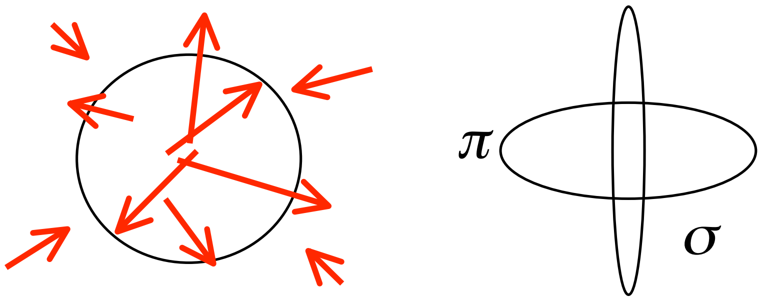
Redshift Distortions

1. Kaiser effect:



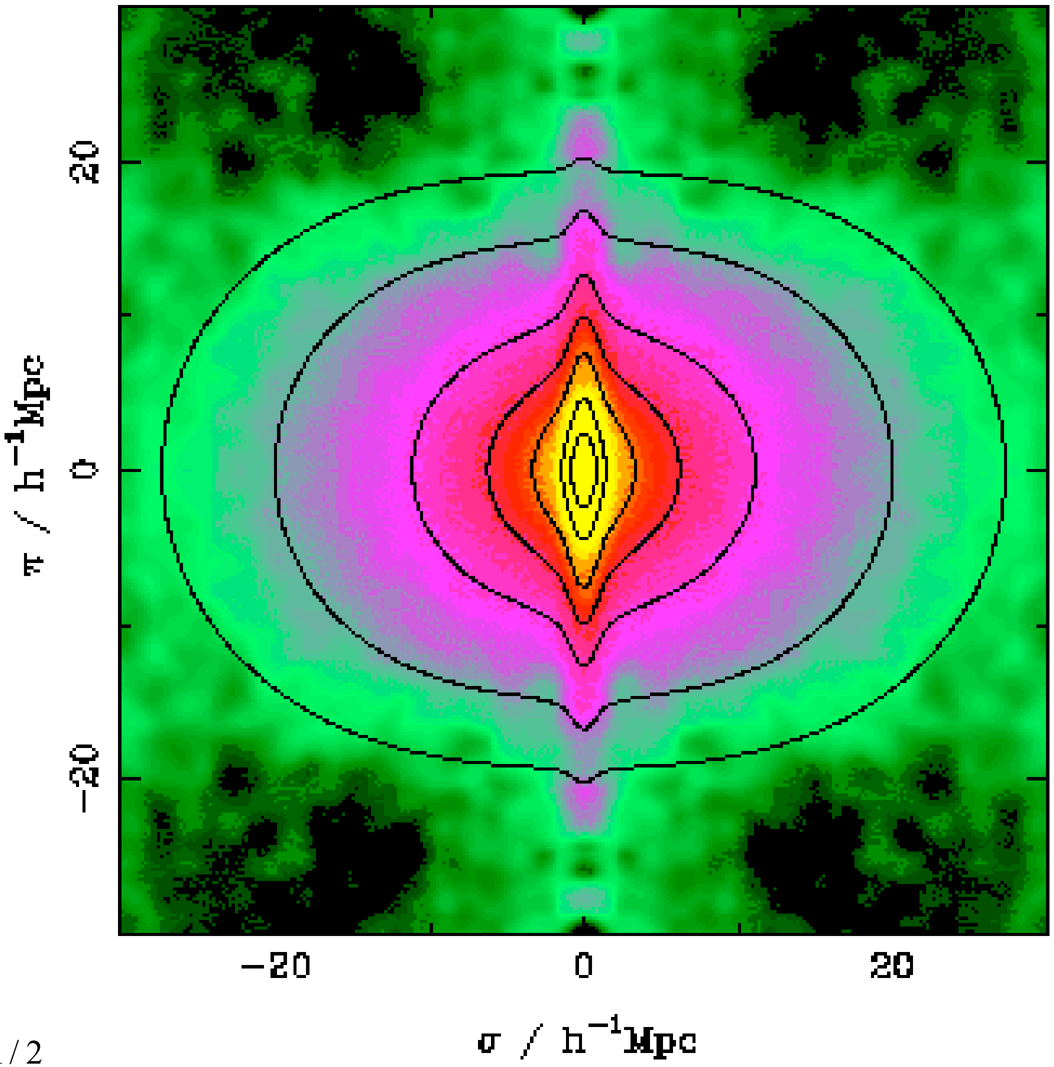
Infall velocities reduce π

2. "Fingers of God"



