AS4022 — Cosmology — Tutorial Sheet 3

1. Write a formula for the "sound horizon", $t_H(z)$, which is the distance travelled by a sound wave starting at time $t = 0$ and finishing at a particular redshift z. Assume a sound speed $c_S = c/\sqrt{3}$ in the photon-baryon plasma prior to recombination. Evaluate the size in Mpc of the sound horizon at $z_R = 1000$. To what size does this region expand at the present epoch ($z = 0$)?

2. After recombination at $z \approx 1000$, the absence of free electrons removes the electron scattering opacity, releasing photons to flow freely. Matter, released from the photons, is now able to fall into dark matter potential wells to form compact structures. Some time later, the first stars appeared, ending the Dark Ages. UV radiation from the first high mass stars re-ionised the intergalactic gas, producing free electrons once again, which scattered some of the background photons, producing polarisation. WMAP detected this polarisation at a strength indiating that $\approx 15\%$ of the background photons were scattered.

   Write a formula for the optical depth to electron scattering along the path of a photon that was emitted at redshift z and observed today. Assuming $h = 0.7$, $\Omega_b = 0.04$, $\Omega_M + \Omega_\Lambda = 1$, and $\Omega_M = 0.3$, at what redshift does the optical depth become 0.15? This is the redshift of re-ionisation, when the first stars formed.