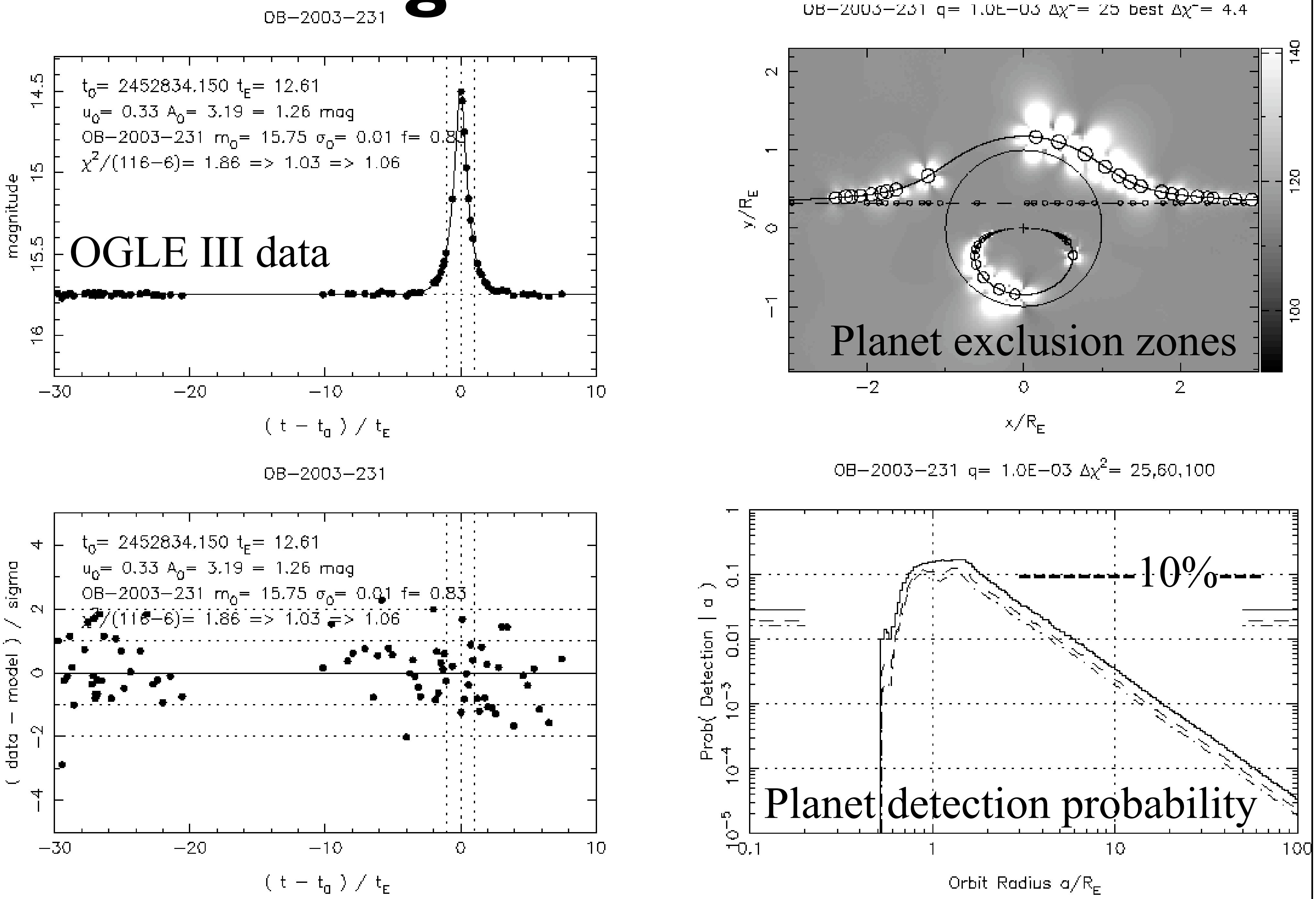
Fitting Data with PLENS



PLENS: web page

http://star-www.st-and.ac.uk/~kdh1/plens/plens.html

PLENS

PLENS is Keith Horne 's fortran code to fit lightcurves of microlensing events.
PLENS uses PGPLOT for graphics. The code is continually under development.

- plens.tar.gz , tarball with all files listed below.
- plens.exe , executable image.
- plens.com , script to compile plens.
- plens.inc , include file with common blocks for global variables.
- plens.for , plens main programme and subroutines.
- lens1.for , single lens subroutines.
- lens2.for , binary lens subroutines.
- asp.for , multiple lens subroutines.
- dzw.for , detection zone area subroutines.
- misc.for , miscellaneous subroutines.
- test390.dat , lightcurve data file for testing.
- plens.fit runs plens.exe to fit and make standard plots.
- plens.fitc for plots with c(x,y).
- plens.fitl for plots with c(u,theta).
- plens.fitv for plots with c(t,f).
- plens.fitp for plots with pdet(a).
- plens.test runs plens.fit* on test390.dat.
- pspl fit: test390.4.(a , b , m , n , r , g , y , z).ps

+blending: test390.5.(a , b , m , n , r , g , y , z , c , l , v , p).ps
+sigmag0: test390.6.(a , b , m , n , r , g , y , z).ps
+erbscl: test390.7.(a , b , m , n , r , g , y , z).ps
2-panel plots: test390.5.(ab , mr , mn , mc , ml , mv , cl , pc , pl).ps
3-panel plots: test390.5.(mrn , mrc , mnv , mrl , mnl , mrv , mnv , pcm , plm).ps

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Keith Horne (kdh1 at st-andrews.ac.uk)

Tarball with all files.

F77 source code

Test dataset test390.dat

Scripts to fit and produce various plots

Plots for the test dataset.

PLENS: PSPL Model

PSPL magnification: $A(t) = \dfrac{u^2 + 2}{u\left(u^2 + 4\right)^{1/2}}$ $\qquad u^2 = {u_0}^2 + \left(\dfrac{t - t_0}{t_E}\right)^2$

model flux (magnified source + blend):

$$\mu_i(t) = f_S\ A(t) + f_B = f_S\left(A(t) - 1\right) + f_0 \qquad f_0 \equiv f_S + f_B$$

$$= f_S\left(A(t) + b\right) = f_0\,\frac{A(t) + b}{1 + b} \qquad b \equiv f_B / f_S \quad b \in (-\infty, \infty)$$

3 nonlinear parameters: $t_0 \quad t_E \quad u_0$

$2N$ linear parameters: $f_S(i) \quad f_B(i)$ for site/filter $i = 1 \ldots N$

from which : $f_0(i) \quad b(i)$ more nearly orthogonal.

Optimal scaling fits for linear parameters.

Amoeba (downhill simplex) for non-linear parameters.

PLENS: Lightcurve Data Files

Lightcurve data files: Multi-column ASCII files.

PLENS recognises several formats:

N	mag	err	HJD	see	sky	code	dwell
mag	err	HJD	see	sky	code	dwell	
HJD	mag	err	see	sky	dwell		
HJD	mag	err	see	sky			
HJD	mag	err	see				
HJD	mag	err					

PLENS decodes site/event/filter from standard file names:
e.g. OOB08300I.dat JOB08300R.dat

Default data file extension is ".dat".
e.g. "JOB08300R" picks up file "JOB08300R.dat".

PLENS: Multiple Data Files

List of lightcurve data files: “@OB08300.lis”

The file OB08300.lis holds a list of lightcurve data files,
e.g. OOB08300I.dat
KOB08300I.dat
HOB08300R.dat
JOB08300R.dat

Best data file (usually OGLE or MOA) should be first (used to guess initial lens parameters).

Default extension is “.lis”.
e.g. “@OB08300” picks up “@OB08300.lis”.

PLENS: Initial Parameters

Subroutine GUESSLENS runs on new input data.
Fast (optimal scaling) fit of gaussian + constant model to observed magnitudes on best (first) dataset.

Grid search in centroid t_0 and rms σ_t :

t_0 range: start to end of data in steps of $\sigma_t / 10$

σ_t range: 2 to 100 days in steps of 1/12 in log σ_t

Reject if: no support (gaussian falls in a data gap).
best-fit gaussian goes down rather than up.

Retain best gaussian + constant model, lowest χ^2 (t_0, σ_t).
[Convert gaussian width σ_t to t_E.]

Evaluate PSPL magnification $A(t)$ and scale to fit source flux f_S for each dataset (no initial blending $b = 0$).

OB-08-300

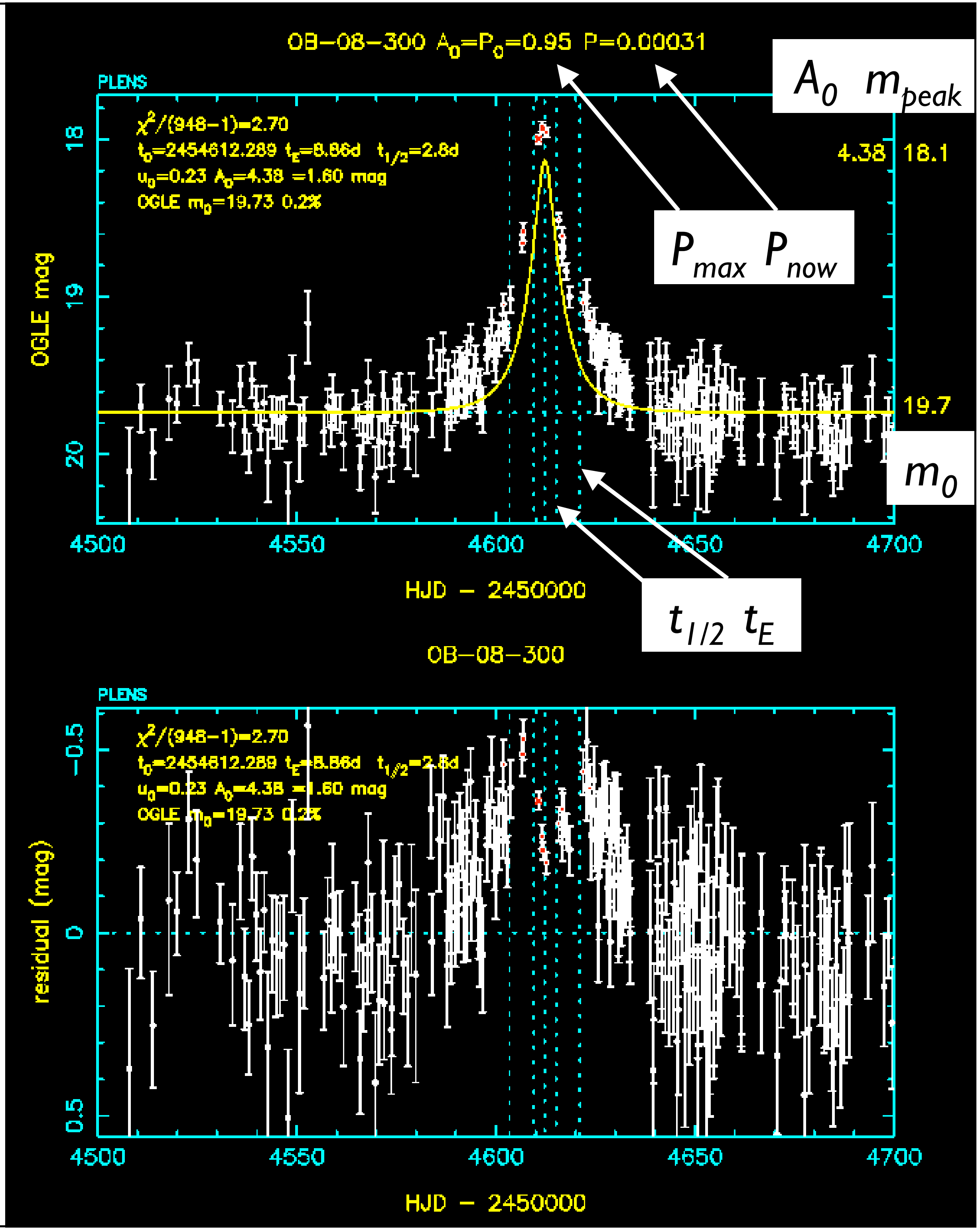
GUESSLENS:

initial PSPL parameters

MR plot
= Magnitudes
+ Residuals

$\chi^2 / (948-1) = 2.70$

Edit parameters by hand (if needed) before fit



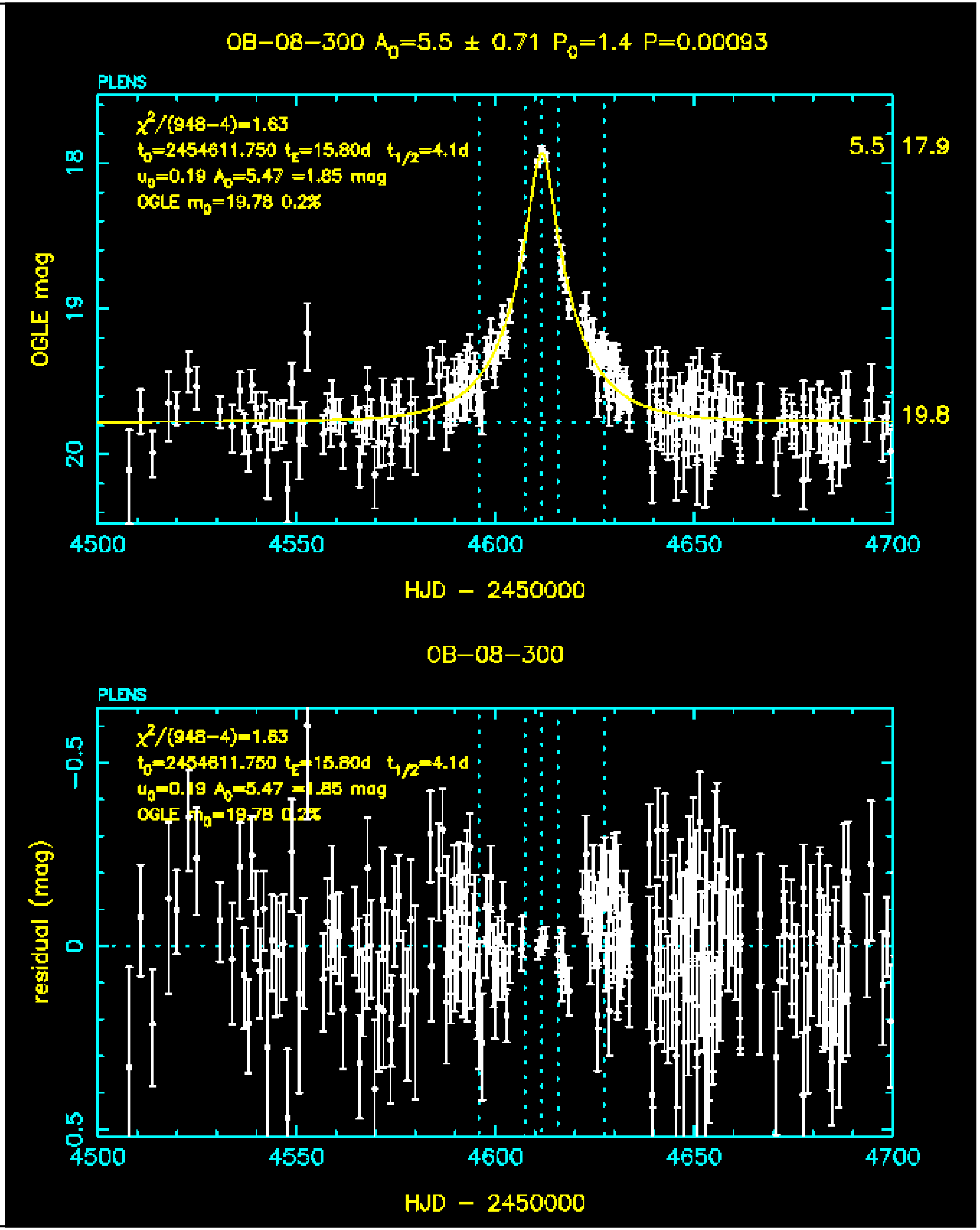
OB-08-300

Fit 4 params:

PSPL with no blending.

$t_0 = 4611.75$
$t_E = 15.80$
$A_0 = 5.47$
$m_0 = 19.78$

$\chi^2 / (948\text{-}4) = 1.63$



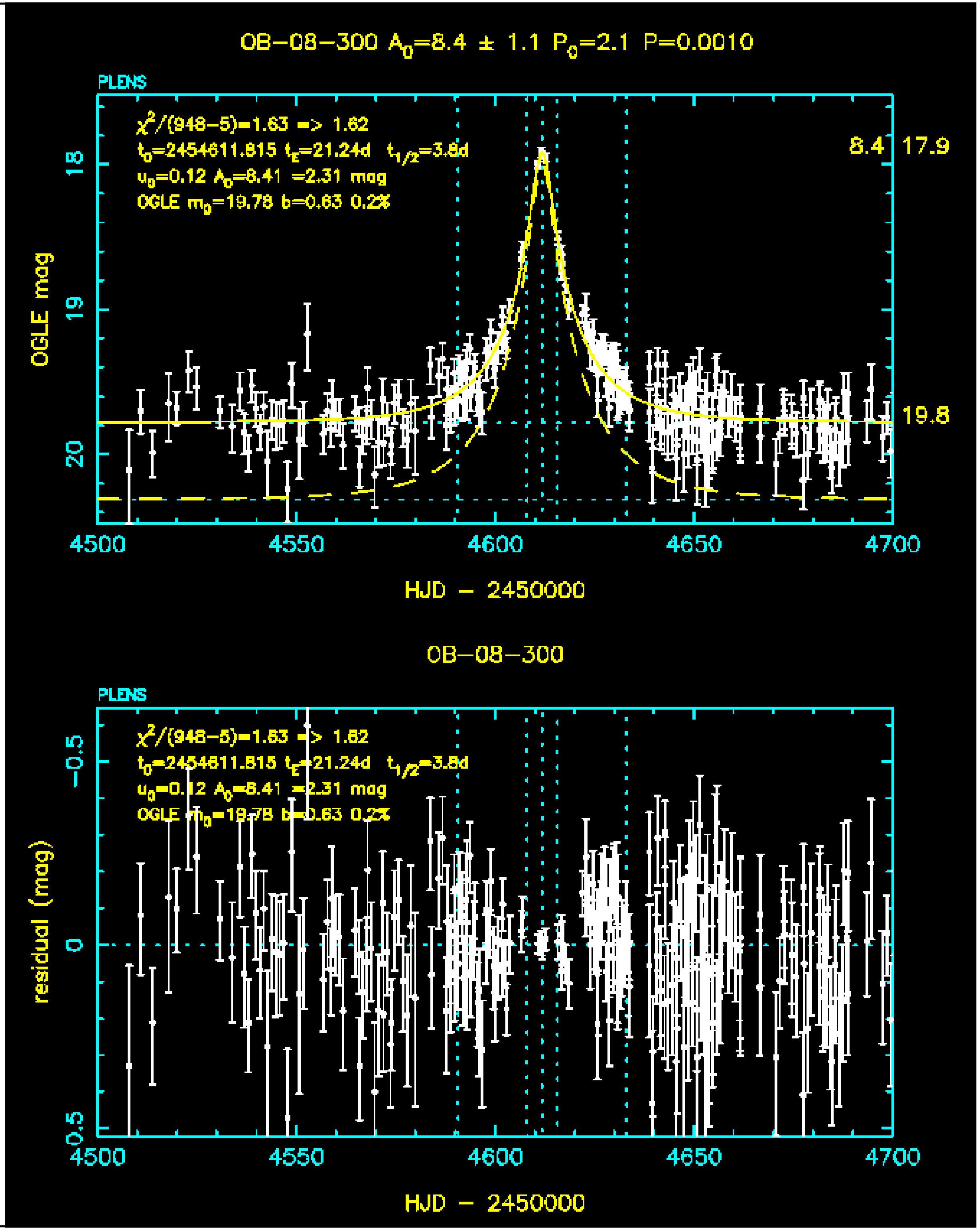
OB-08-300

Fit 5 params:

PSPL with blending.