Virialised cluster cores

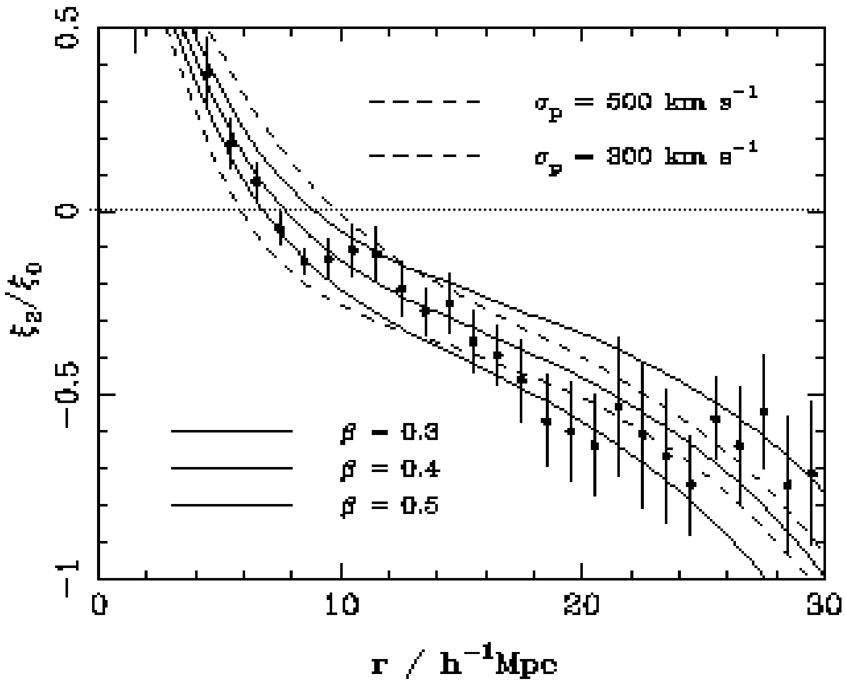
$$V \sim \left(\frac{G\,M_c}{r_c} \right)^{1/2}$$



