AS4022 — Cosmology — Tutorial Sheet 1

1. Write a metric in spherical polar coordinates for a 3-dimensional closed Universe with constant radius of curvature $R = 10$ Mpc. Use your metric to calculate (a) the surface area and (b) the volume for a sphere with radius $D = 10$ Mpc.
   By what factor are these results larger (or smaller) than they would be in a flat Euclidean geometry?

2. Write the Robertson-Walker (pseudo)metric in terms of a dimensionless radial coordinate (e.g. u or χ) and the conformal time coordinate $\eta = \int \frac{c\,dt}{R(t)}$.

3. A high-energy cosmologist travels in a starship to a nearby star 9 light years away, spends 10 years exploring the region, and then returns home. Upon returning, she finds that 30 years have elapsed on Earth. Draw a space-time diagram of the journey, assuming that the starship travels at constant speed V, and use the Minkowski (pseudo)metric to compute the proper time on each leg of the journey.
   (a) What was the speed V in units of the speed of light?
   (b) At the end of the journey, how much older (or younger) is the traveller than her twin who remained at home.

4. 10^4 galaxies with redshifts z between 0.3 and 0.4 are counted in a 1 square degree field of view. Calculate the co-moving volume of this survey, and hence the number density of galaxies per *co-moving* cubic Megaparsec. (The co-moving volume is the survey volume expanded to the current epoch t_0.) Assume a flat universe with $h = 0.7$ and $\Omega_M = 1.0$. For a challenge, do the same using the Concordance model $(h, \Omega_M, \Omega_\Lambda) = (0.7, 0.3, 0.7)$.

5. A radio jet emerging from the nucleus of a quasar at redshift $z = 0.2$ is observed to be 3 arcseconds long. Assuming that the jet is perpendicular to the line of sight, calculate its physical length in pc for two cosmological models with parameters $(h, \Omega_M, \Omega_\Lambda) = (0.7, 0.3, 0.7)$ and $(0.7, 0.3, 0.0)$.

6. Write a metric in spherical polar coordinates for a 4-dimensional space with constant radius of curvature R. Use this metric to write the integral expressions needed to compute (a) the 3-dimensional surface area A, and (b) the 4-dimensional volume V, for a sphere of radius D. Evaluate your expressions.