$b = f_B / f_S = 0.63$
$t_E = 15.8 \Rightarrow 21.2$
$A_0 = 5.5 \Rightarrow 8.41$

$\chi^2 / (948-4) = 1.63$
$\chi^2 / (948-5) = 1.62$



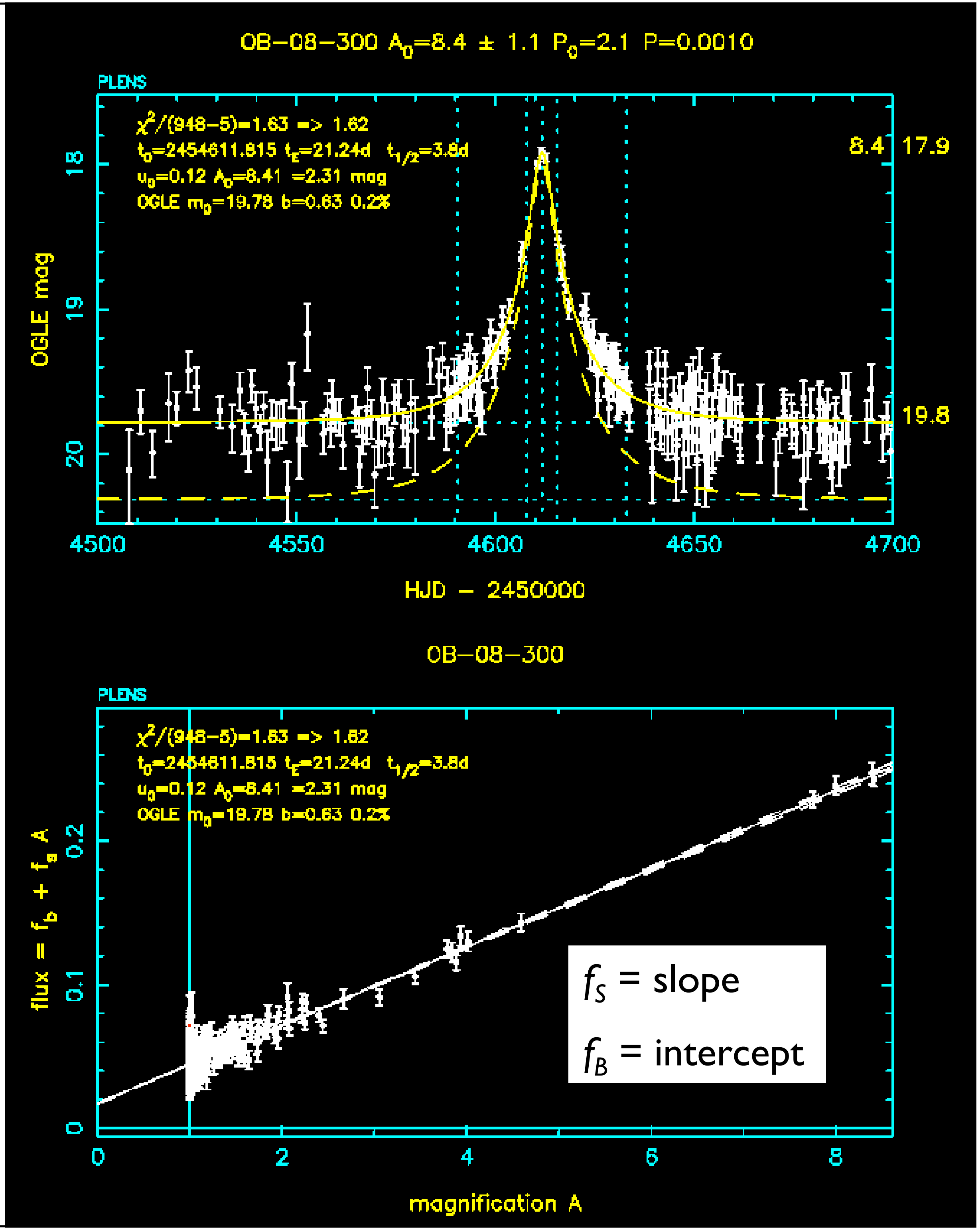
OB-08-300

Fit 5 params:

PSPL with blending.

MB plot = Mags + Blend fit

$\chi^2 / (948\text{-}4) = 1.63$
$\chi^2 / (948\text{-}5) = 1.62$



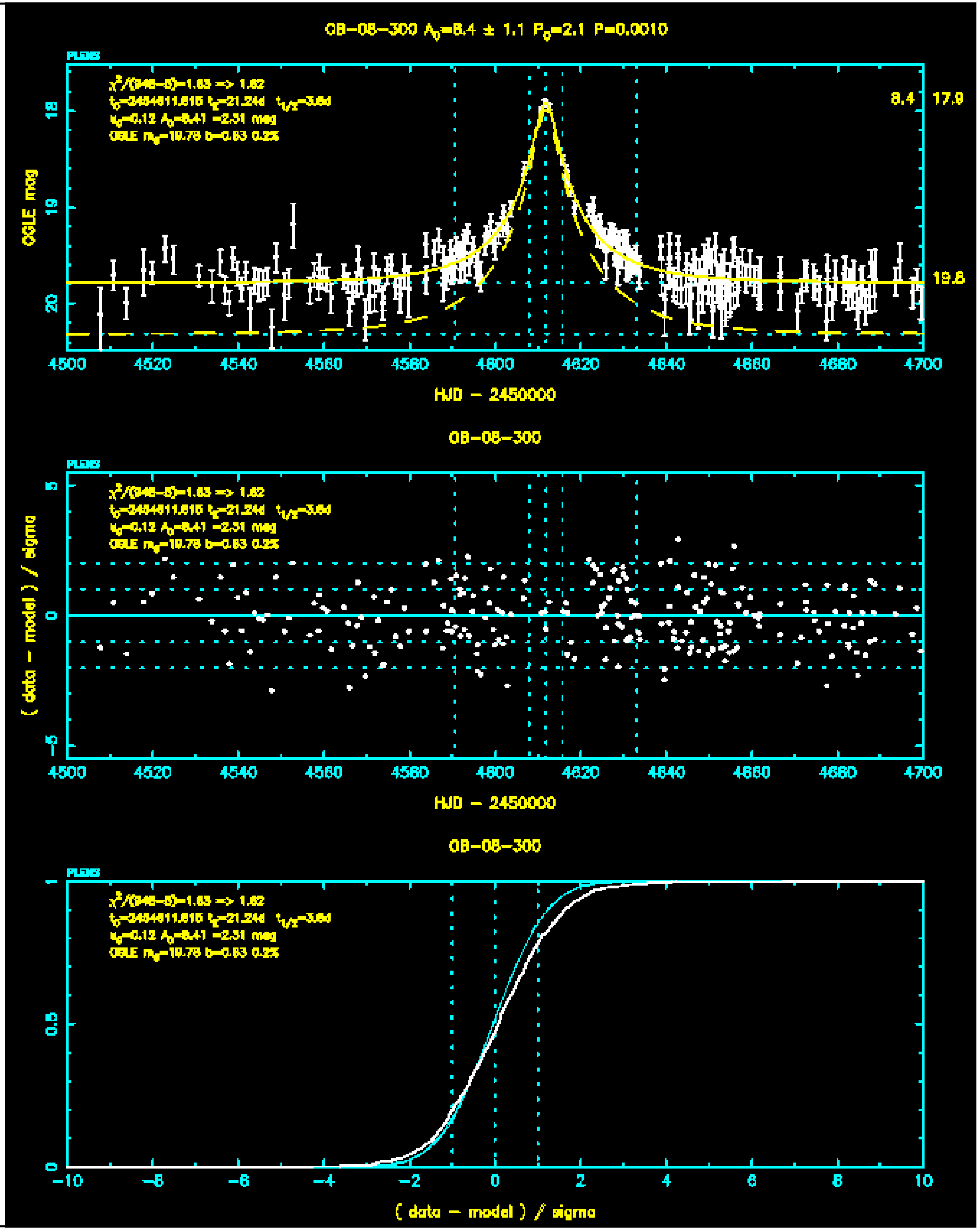
OB-08-300

Fit 5 params:

PSPL with blending.

MNG = Mags
+ Normalised
. residuals
+ Gaussian test

$\chi^2 / (948\text{-}4) = 1.63$
$\chi^2 / (948\text{-}5) = 1.62$



PLENS: Noise Model

OGLE, MOA data: often show more scatter than expected in the baseline flux (due to blending with nearby stars ?) Simple scaling of error bars not appropriate. Increase flux error bar (add variances), not magnitude error bar.

Flux error bars modified:

$$\sigma_i^{\,2} \quad \Rightarrow \quad \sigma_0^{\,2} + f^2\, \sigma_i^{\,2} + \sigma_{sys}^{\,2} \mu_i^{\,2}$$

Noise model parameters:

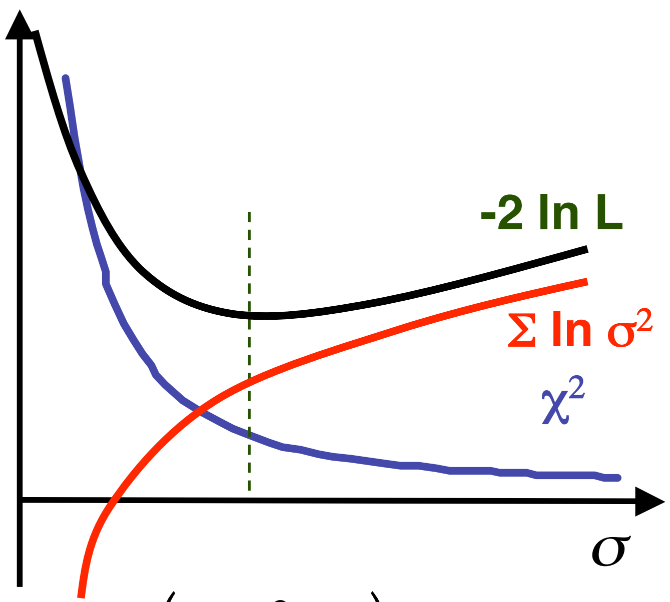
σ_0 = baseline rms flux error (e.g. 0.02 mag relative to f_0)

f = error bar scale factor (e.g. 1.2)

σ_{sys} = scale errors (e.g. 0.2%)

PLENS: Maximum Likelihood

$$\chi^2 = \sum_{i=1}^{N} \frac{(D_i - \mu_i)^2}{\sigma_0^{\,2} + f^2 \sigma_i^{\,2}}$$



χ^2 minimisation fails :

$$\chi^2 \to 0 \quad \text{as } \sigma \to \infty$$

Maximum likelihood :

$$L(\text{model}) = P(\text{data} \mid \text{model}) = \frac{\exp\left(-\chi^2/2\right)}{(2\pi)^{N/2} \prod_{i=1}^{N} \left(\sigma_0^{\,2} + f^2 \sigma_i^{\,2}\right)^{1/2}}$$

$$-2 \ln L = \chi^2 + \sum_i \ln\left(\sigma_0^{\,2} + f^2 \sigma_i^{\,2}\right) + N \ln(2\pi)$$

Amoeba (Downhill simplex) minimises -2 *ln L*

PLENS: Bayesian Priors

$$L(\text{model}) = P(\text{data}\,|\,\text{model}) = \exp\left(-\chi^2/2\right) \Big/ \prod \left(2\pi\,\sigma^2\right)^{1/2}$$

$$P(\text{model}\,|\,\text{data}) = \frac{P(\text{data}\,|\,\text{model})\; P(\text{model})}{\int P(\text{data}\,|\,\text{model})\; P(\text{model})\; \text{d}(\text{model})}$$

$$\text{-}2\ln(L \times \text{Prior}) = \chi^2 + \sum \ln\left(\sigma_0{}^2 + f^2 \sigma_i{}^2\right) - 2\ln P(\text{model}) + const$$

Uniform in log A_0 , log f , log t_E .
Optional: gaussian in log f, log t_E .
Optional: exponential in A_0 .

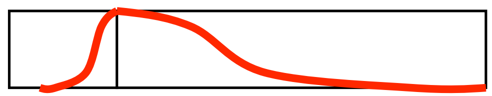
Uniform with gaussian taper for t_0
(avoids t_0 “too far” off ends of data).
Split Gaussian (avoids $b << 0$).
e.g. $\sigma_b = 0.01$ for $b < 0$, 0.1 for $b > 0$

$$P(t_E) \propto \frac{1}{t_E} e^{-\frac{1}{2}\left(\frac{\log t_E - \langle \log t_E \rangle}{\sigma(\log t_E)}\right)^2}$$

$$P(f) \propto \frac{1}{f} e^{-\frac{1}{2}\left(\frac{\log f - \langle \log f \rangle}{\sigma(\log f)}\right)^2}$$

$$P(A_0) \propto \frac{1}{A_0} e^{-\left(A_0 / \langle A_0 \rangle\right)}$$

$P(t_0) \propto$ 

$P(b) \propto$ 

OB-08-300

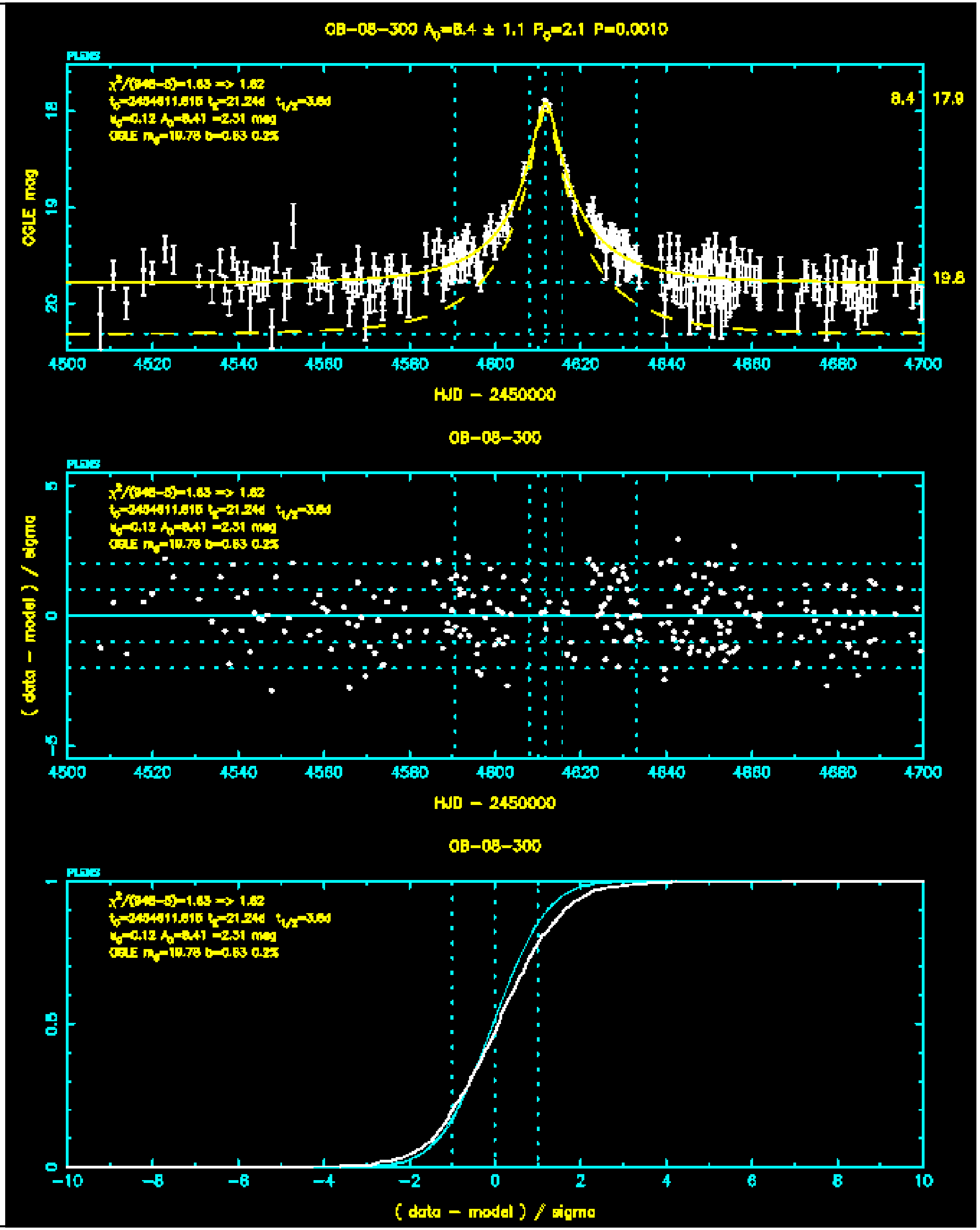
Fit 5 params:

PSPL with blending.

$\sigma_0 = 0.0$

$f = 1.0$

$\chi^2 / (948-4) = 1.63$
$\chi^2 / (948-5) = 1.62$



OB-08-300

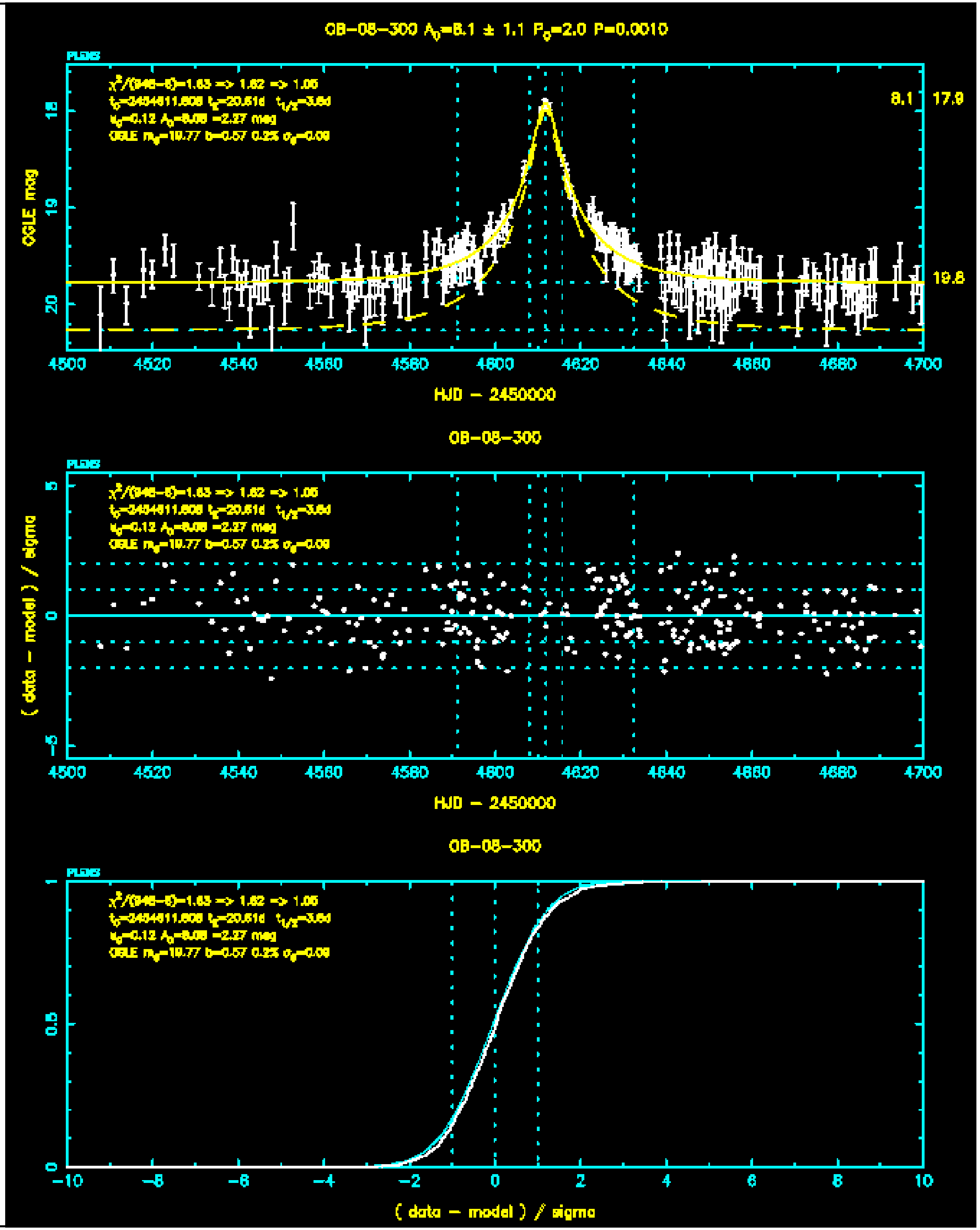
Fit 6 params:

PSPL with blending.