Hawkins et al. 2002

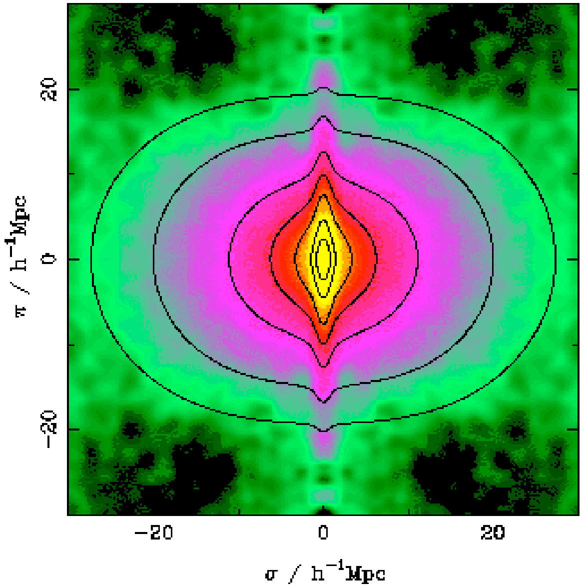
Kaiser Effect

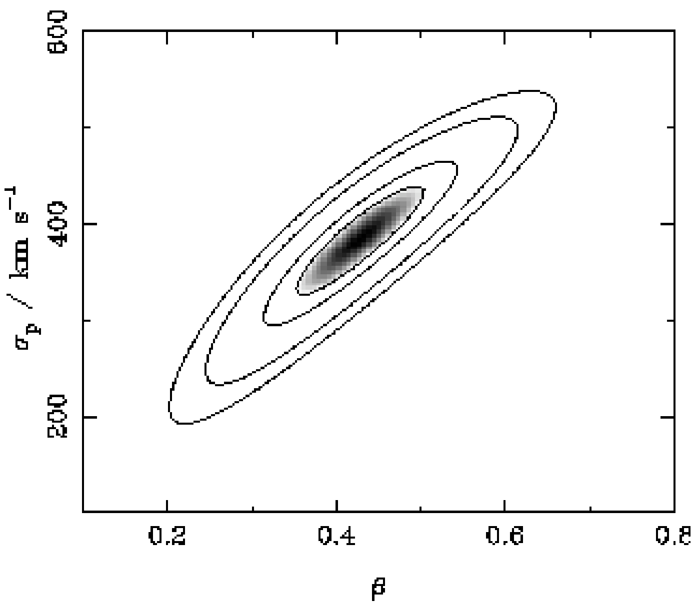
Flattening vs size of $\sigma - \pi$ contours



