AS4022 — Cosmology — Tutorial Sheet 4

1. Consider a micro-cosmos of N ants inhabiting an expanding sphere of radius $R = R_0\ (t/t_0)^q$. At the current time $t_0 = 1$ yr, the radius is $R_0 = 1$ m. Let $q = 1/2$ and $N = 100$. The ants have a cross-length $\sigma = 1$ cm for encountering other ants, and they move in random directions with a velocity $v(t)$ that conserves their angular momentum, $J = R(t)\,v(t) = 1\ \mathrm{m}^2/\mathrm{yr}$, with respect to the centre of expansion. a) Write expressions as functions of time t for the ant-world Hubble expansion rate H, the ant speed v, and the ant surface density n.
   b) Light emitted by ant A travels half-way around the sphere where it is observed by ant B at time $t = t_0$. What redshift z does ant B observe?
   c) Write expressions for the distance travelled by an ant and for the probability that it encounters another ant, between times t_1 and t_2. Calculate the distance and encounter probability for $t_1 = 1/2$ yr and $t_2 = 2$ yr.

2. Assume that Dark Matter particles are being emitted isotropically with a mildly relativistic speed $v \approx c$ from the centre of a spiral galaxy. Derive an expression for the resulting DM energy density as a function of radius R. Show that this component makes the galaxy's rotation curve $V(R)$ asymptotically flat. What mass-loss rate $\dot{M}$, in solar masses per year, is required to make the circular velocity at large distances $V_0 = 250\ \mathrm{km\ s^{-1}}$, as observed in large spirals like the Milky Way. Why is this, or is this not, a viable explanation for the Dark Matter problem in spiral galaxies?

3. Verify the low-redshift expansions for the angular diameter distance

$$D_A(z) = \frac{c\,z}{H_0}\left(1 + \frac{3 - q_0}{2}\,z + ...\right)$$

   and the luminosity distance

$$D_L(z) = \frac{c\,z}{H_0}\left(1 + \frac{1 - q_0}{2}\,z + ...\right)$$

   in terms of the Hubble parameter H_0 and the deceleration parameter q_0.