$\sigma_0 = 0.08$ mag

$f = 1.0$

$\chi^2 / (948-4) = 1.63$
$\chi^2 / (948-5) = 1.62$
$\chi^2 / (948-6) = 1.06$



OB-08-300

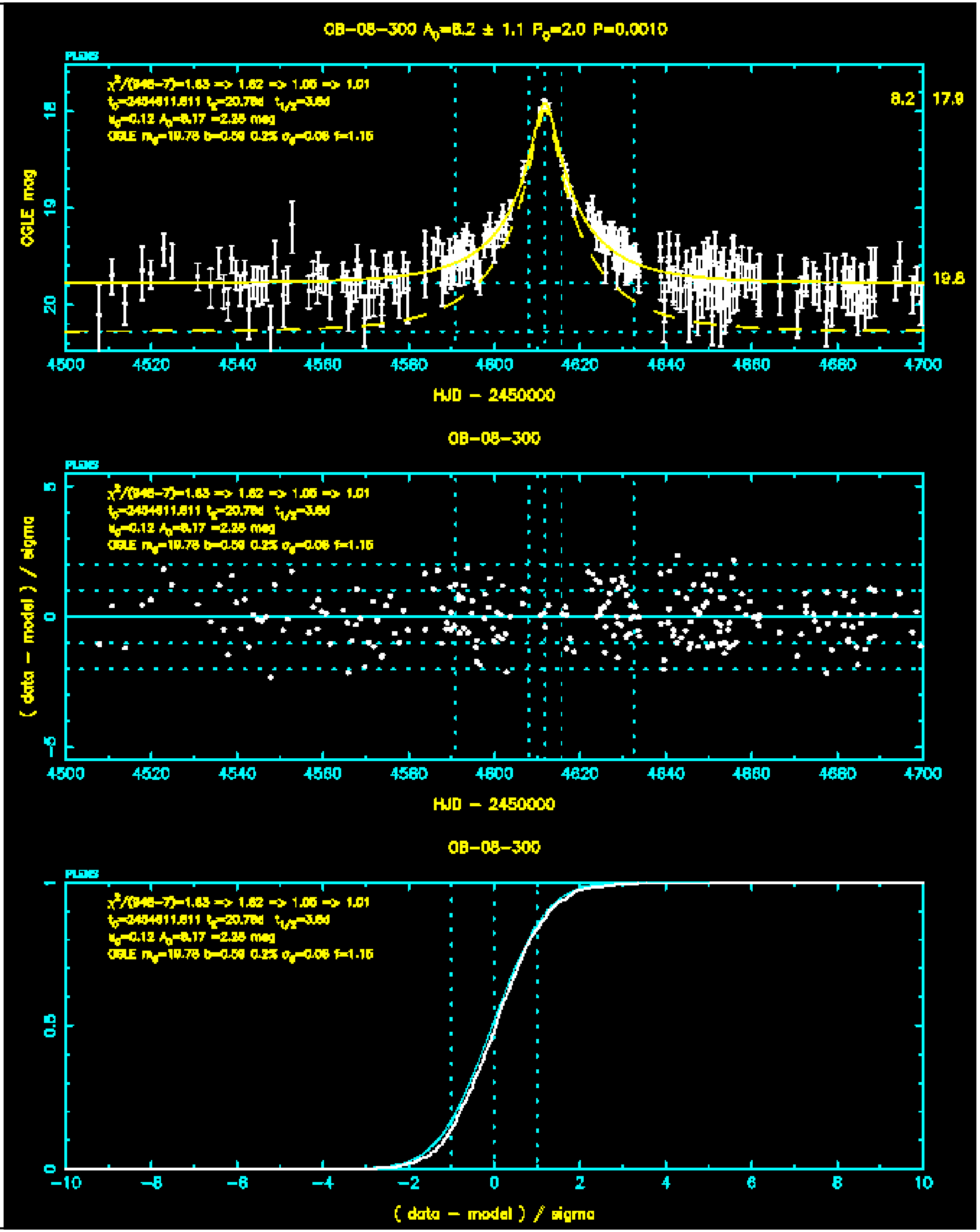
Fit 7 params:

PSPL with blending.

$\sigma_0 = 0.06$ mag

$f = 1.15$

$\chi^2 / (948-4) = 1.63$
$\chi^2 / (948-5) = 1.62$
$\chi^2 / (948-6) = 1.06$
$\chi^2 / (948-7) = 1.01$



OB-08-300

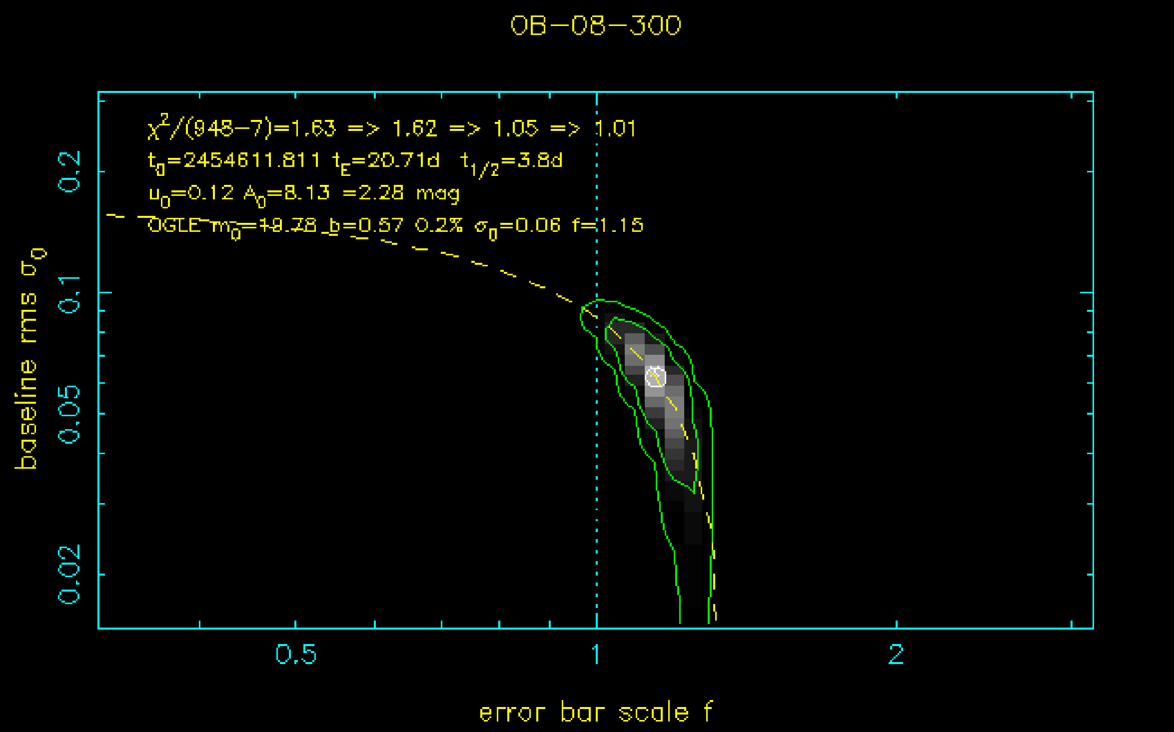
Fit 7 params:

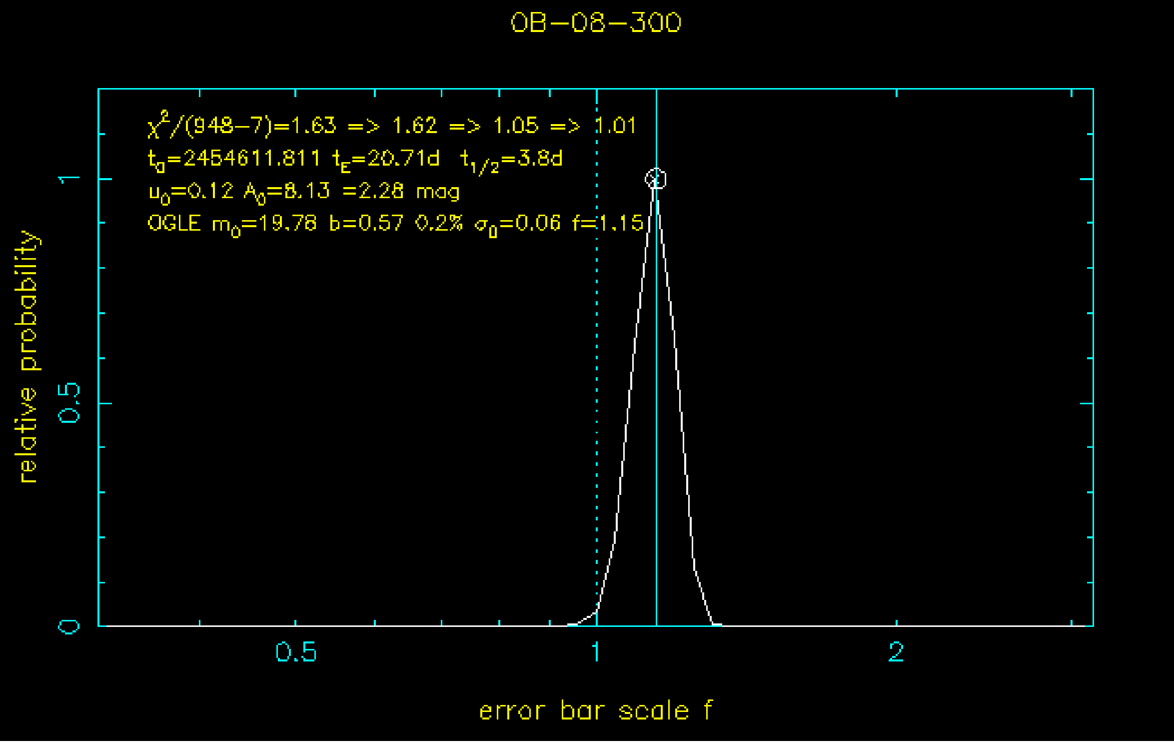
PSPL with blending.

$\sigma_0 = 0.06$ mag

$f = 1.15$

$\chi^2 / (948-4) = 1.63$
$\chi^2 / (948-5) = 1.62$
$\chi^2 / (948-6) = 1.06$
$\chi^2 / (948-7) = 1.01$





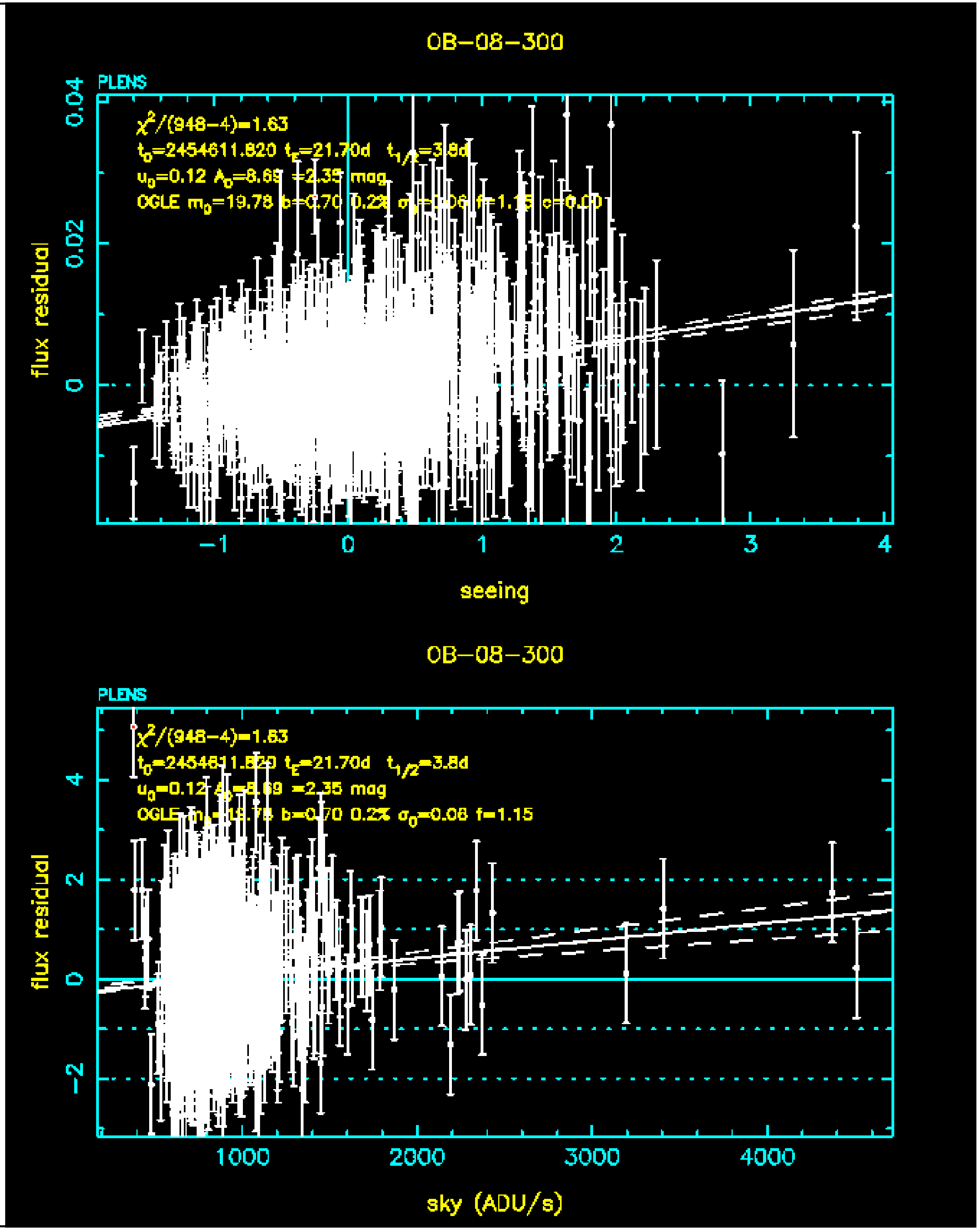
OB-08-300

Fit 7 params:

Residuals vs
Seeing and Sky

Seeing normalised
to mean and rms
for each site.

χ^2 / (948-4) = 1.63
χ^2 / (948-5) = 1.62
χ^2 / (948-6) = 1.06
χ^2 / (948-7) = 1.01



PLENS: Parameter Error Bars:

Parameter Covariance Matrix = inverse of Hessian Matrix

```
-------- Chi^2/( 127 - 4 )=  11.7392187
-------- Chi^2/( 127 - 5 )=  11.6938019
sigmag0=  0.=>  6.55033E-05
-------- Chi^2/( 127 - 6 )=  1.65649915
erbscl=  1.=>  3.15491009
-------- Chi^2/( 127 - 7 )=  1.61892235
Hessian Determinant  0.0145665174
Unit matrix:  Max error  2.18087267E-15 OK :)
Covariances above  0.5 : 2 of 10
  Cov(  1  3 ) =   0.977    t0 vs u0
  Cov(  2  4 ) =  -0.679    tE vs fs(test390)
Covariances:
 1  1.00  0.08  0.98  0.00 -0.03
 2  0.08  1.00  0.06 -0.68 -0.36
 3  0.98  0.06  1.00 -0.02 -0.02
 4  0.00 -0.68 -0.02  1.00  0.00
 5 -0.03 -0.36 -0.02  0.00  1.00
      1     2     3     4     5
Parameters: 5
1 t0  3582.73828 +/-  0.0278527793
2 tE  11.7023048 +/-  0.0237461254
3 u0  0.330508411 +/-  0.003183953
4 fs(test390)  1.96691978 +/-  0.00258736056
5 f0(test390)  2.15972519 +/-  0.000888774695

Peak magnification A0  3.14819837 +/-  0.0279930346
Baseline magnitude m0  15.564003 +/-  0.00044680445
```

$$\chi^2(\alpha) = \chi^2(\hat{\alpha}) + \sum_{j,k} \left(\alpha_j - \hat{\alpha}_j\right) H_{jk} \left(\alpha_k - \hat{\alpha}_k\right) + \ldots$$

$$H_{ij} \equiv \left(\frac{1}{2} \frac{\partial^2 \chi^2}{\partial \alpha_k \partial \alpha_j} \right) \Bigg|_{\alpha=\hat{\alpha}}$$

$$\mathrm{Cov}(a_k, a_j) = \left[\frac{1}{2} \frac{\partial^2 \chi^2}{\partial \alpha_k \partial \alpha_j} \right]^{-1} = \left[H^{-1} \right]_{kj}$$

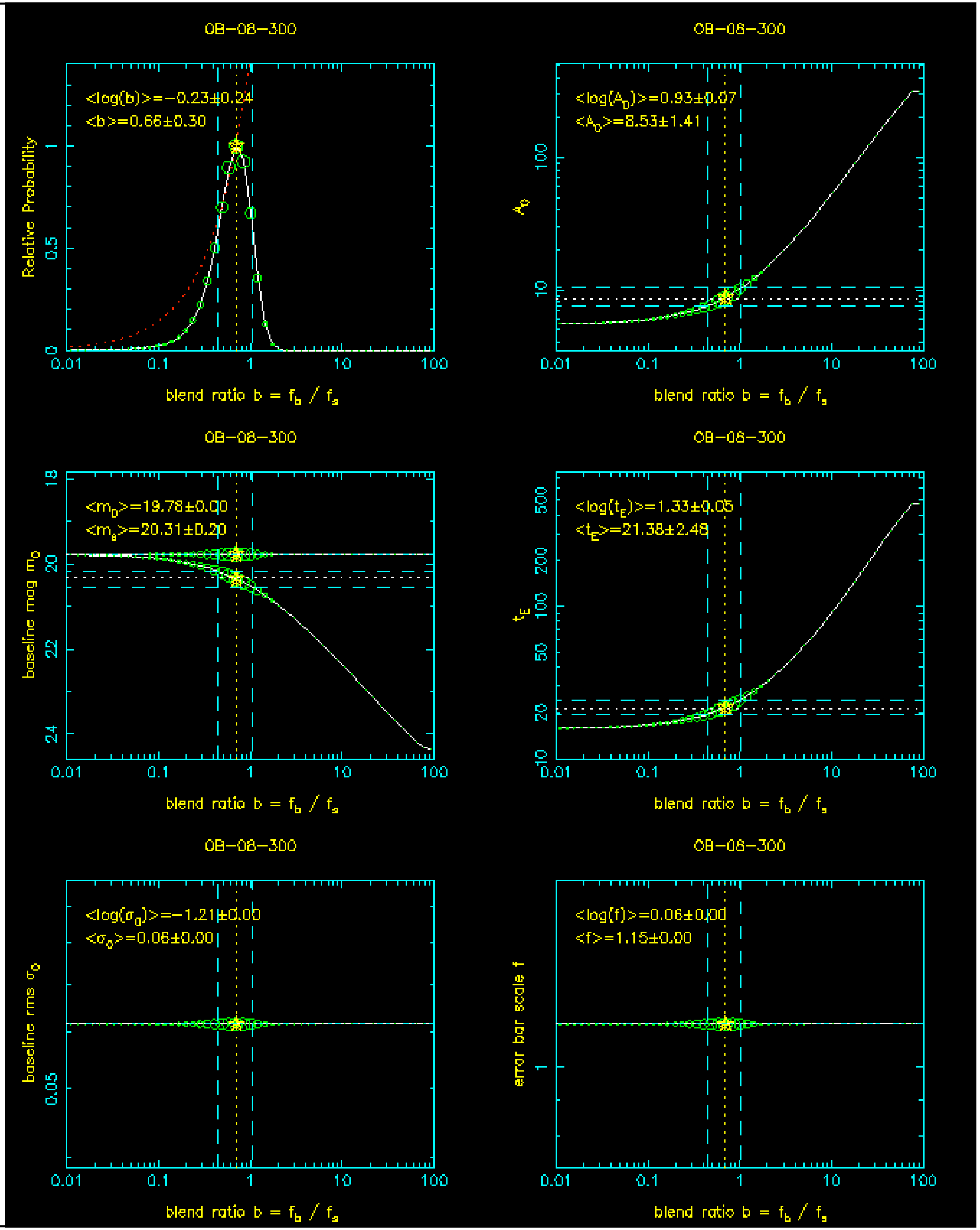
OB-08-300

Fit 7 params:

BB plot = Bayesian Blend analysis

log *b* grid forces *b > 0.*

χ^2 / (948-4) = 1.63
χ^2 / (948-5) = 1.62
χ^2 / (948-6) = 1.06
χ^2 / (948-7) = 1.01



OB-08-300

Fit 7 params:

BB plot = Bayesian Blend analysis

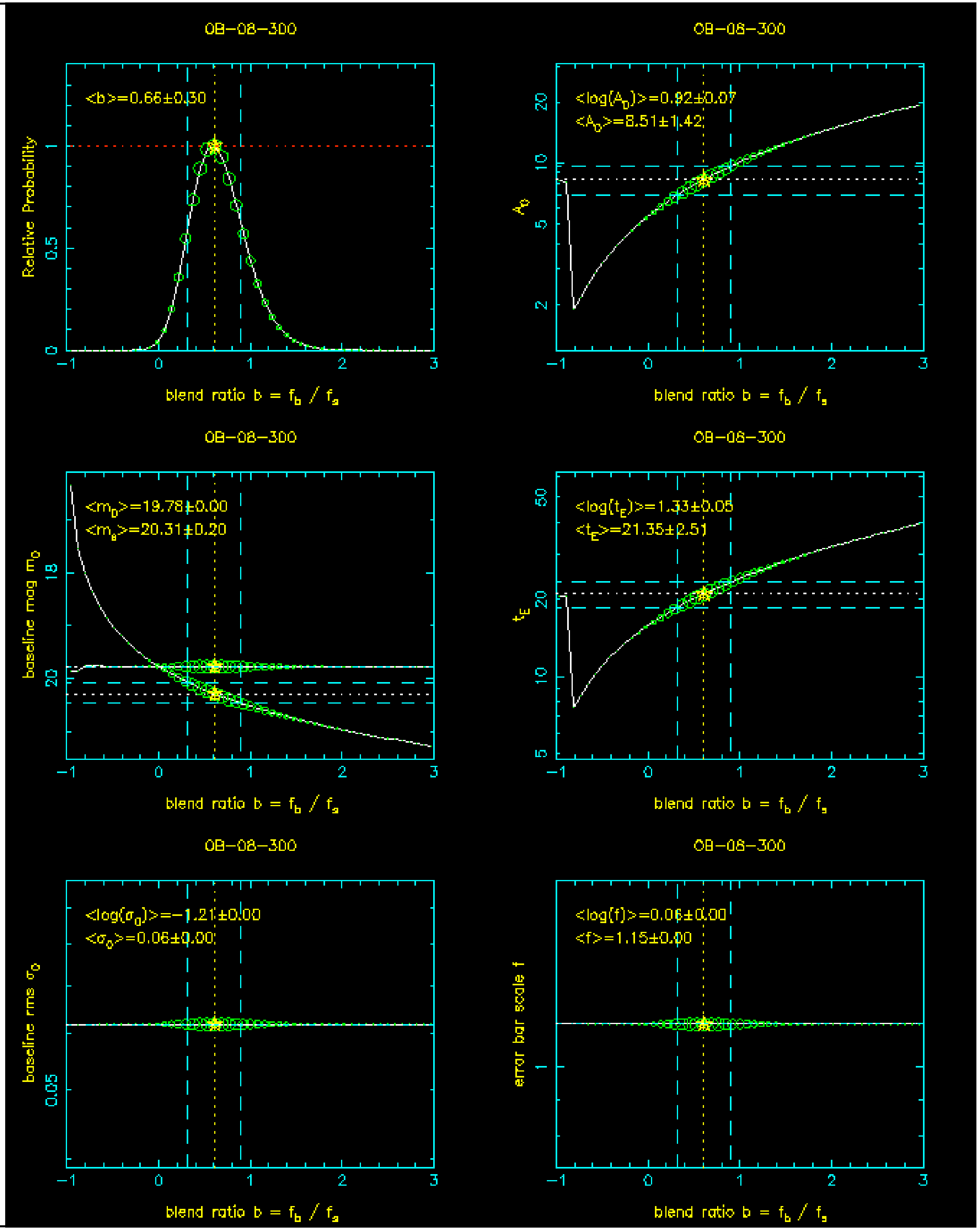
b grid allows *b* < *0*.

$\chi^2 / (948-4) = 1.63$

$\chi^2 / (948-5) = 1.62$

$\chi^2 / (948-6) = 1.06$

$\chi^2 / (948-7) = 1.01$



OB-08-300

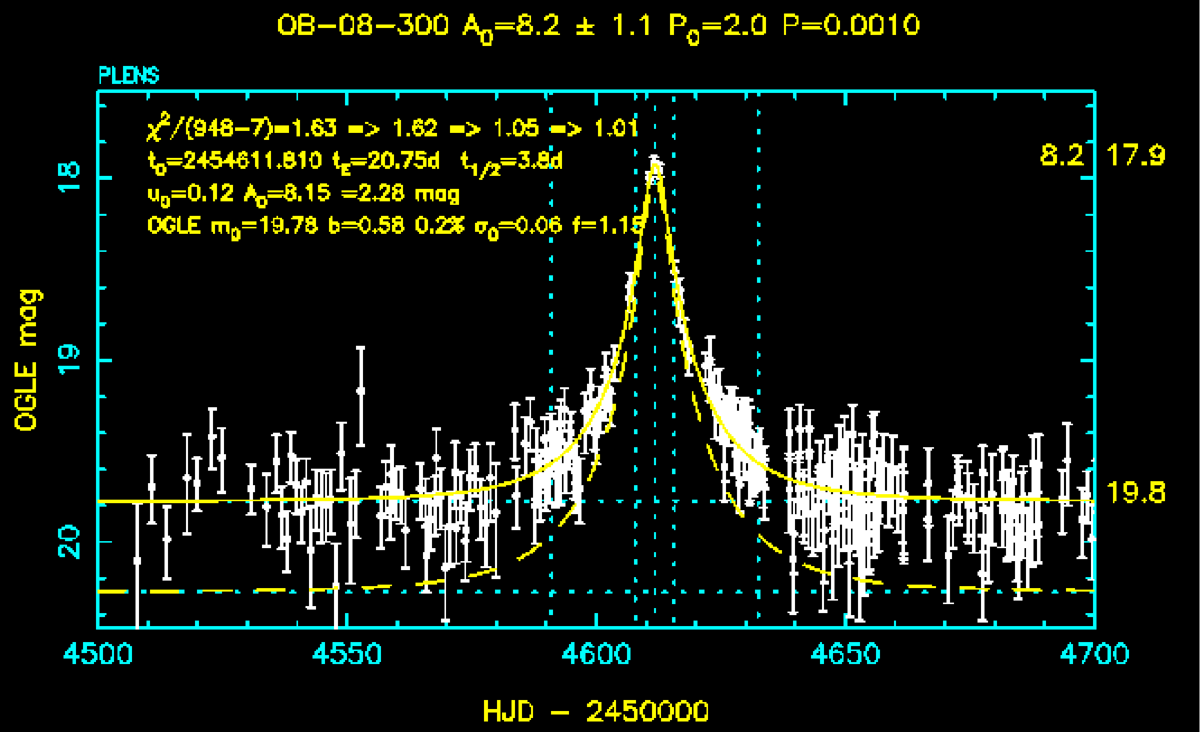
Fit 7 params:

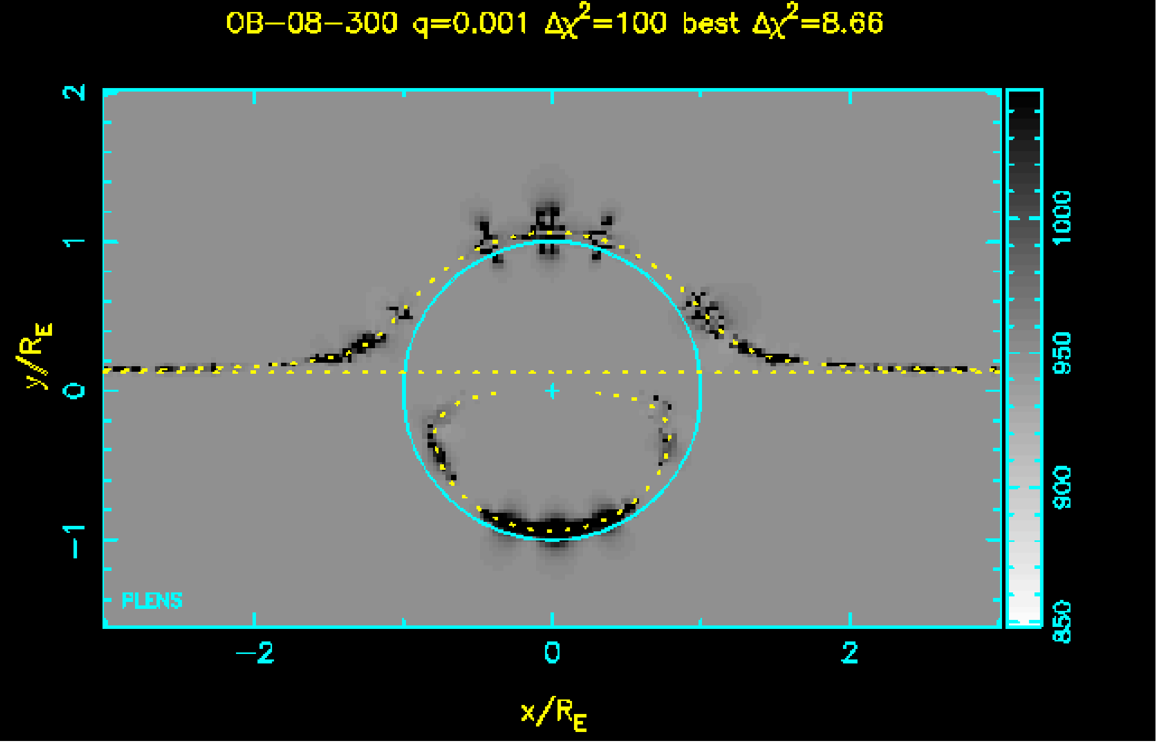
MC plot =
Mags + $\chi^2(\,x,\,y\,)$

Planet position
$(\,x\,,\,y\,)$ in R_E units

$q = 0.001$
$\Delta\chi^2 = 8.7$

$\chi^2\,/\,(948\text{-}4) = 1.63$
$\chi^2\,/\,(948\text{-}5) = 1.62$
$\chi^2\,/\,(948\text{-}6) = 1.06$
$\chi^2\,/\,(948\text{-}7) = 1.01$





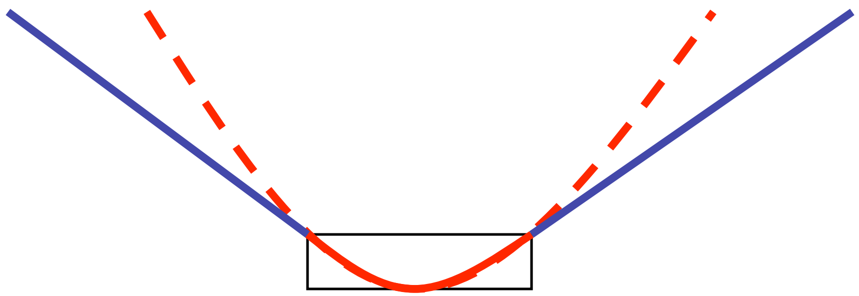
PLENS: "Clipping" Outliers

maximum likelihood fit minimises

$$\chi^2 = \sum \left(\frac{\text{data} - \text{model}}{\text{error bar}} \right)^2$$

median fit minimises

$$\text{SAD} = \sum \left| \text{data} - \text{model} \right|$$



PLENS: "Badness-of-Fit" is linear (rather than quadratic) for residuals larger than k σ

PLENS parameters k+ and k- set the k σ clip thresholds. (e.g. +5 σ to -5 σ)

OB-05-390

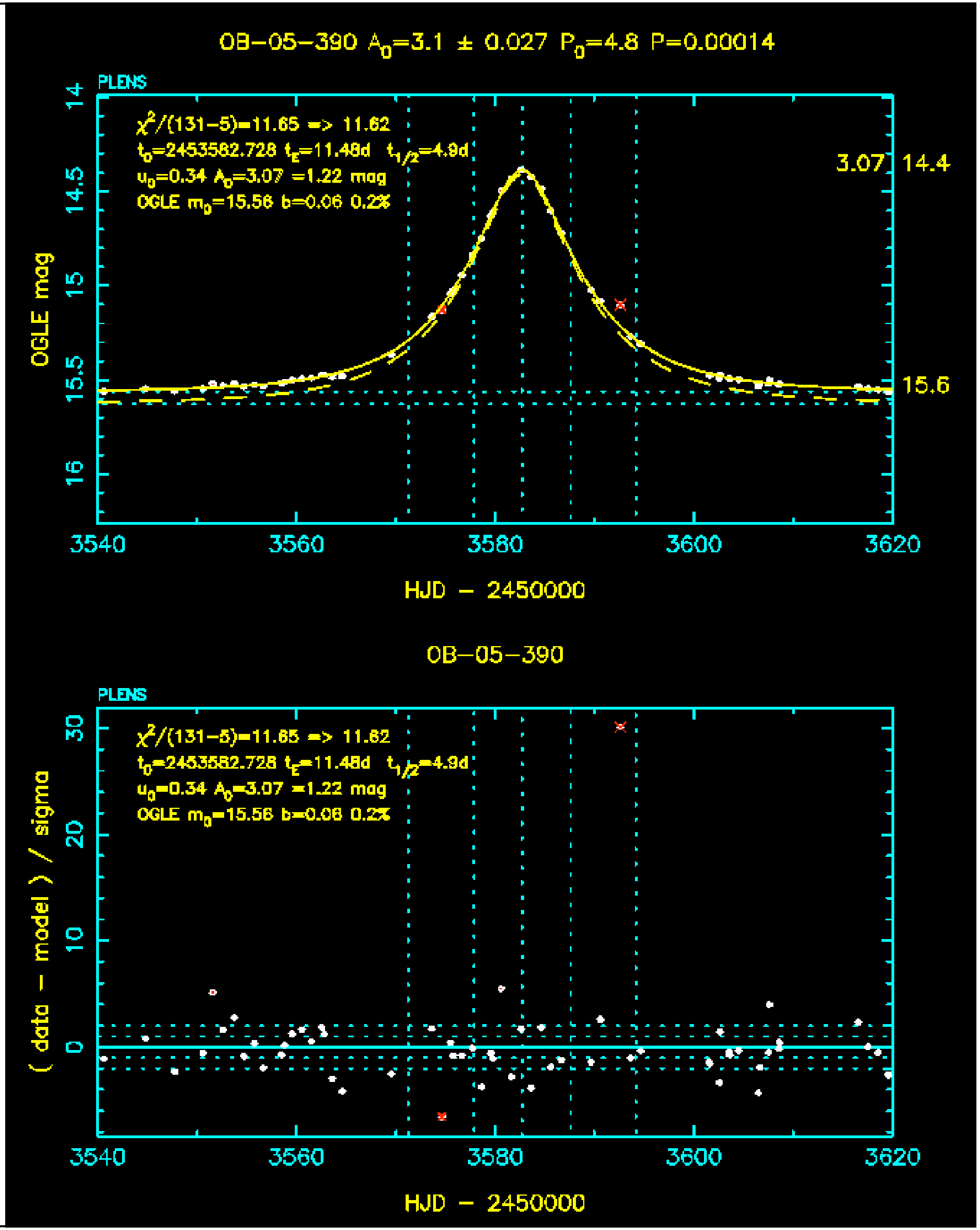
Fit 5 params:

MN = Magnitudes
+ Normalised
. residuals

30 σ outlier

Dinimished data marked by red X

$\chi^2 / (127-4) = 11.65$
$\chi^2 / (127-5) = 11.62$



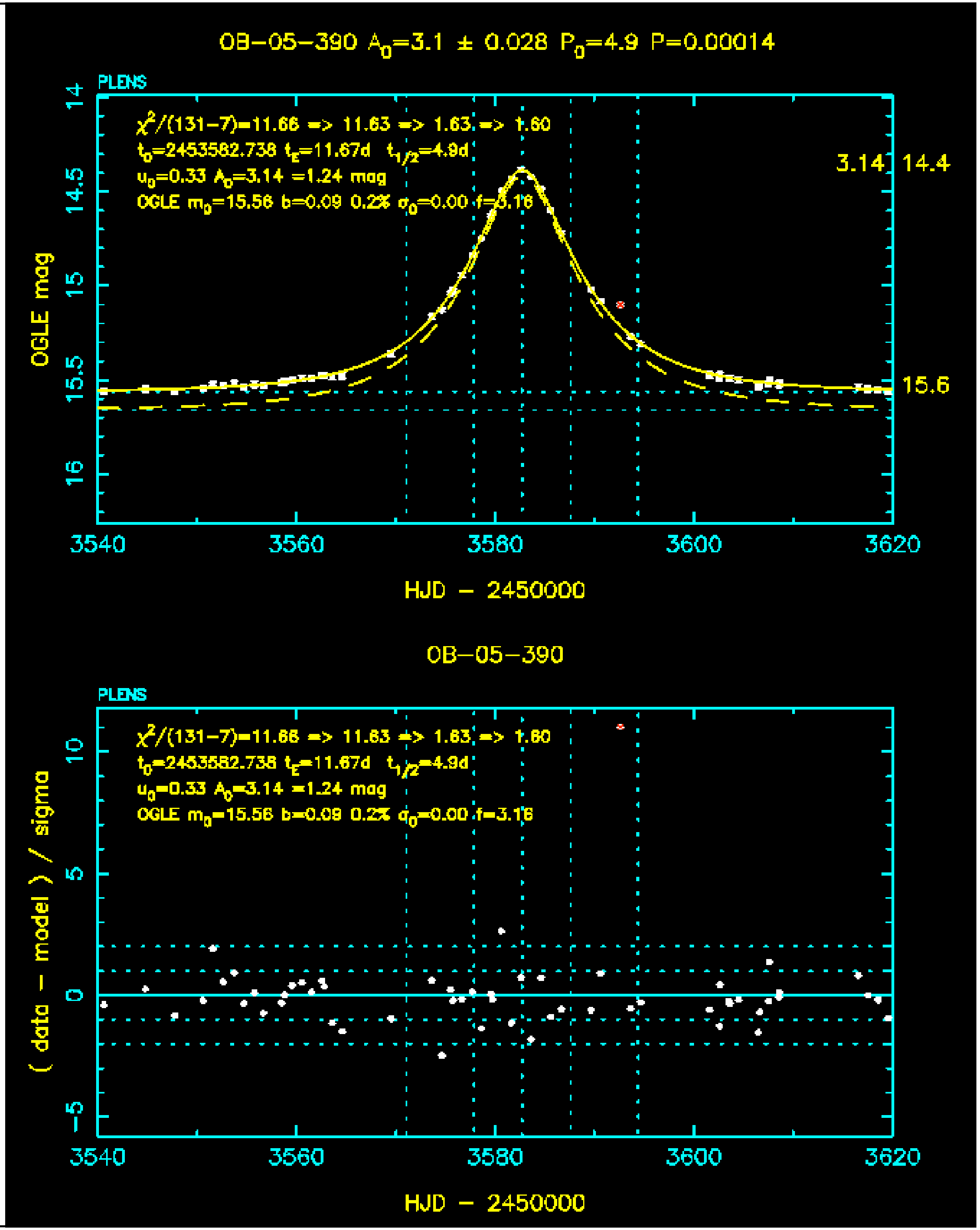
OB-05-390

Fit 7 params:

MN = Magnitudes
+ Normalised
. residuals

f = *1.0 => 3.2*
30 σ => 11σ
outlier

χ^2 / (127-4) = 11.66
χ^2 / (127-5) = 11.63
χ^2 / (127-6) = 1.63
χ^2 / (127-7) = 1.60



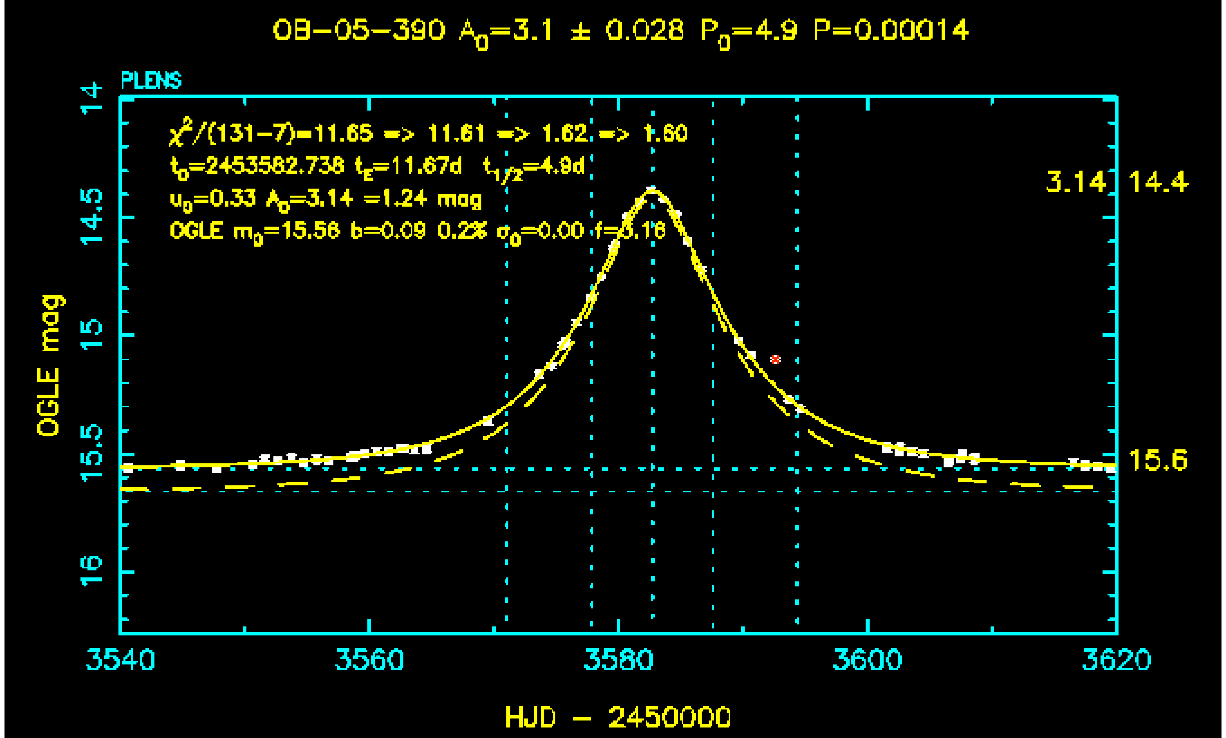
OB-05-390

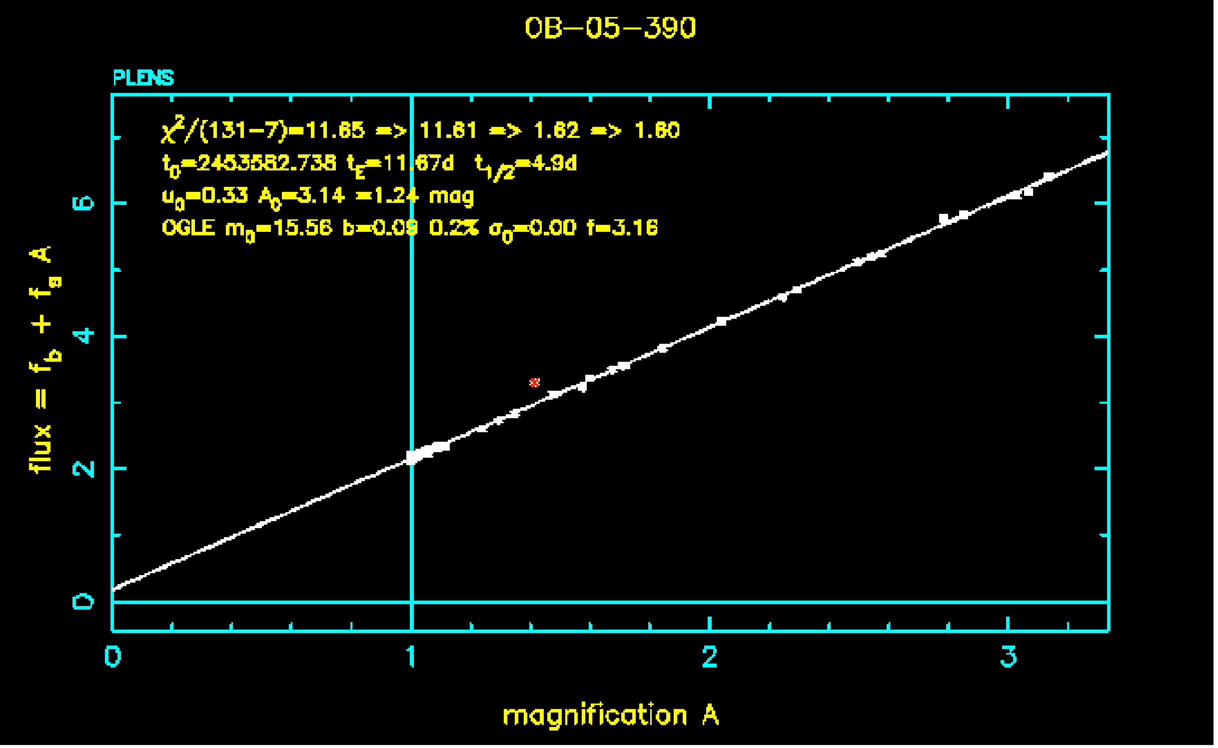
Fit 7 params:

MB = Magnitudes
+ Blend fit

11σ outlier

χ^2 / (127-4) = 11.66
χ^2 / (127-5) = 11.63
χ^2 / (127-6) = 1.63
χ^2 / (127-7) = 1.60





OB-05-390

Fit 7 params:

BB = Bayesian Blend Analysis

log b grid:

$< b > = 0.11$

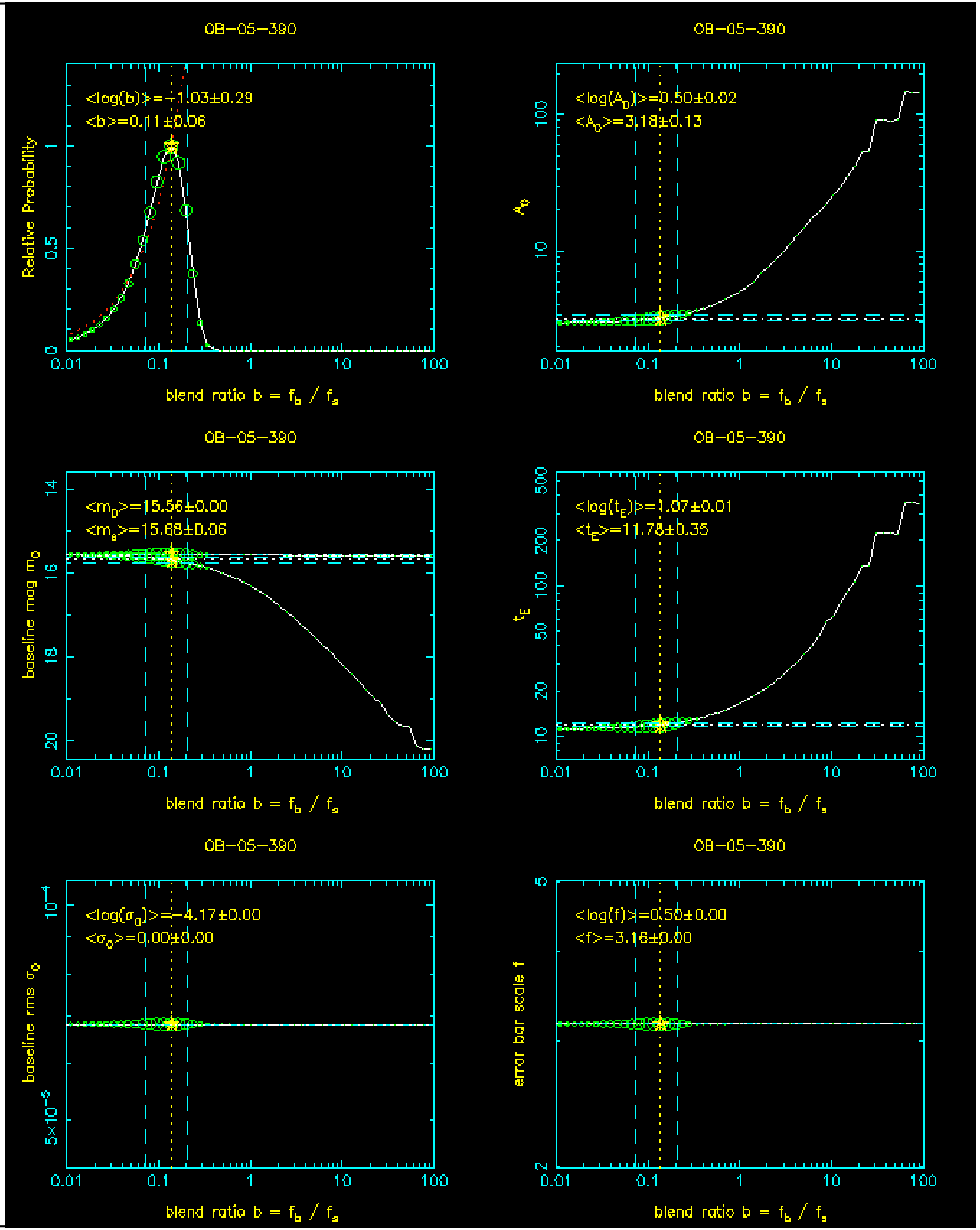
$\sigma(b) = 0.05$

$\chi^2 / (127-4) = 11.66$

$\chi^2 / (127-5) = 11.63$

$\chi^2 / (127-6) = 1.63$

$\chi^2 / (127-7) = 1.60$



OB-05-390

Fit 7 params:

BB = Bayesian Blend Analysis

b grid:

$< b > = 0.10$

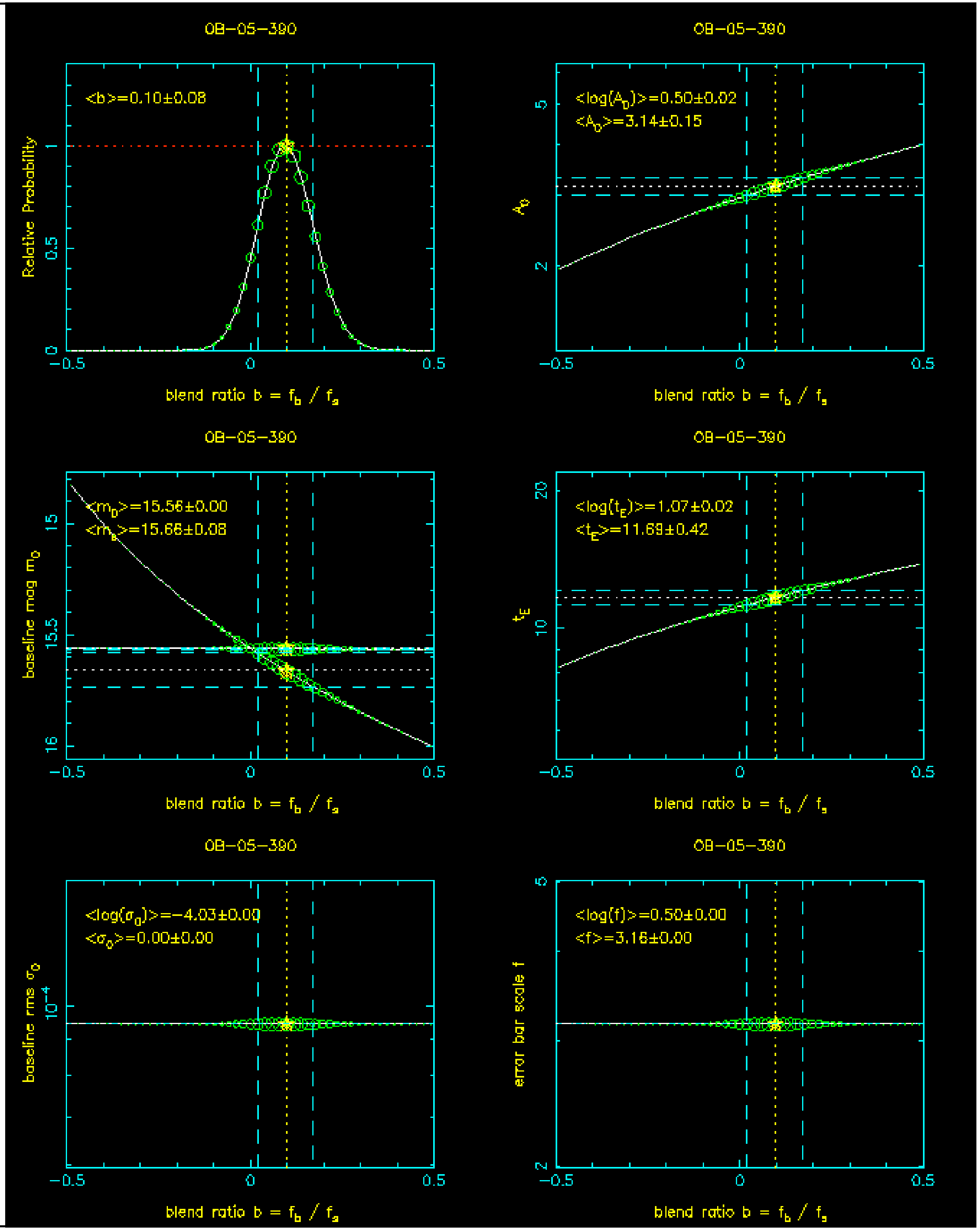
$\sigma(b) = 0.08$

$\chi^2 / (127\text{-}4) = 11.66$

$\chi^2 / (127\text{-}5) = 11.63$

$\chi^2 / (127\text{-}6) = 1.63$

$\chi^2 / (127\text{-}7) = 1.60$



OB-05-390

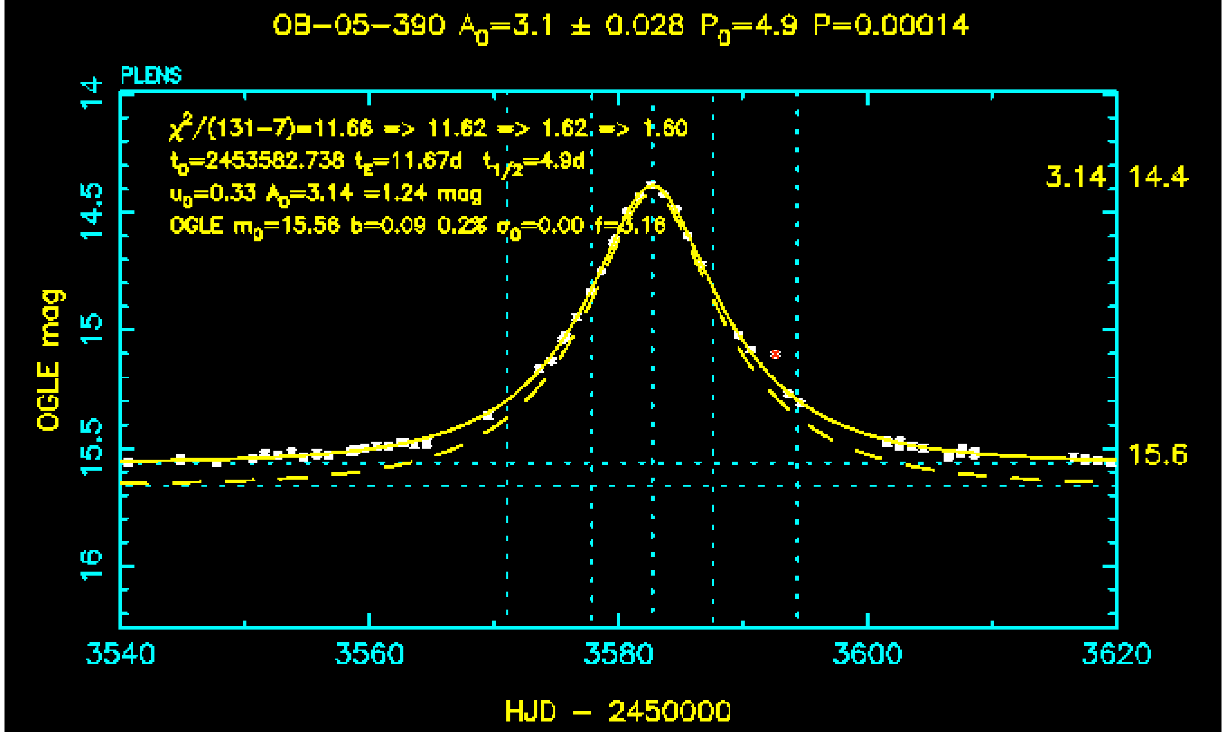
Fit 7 params:

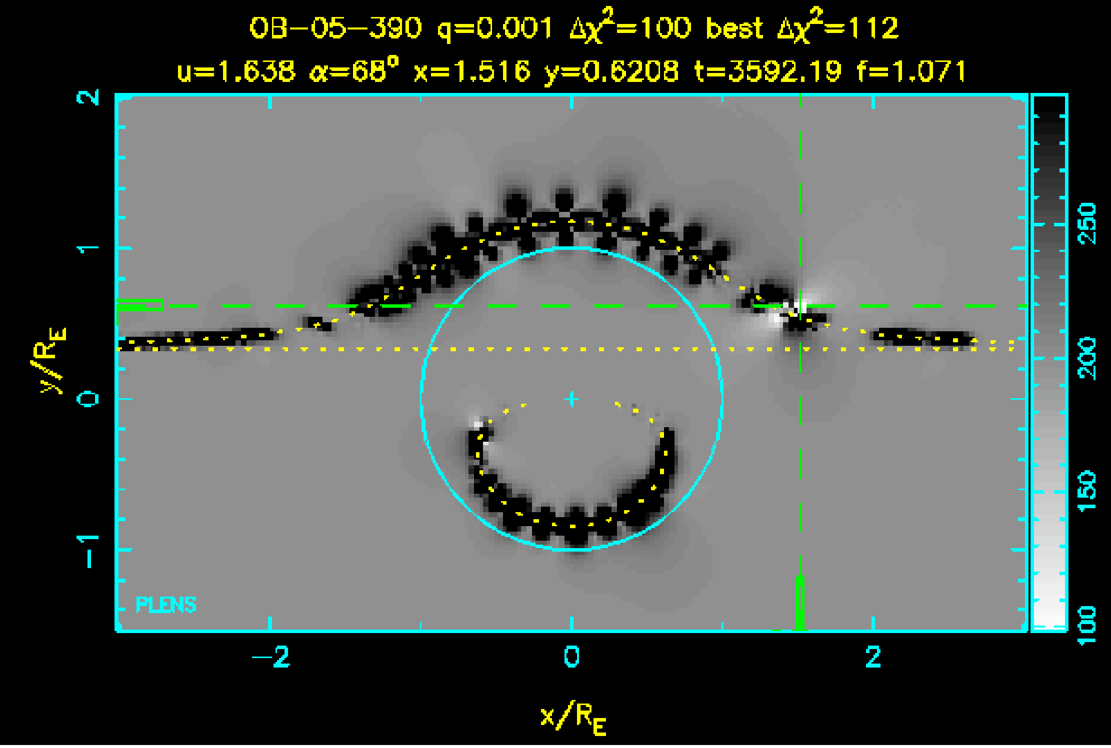
MC plot = Mags + $\chi^2(x, y)$

$q = 0.001$
$\Delta\chi^2 = 112$

Planet params given on plot if $\Delta\chi^2 > 100$

$\chi^2 / (127-4) = 11.66$
$\chi^2 / (127-5) = 11.62$
$\chi^2 / (127-6) = 1.62$
$\chi^2 / (127-7) = 1.60$





Alternative Planet Position Parameterisations

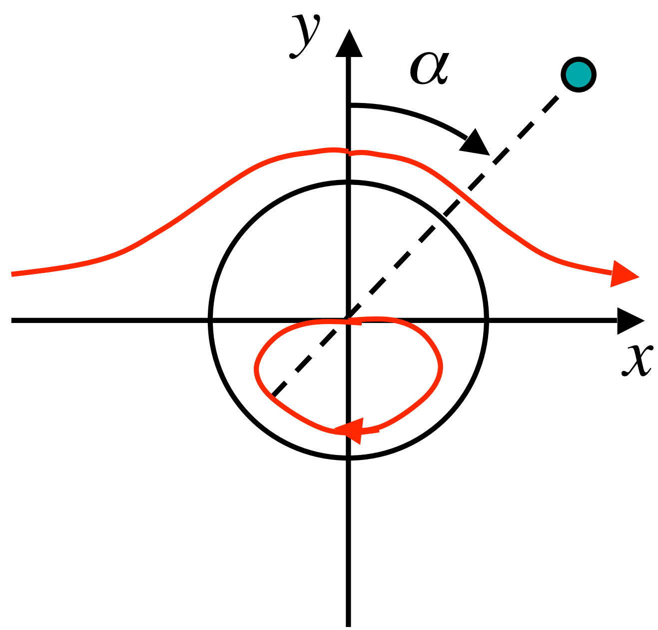
(x, y) in R_E units

$(\ln u, \alpha)$ $\quad u^2 = x^2 + y^2$

$$\alpha = \tan^{-1}(x / y)$$

(t, f) $\quad t =$ time corresponding to α

$$f \equiv \frac{u}{u_\pm} \text{ in PSPL image units}$$



Planet abundance approx uniform in (lnu, α)

(t , f) better constrained by data (more orthogonal to PSPL params).

f = +1 at major image, -1 at minor image

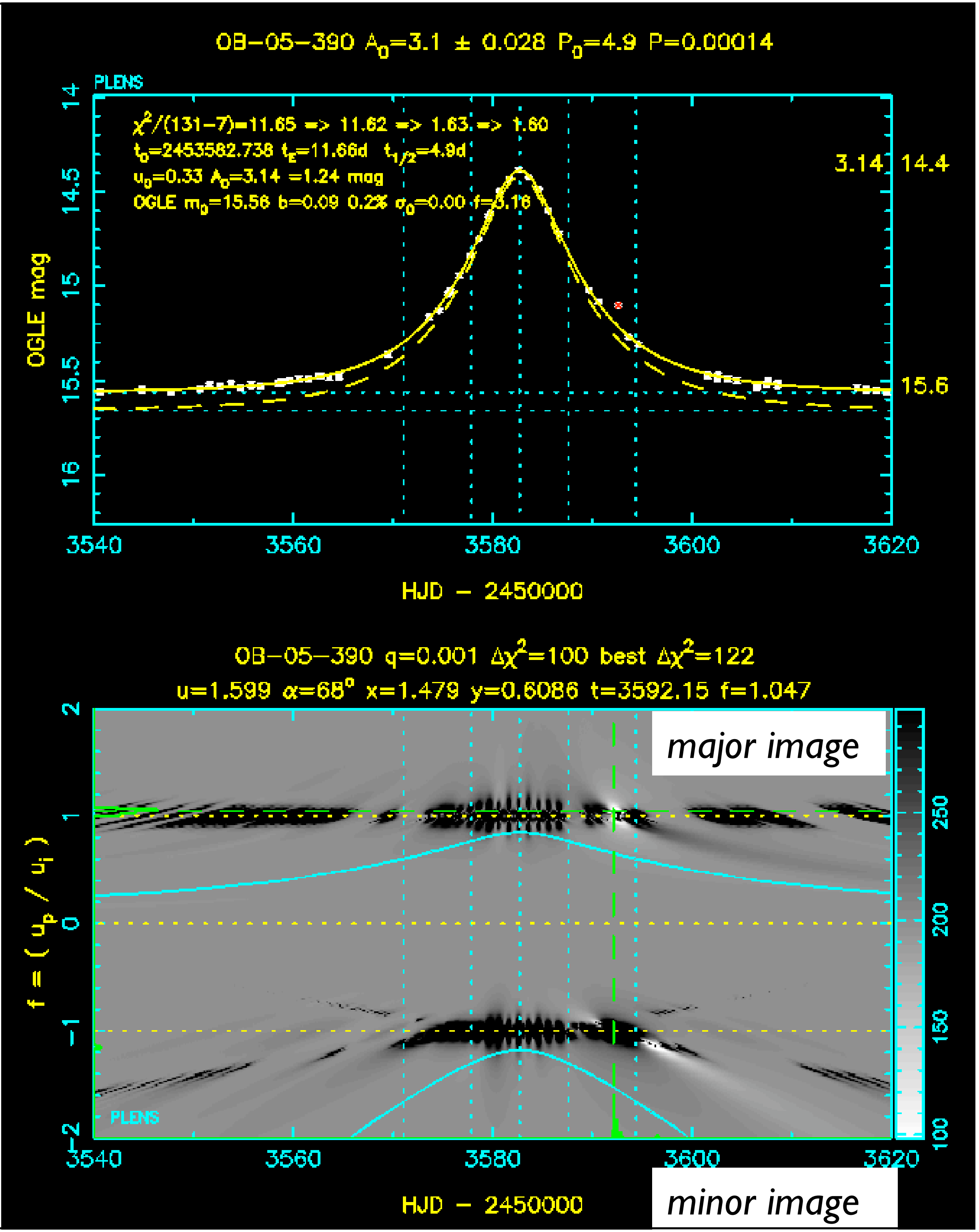
OB-05-390

Fit 7 params:

MV plot =
Mags + $\chi^2(t, f)$

$q = 0.001$
$\Delta\chi^2 = 122$

$\chi^2 / (127-4) = 11.66$
$\chi^2 / (127-5) = 11.62$
$\chi^2 / (127-6) = 1.62$
$\chi^2 / (127-7) = 1.60$



OB-05-390

Fit 7 params:

$\chi^2(\, x, y \,)$

$\chi^2(\, \ln u, \alpha \,)$

$q = 0.001$

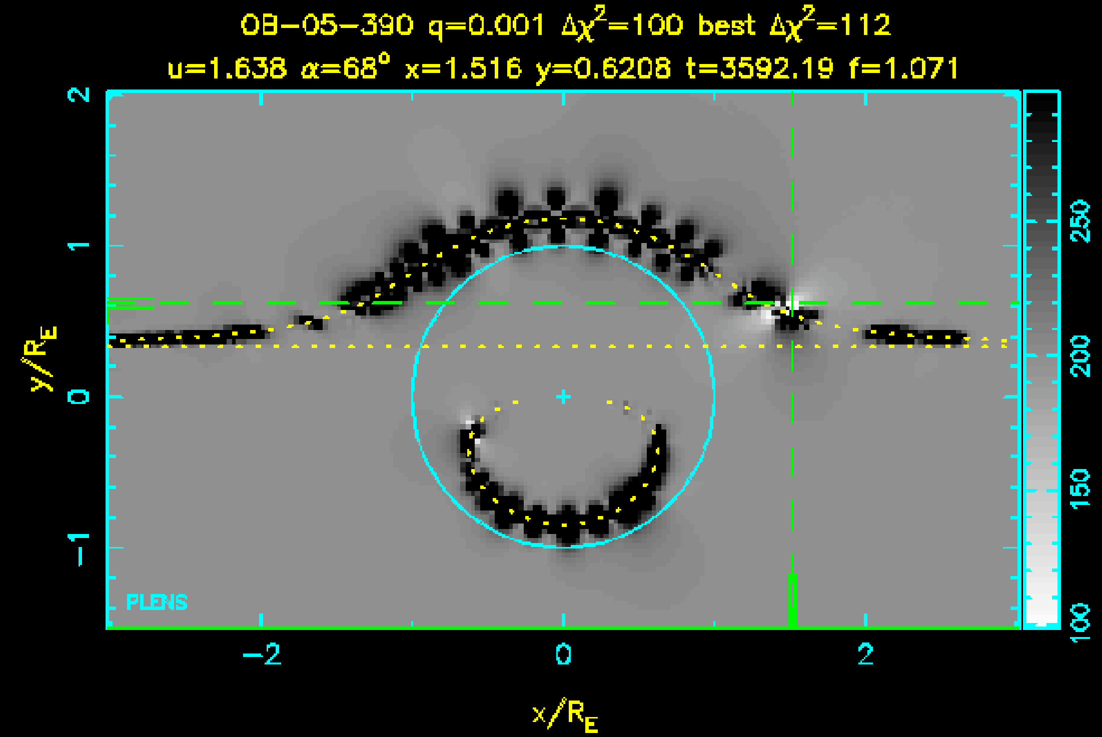
$\Delta\chi^2 = 118$

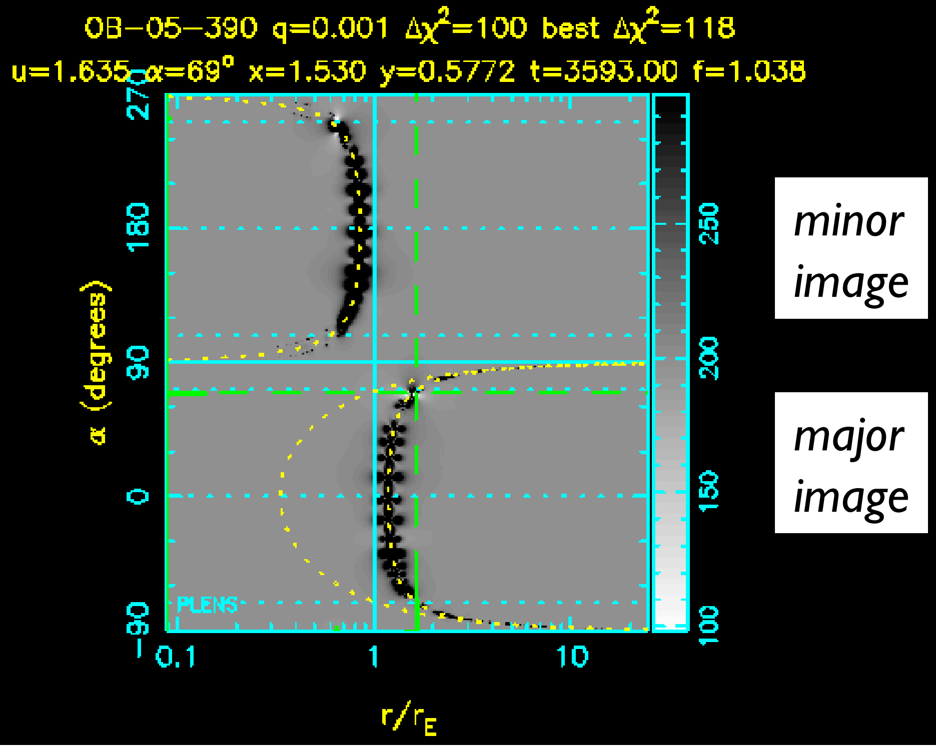
$\chi^2 / (127-4) = 11.74$

$\chi^2 / (127-5) = 11.68$

$\chi^2 / (127-6) = 1.65$

$\chi^2 / (127-7) = 1.62$



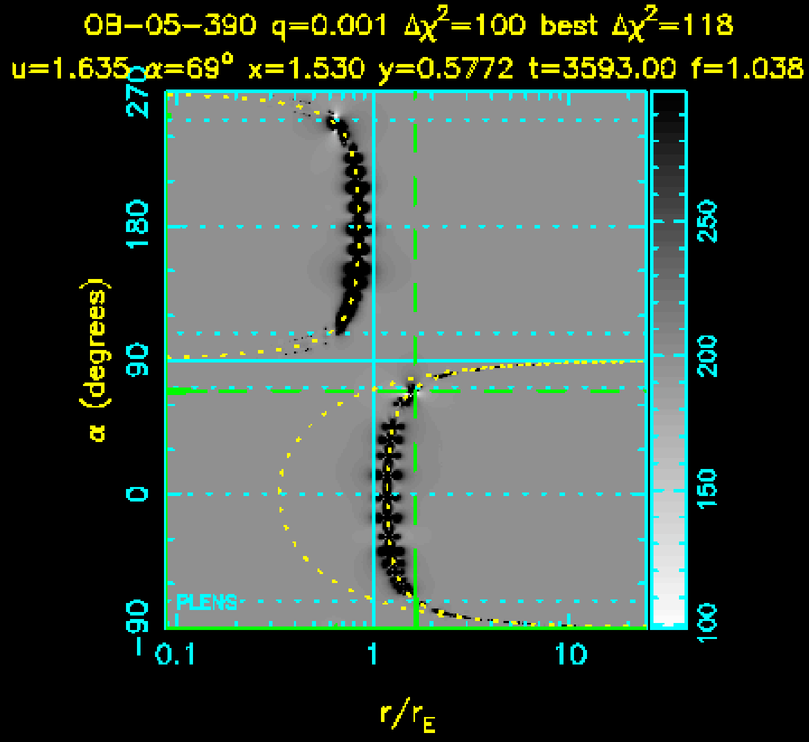


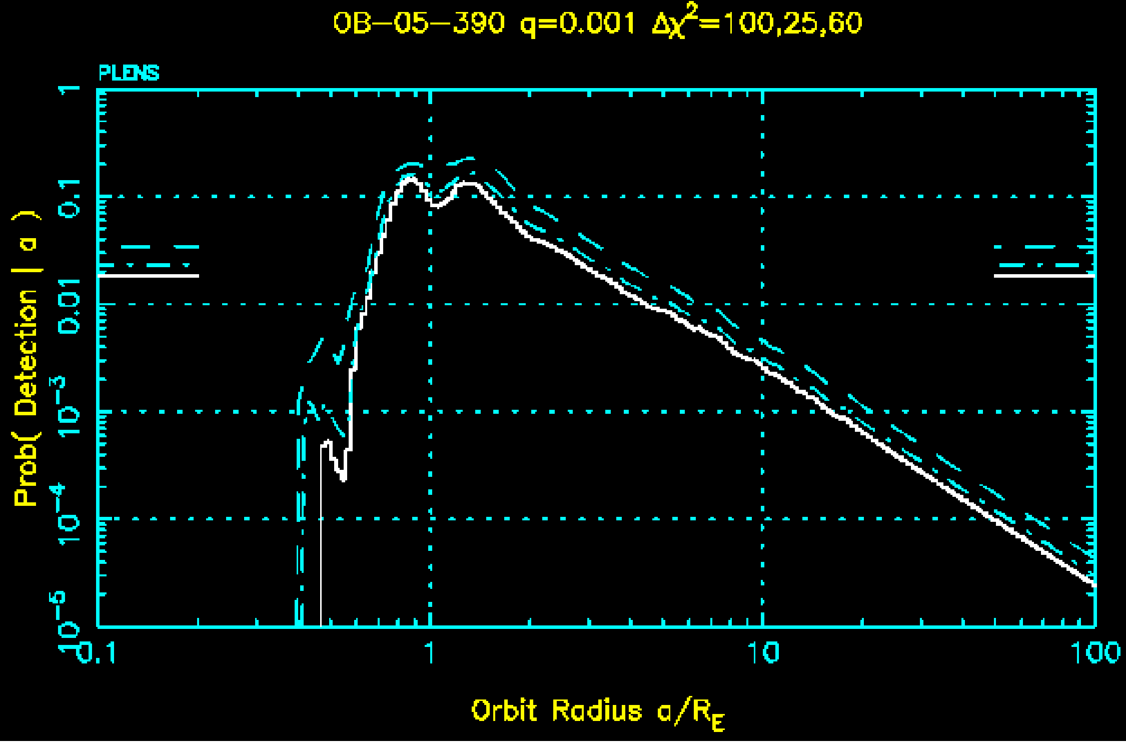
OB-05-390

Fit 7 params:
$\chi^2(lnu, \alpha)$
$P(\text{det} \mid a)$

$q = 0.001$
$\Delta\chi^2 = 118$

$\chi^2 / (127\text{-}4) = 11.74$
$\chi^2 / (127\text{-}5) = 11.68$
$\chi^2 / (127\text{-}6) = 1.65$
$\chi^2 / (127\text{-}7) = 1.62$