$$\beta \equiv \frac{\Omega_M^{\ 0.6}}{b} = 0.43 \pm 0.07$$

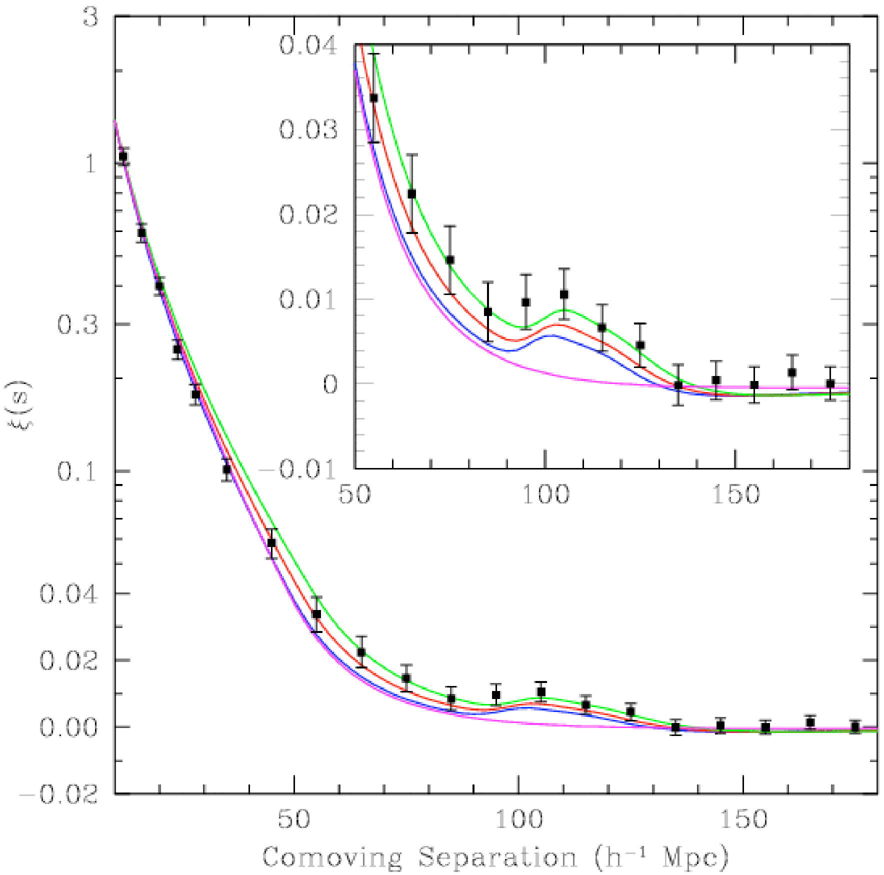
b = bias parameter > 1

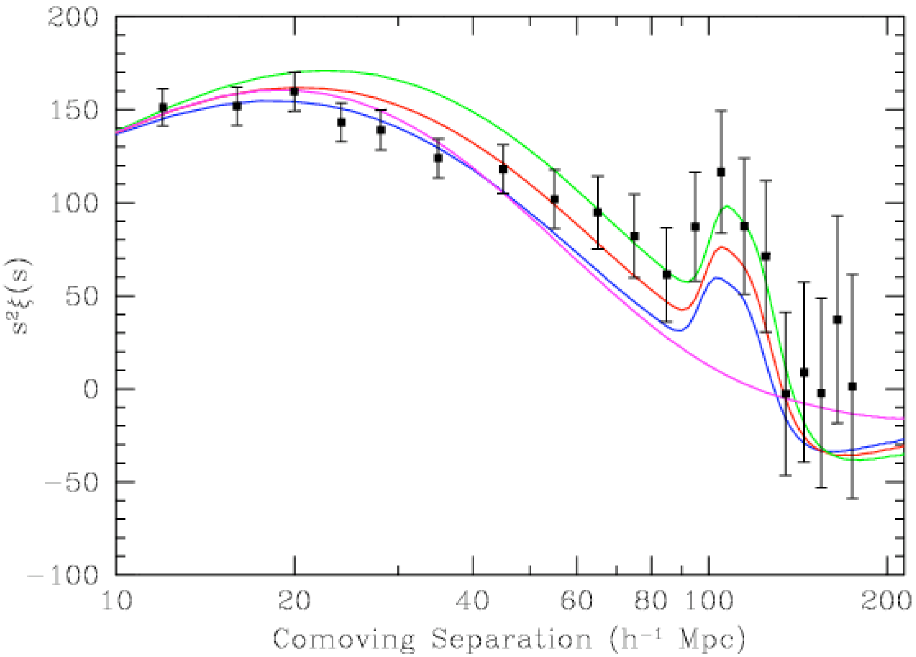


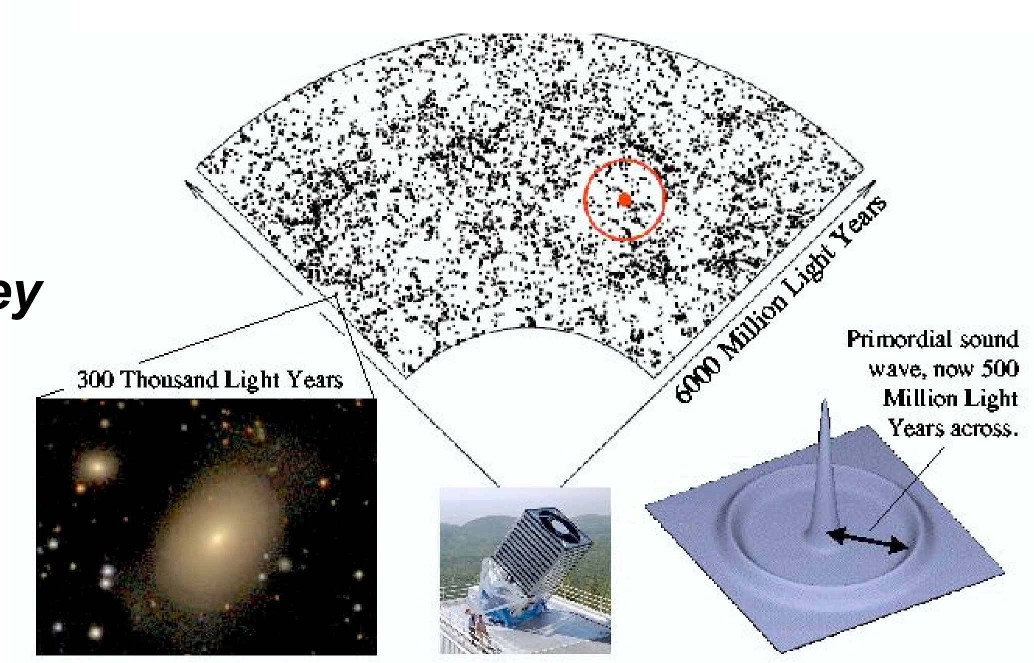


σ_p = dispersion of galaxy peculiar velocities.