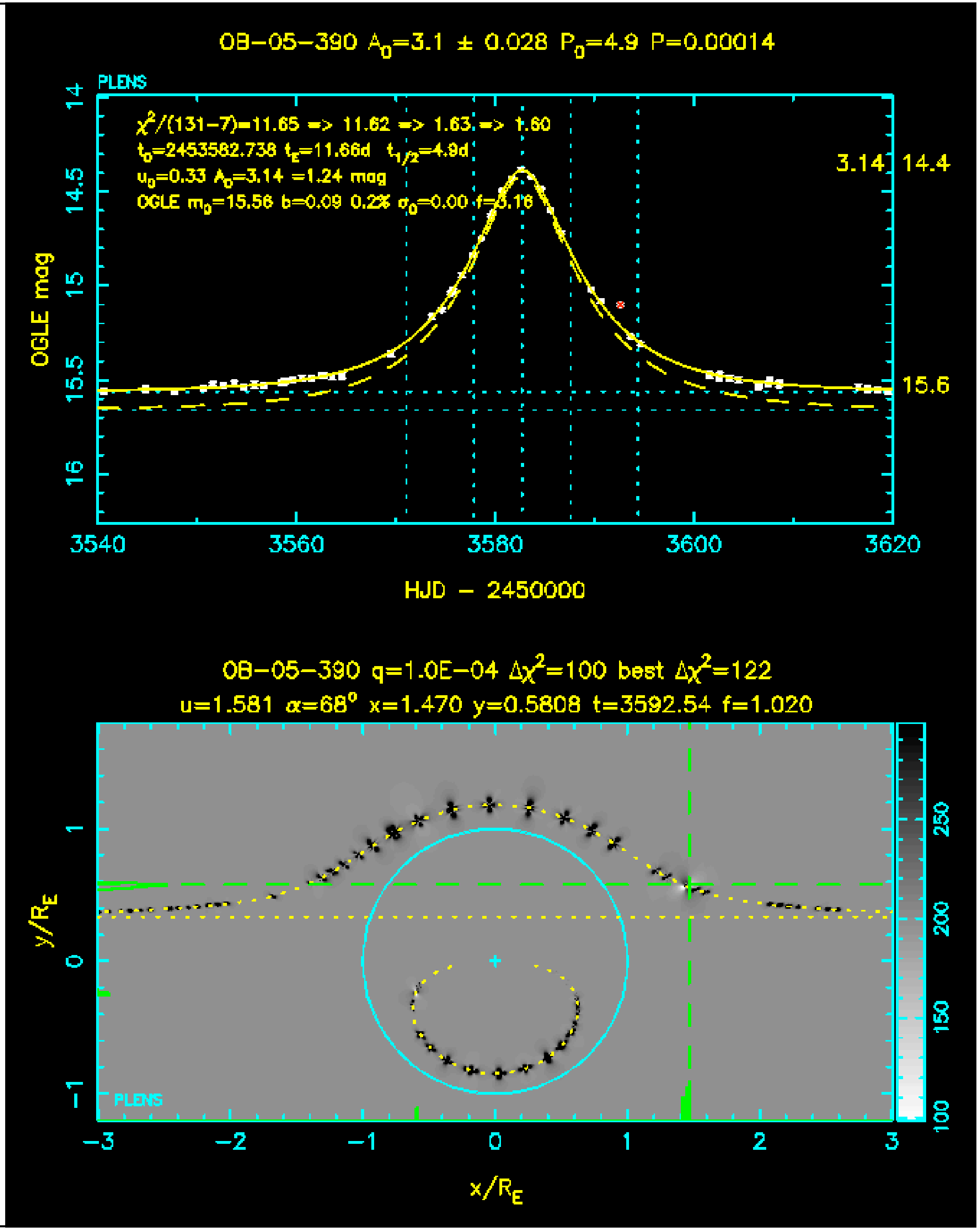


OB-05-390

Fit 7 params:

MC plot = Mags + $\chi^2(x, y)$

$q = 0.0001$
$\Delta\chi^2 = 122$



$\chi^2 / (127\text{-}4) = 11.65$
$\chi^2 / (127\text{-}5) = 11.62$
$\chi^2 / (127\text{-}6) = 1.63$
$\chi^2 / (127\text{-}7) = 1.60$

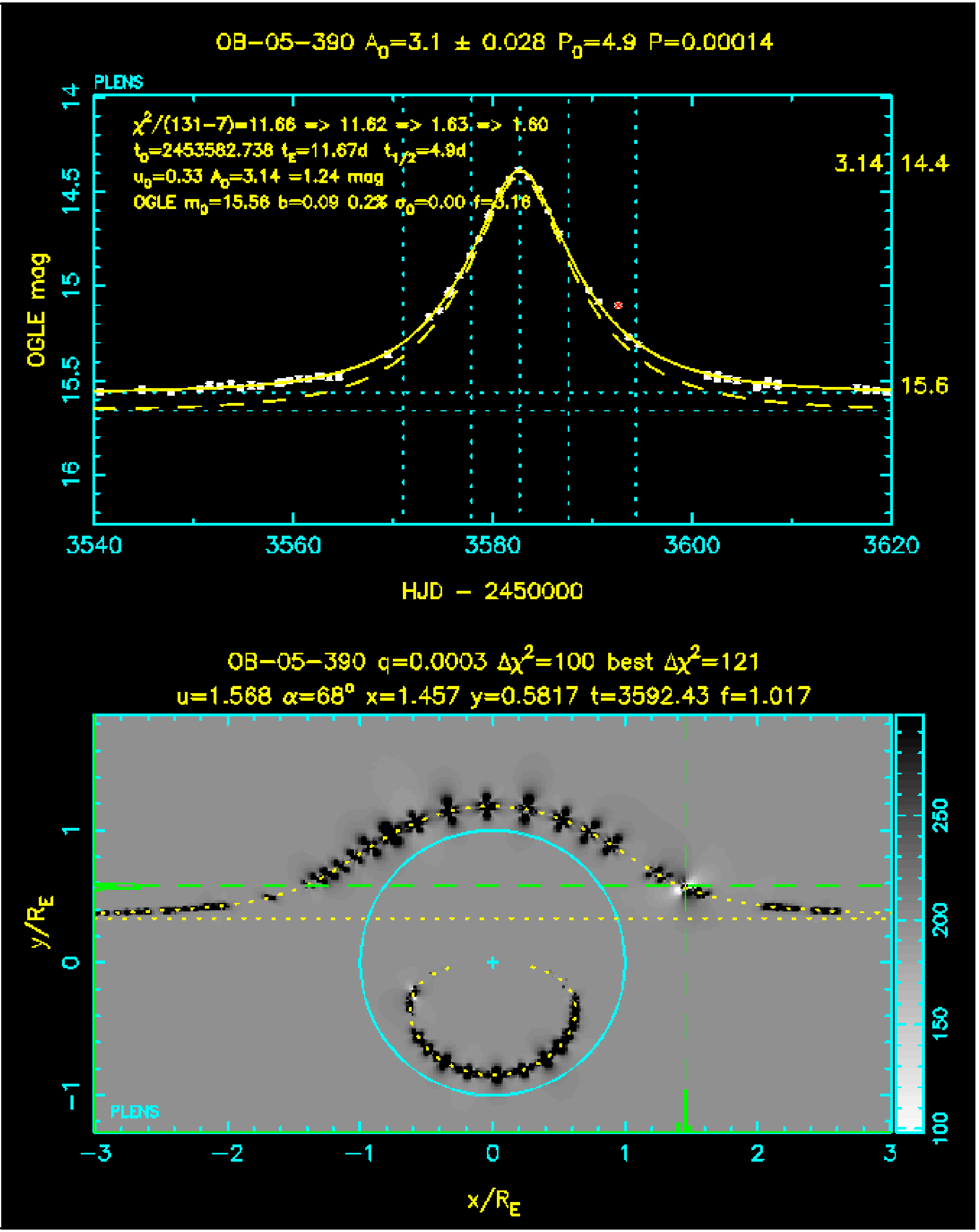
OB-05-390

Fit 7 params:

MC plot =
Mags + $\chi^2(x, y)$

$q = 0.0003$
$\Delta\chi^2 = 121$

$\chi^2 / (127-4) = 11.66$
$\chi^2 / (127-5) = 11.62$
$\chi^2 / (127-6) = 1.63$
$\chi^2 / (127-7) = 1.60$



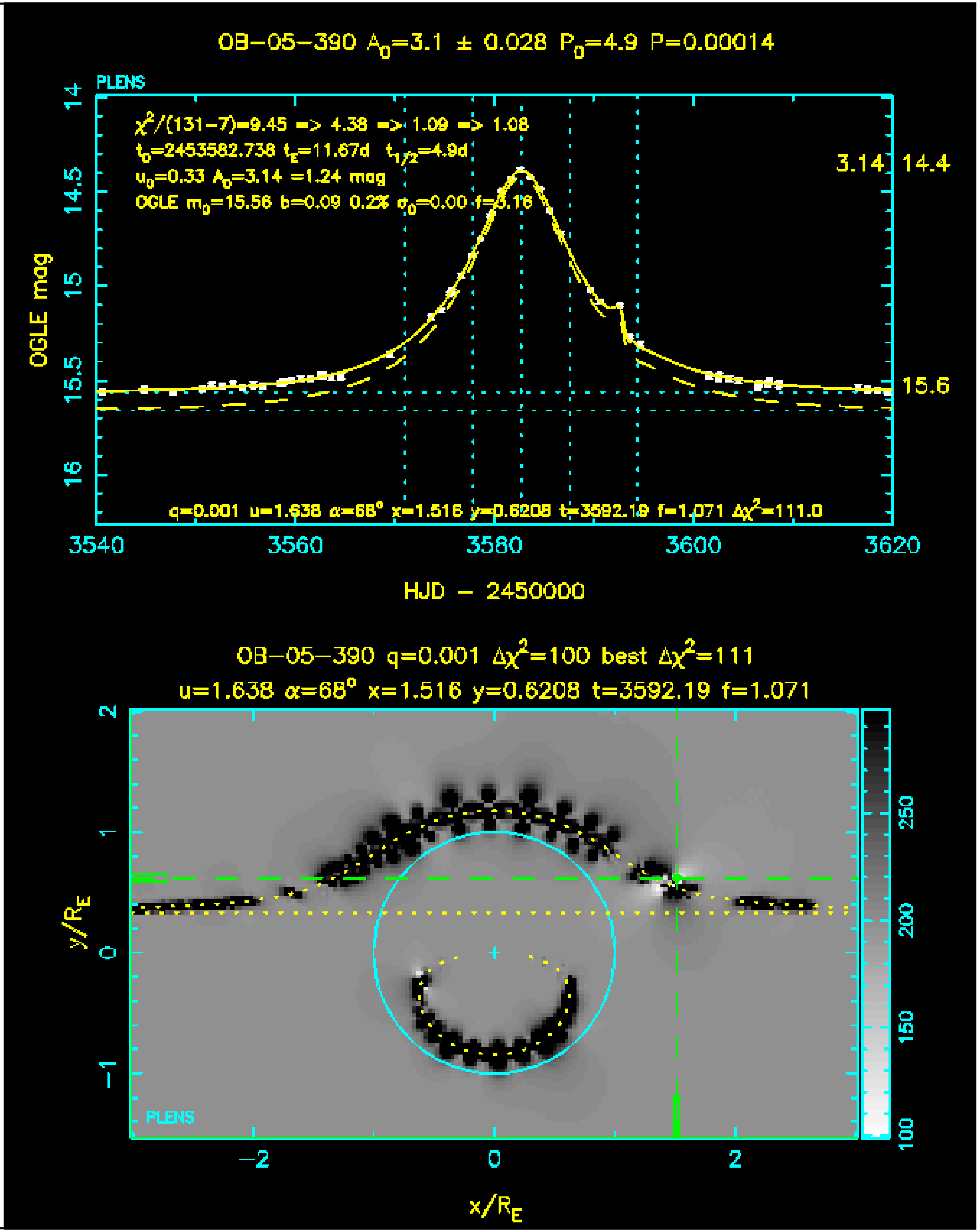
OB-05-390

Fit 7 params:

MC plot =
Mags + $\chi^2(x, y)$

$q = 0.001$
$\Delta\chi^2 = 111$

$\chi^2 / (127-4) = 9.45$
$\chi^2 / (127-5) = 4.38$
$\chi^2 / (127-6) = 1.09$
$\chi^2 / (127-7) = 1.08$



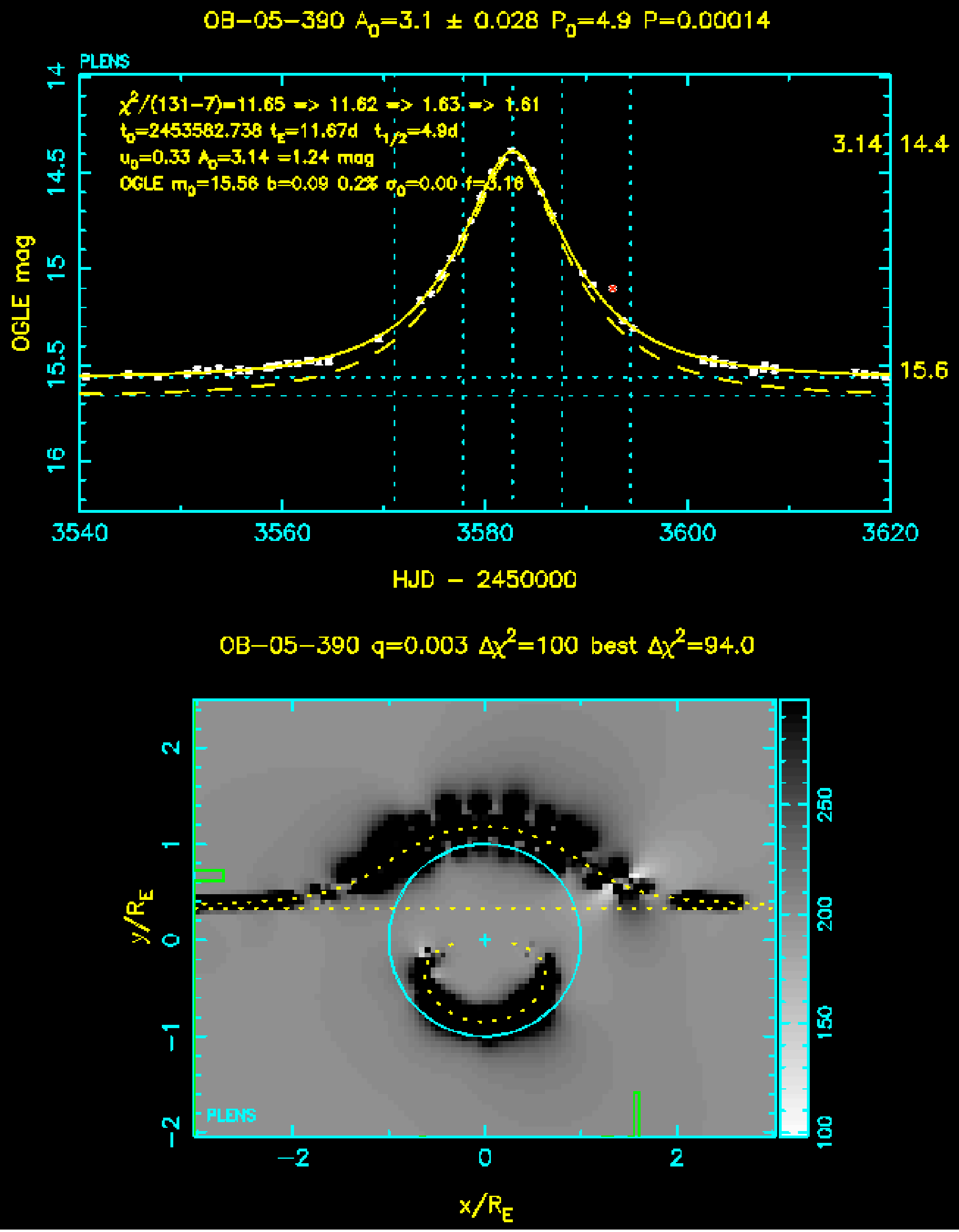
OB-05-390

Fit 7 params:

MC plot =
Mags + $\chi^2(x, y)$

$q = 0.003$
$\Delta\chi^2 = 94$

$\chi^2 / (127\text{-}4) = 11.65$
$\chi^2 / (127\text{-}5) = 11.62$
$\chi^2 / (127\text{-}6) = 1.63$
$\chi^2 / (127\text{-}7) = 1.61$



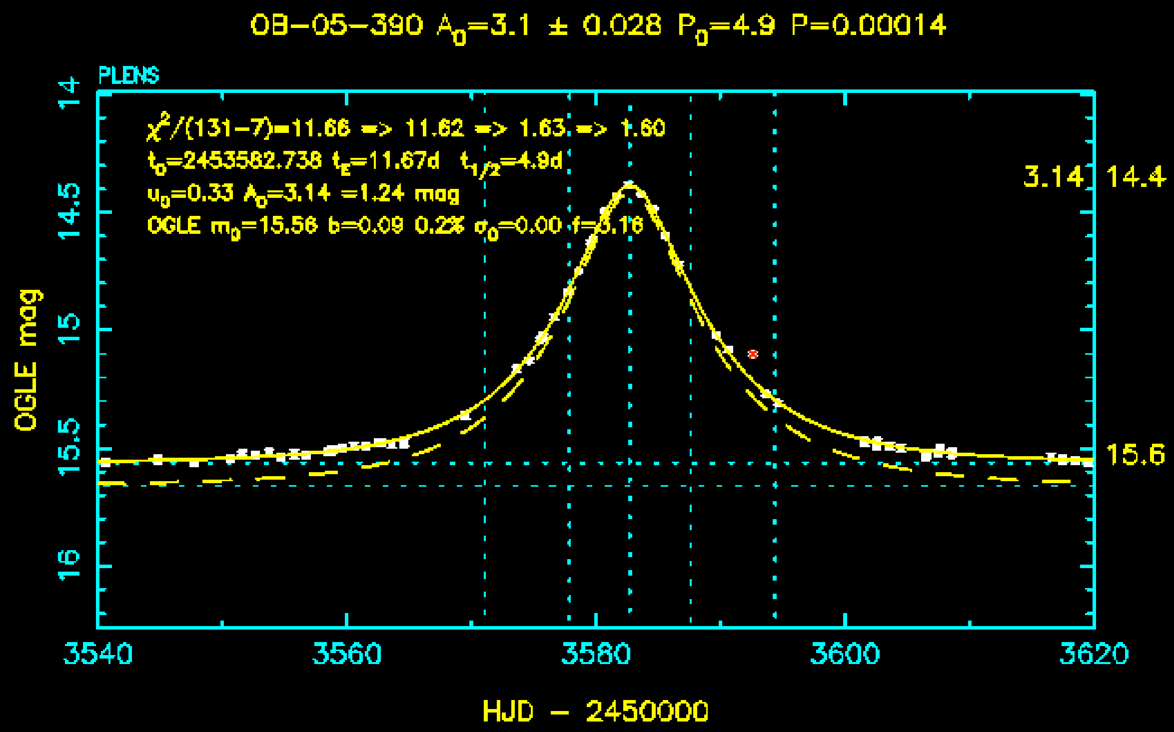
OB-05-390

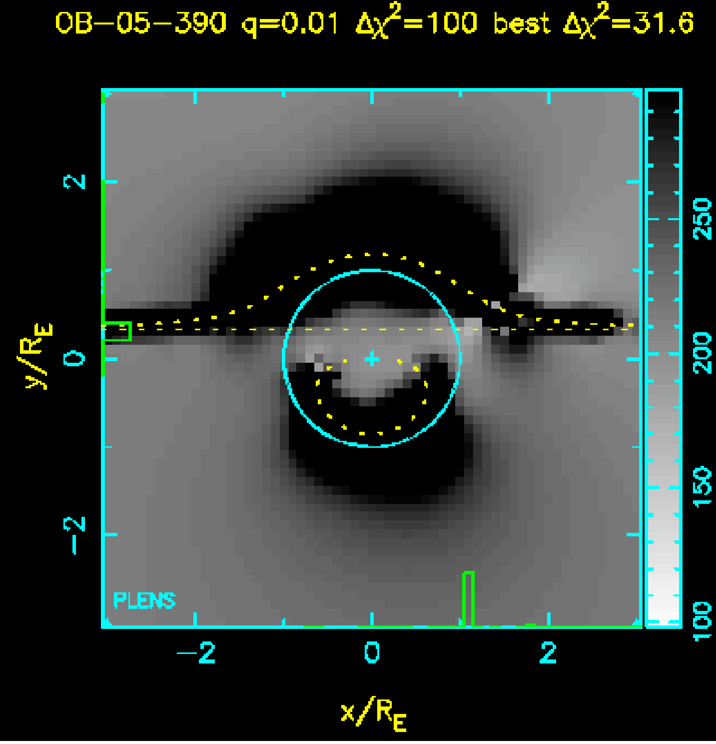
Fit 7 params:

MC plot =
Mags + $\chi^2(x, y)$

$q = 0.01$
$\Delta\chi^2 = 31.6$

$\chi^2 / (127\text{-}4) = 11.66$
$\chi^2 / (127\text{-}5) = 11.62$
$\chi^2 / (127\text{-}6) = 1.63$
$\chi^2 / (127\text{-}7) = 1.60$

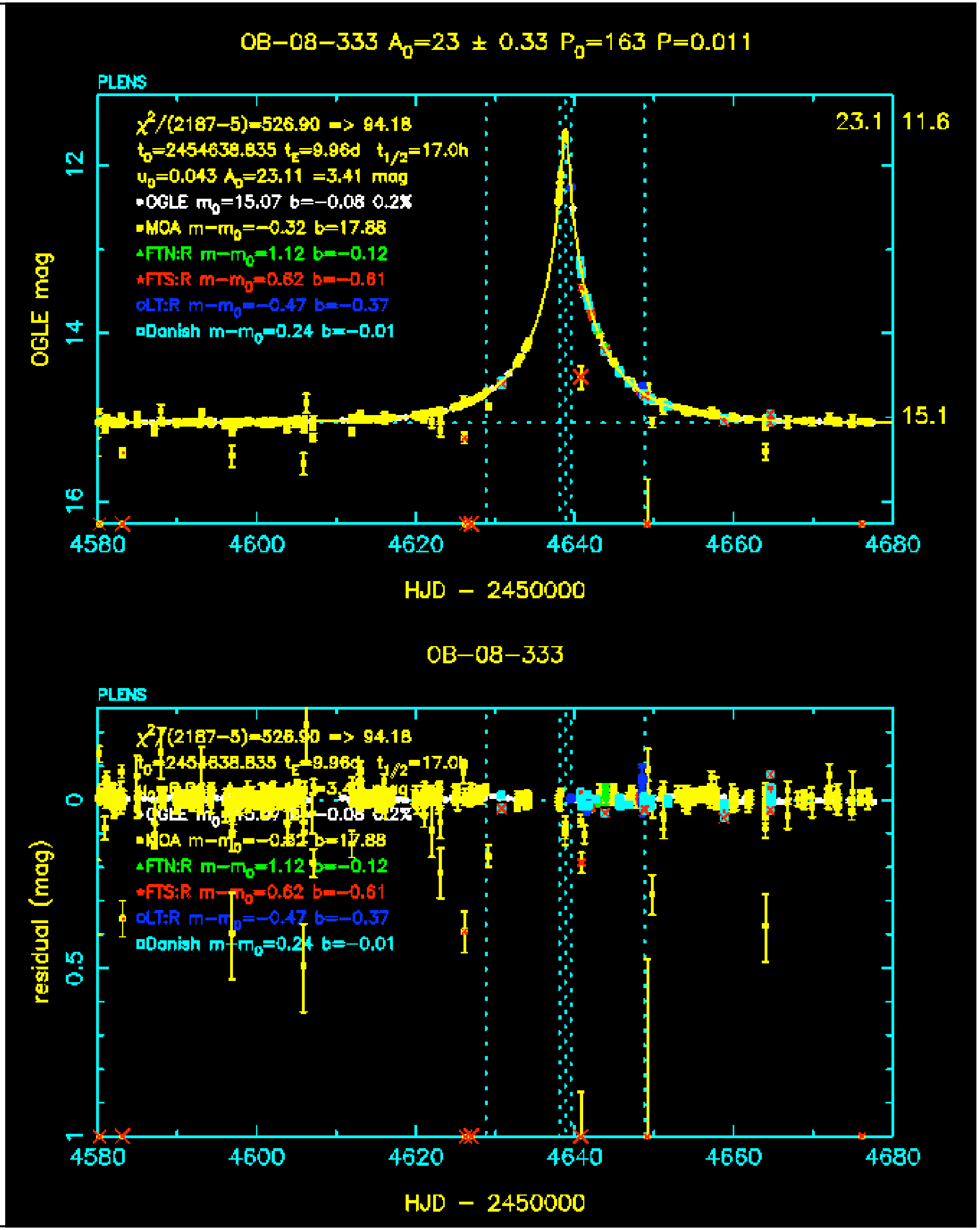




OB-08-333

Fit 5 params:

6 sites



$\chi^2 / (2187\text{-}4) = 526.9$

$\chi^2 / (2187\text{-}5) = 94.18$

OB-08-333

Fit 5 params:

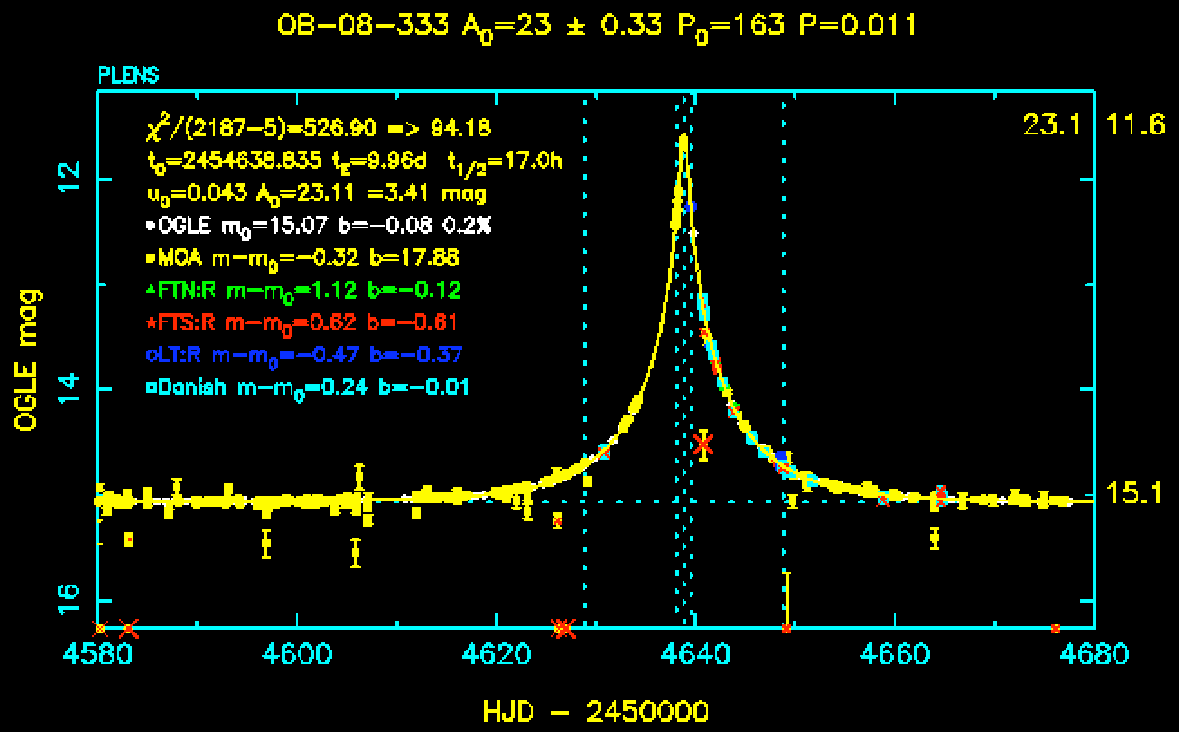
6 sites

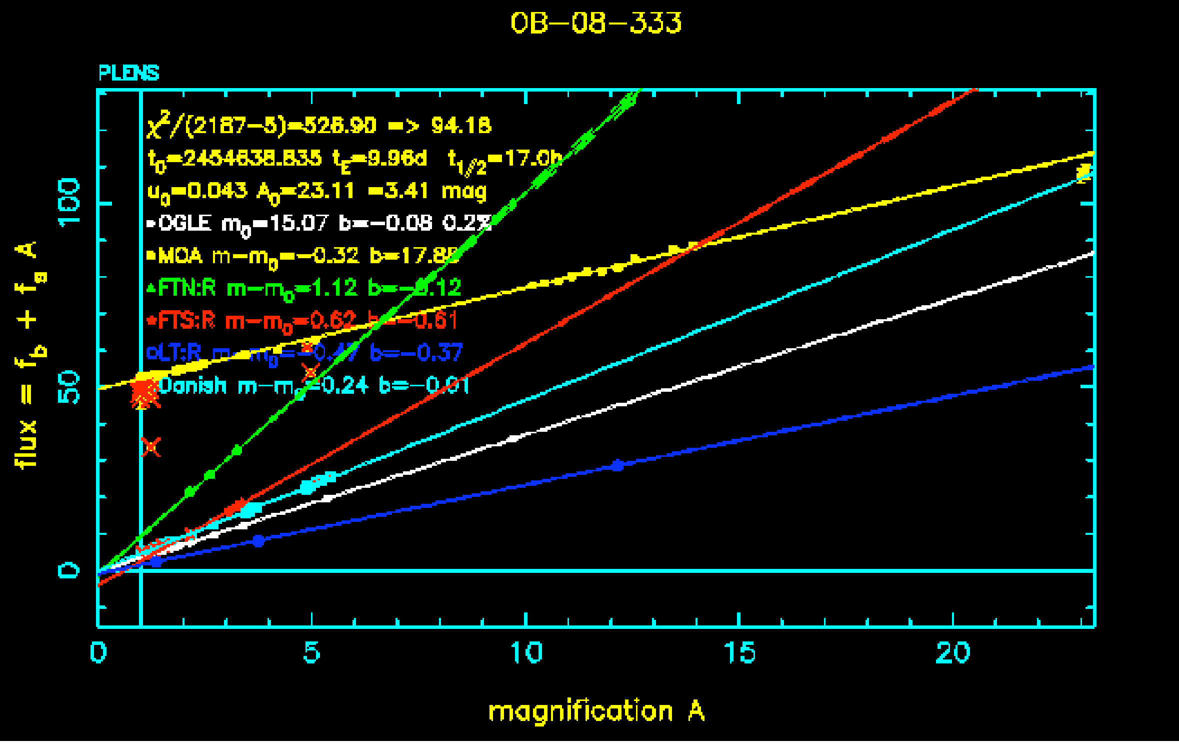
Blend fits

MOA: *b* = *-0.32*

$\chi^2 / (2187-4) = 526.9$

$\chi^2 / (2187-5) = 94.18$





OB-08-333

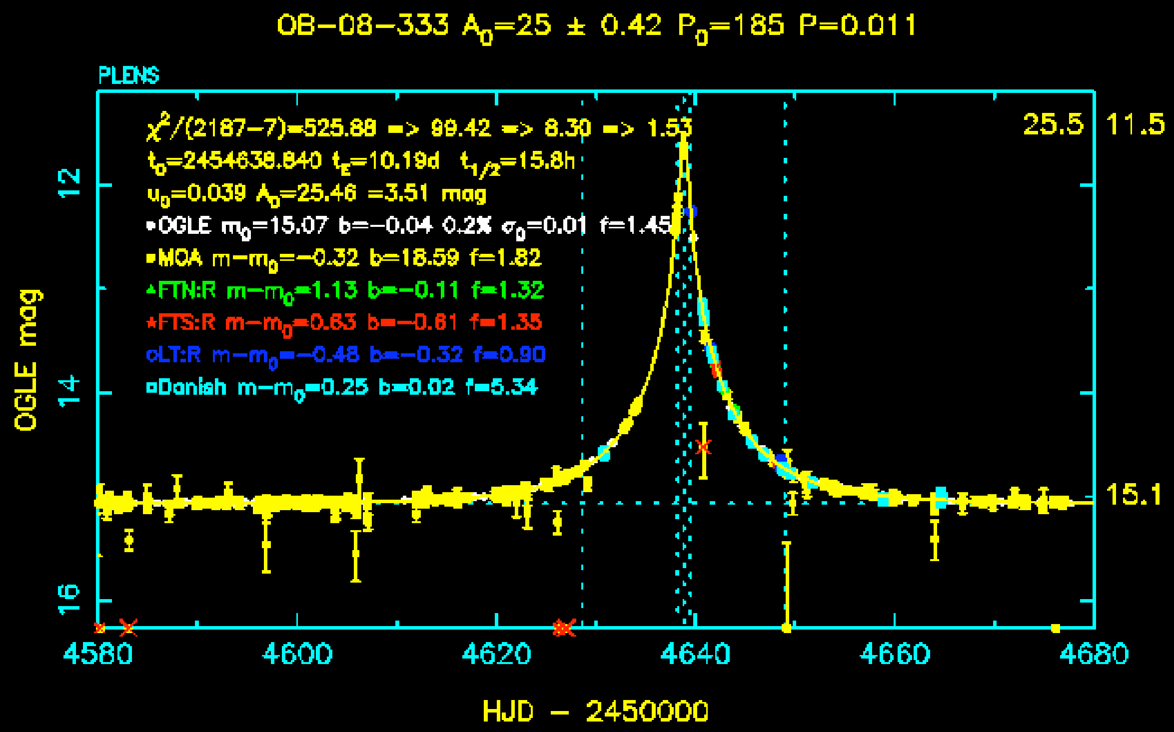
Fit 7 params:

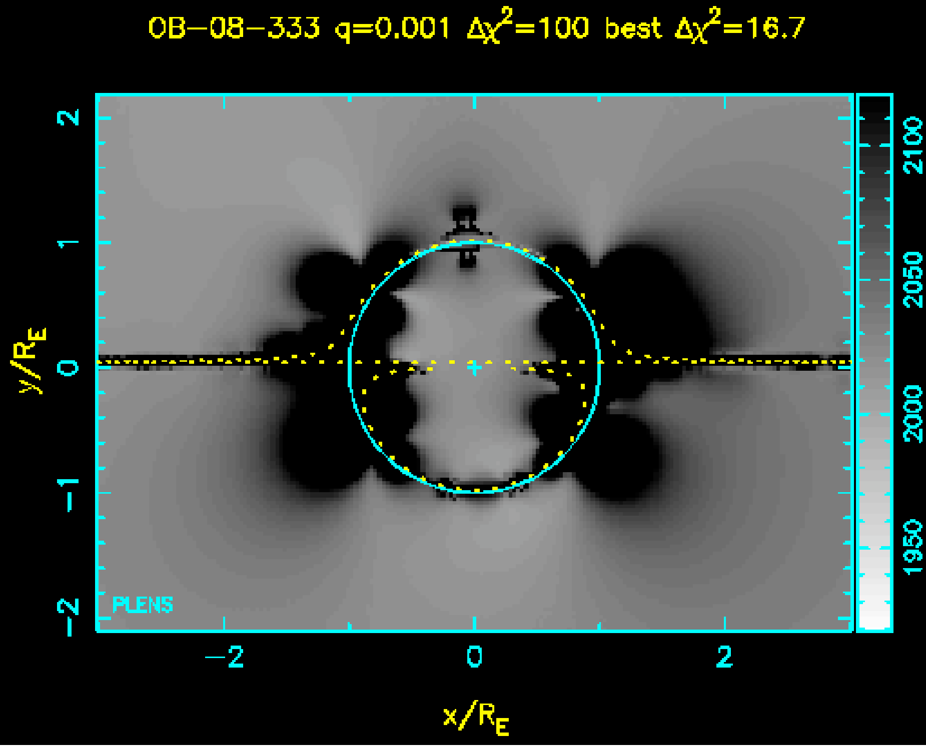
6 sites

Blended fits

$\chi^2(x, y)$

χ^2 / (2187-4) = 525.9
χ^2 / (2187-5) = 99.42
χ^2 / (2187-6) = 8.30
χ^2 / (2187-7) = 1.53





OB-07-472

Fit 5 params:

OGLE+Dan+UTas

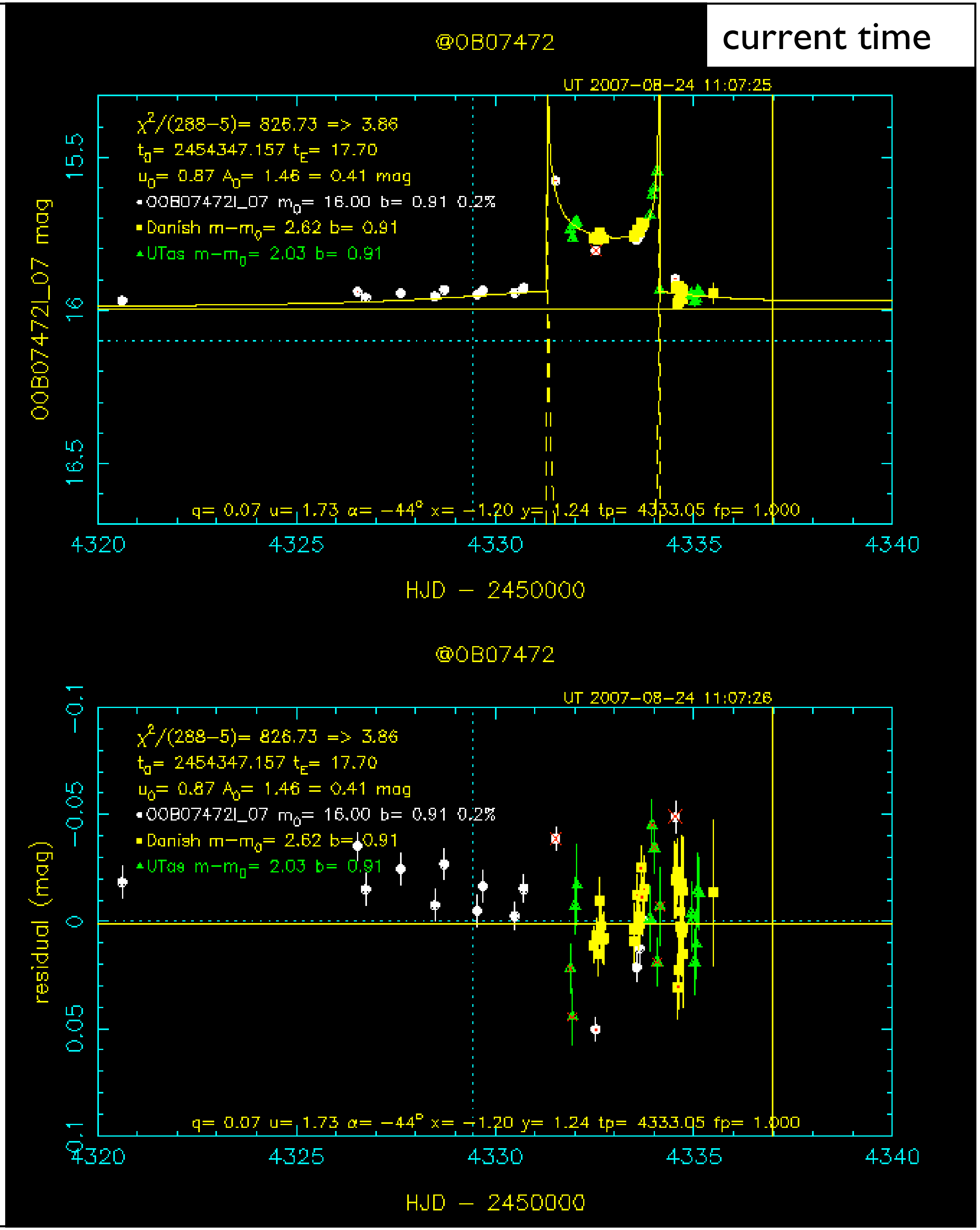
caustic entry/exit

t = 4335.05

f = 1.0 q = 0.07

χ^2 / (2187-4) = 826.7

χ^2 / (2187-5) = 3.86



PLENS: limitations

Fits sometimes fail (A_0 => 0) for MOA-only events.

No finite-source, binary source, parallax / xallarap effects. Comparison of alternative models.

Full noise model / blend analysis implemented only for first dataset.

Covarances from inverse Hessian matrix (could use MCMC).

$\chi^2(x , y)$ slices for fixed q. Starting points for follow-up with Amoeba (and/or MCMC) to locate and characterise all viable local minima.