Baryon Acoustic Oscillations

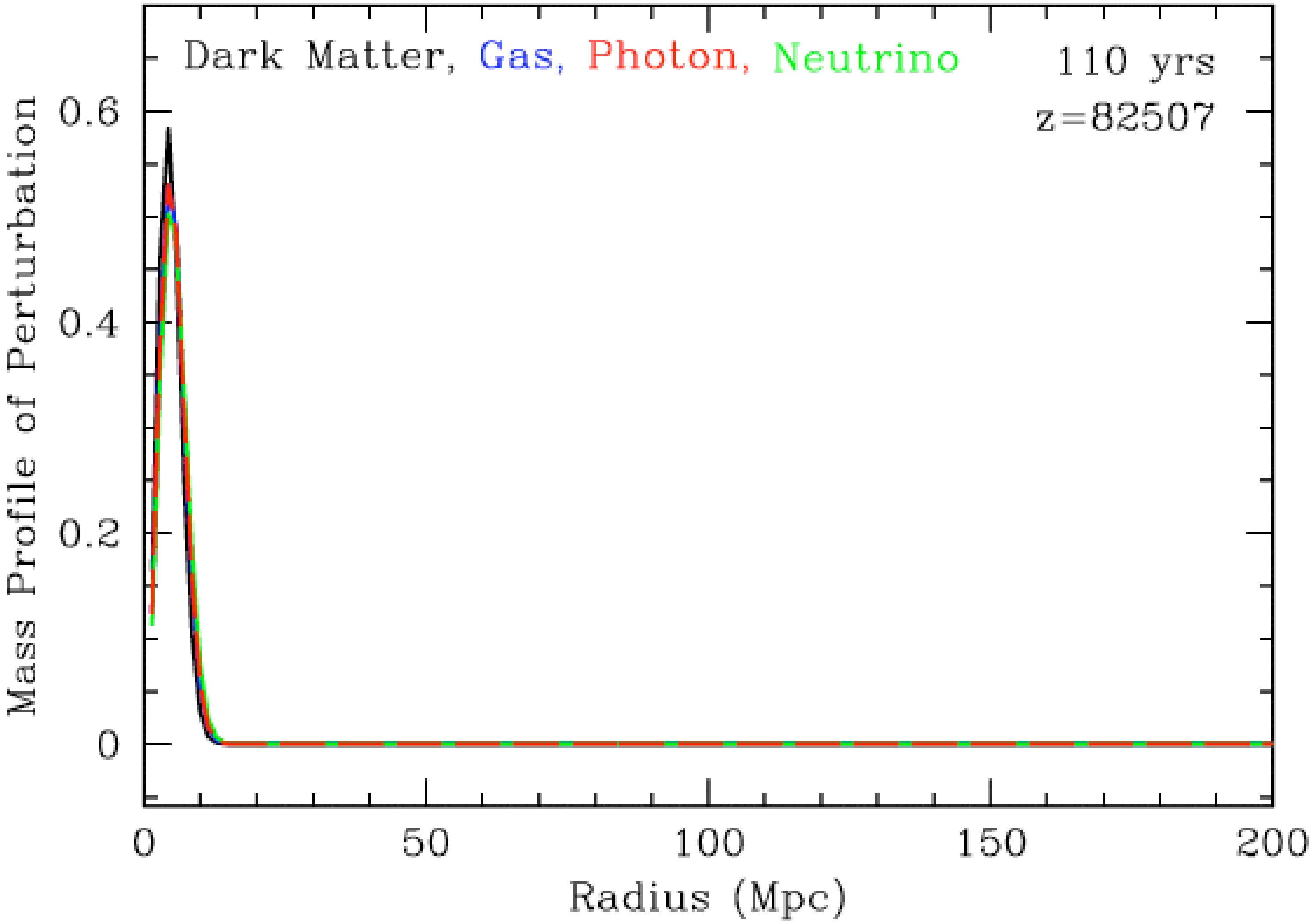






SDSS = Sloan Digital Sky Survey

Eisenstein et al. 2001.


Dark Matter, Gas, Photon, Neutrino
110 yrs
z=82507
Mass Profile of Perturbation
0.6
0.4
0.2
0
0
50
100
150
200
Radius (Mpc)

Baryon Acoustic Oscillations