Stars AS4023:
Stellar Atmospheres (13)
Stellar Structure & Interiors (11)

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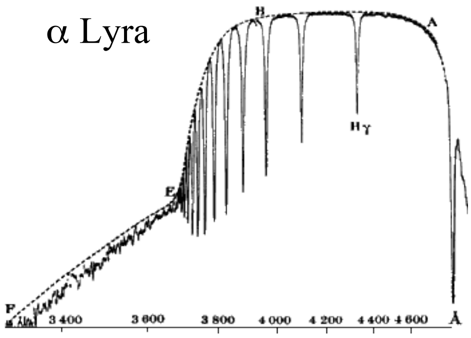
What is a Stellar Atmosphere?

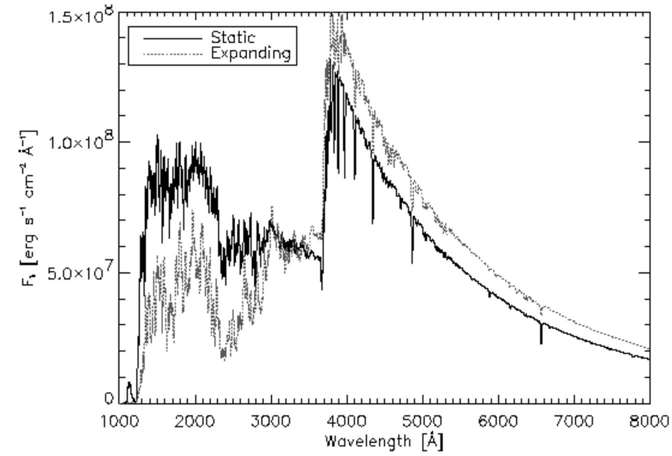
- Transition from dense stellar interior to interstellar medium.
- Region that produces the stellar spectrum. The physical depths in the atmosphere where the spectral features form depend on the atmospheric conditions: temperature, density, level populations, optical depth…
- We see radiation from the "optical depth one surface"
- Goal: From analysis of spectral lines and continua, determine physical conditions, chemical abundances, mass loss rates…

Balmer Lines & Balmer Jump



- H-opacity ~ λ^3
- Jump at Balmer "series limit"
- Size of jump: estimate temperature & pressure

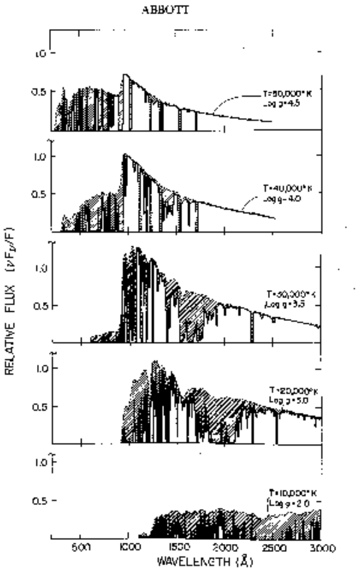
Spectrum from Model Atmosphere



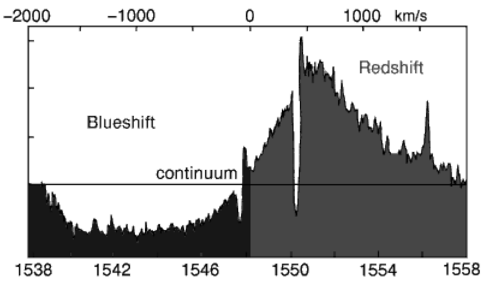
- Expect "saw tooth" due to H opacity
- Line blanketing…

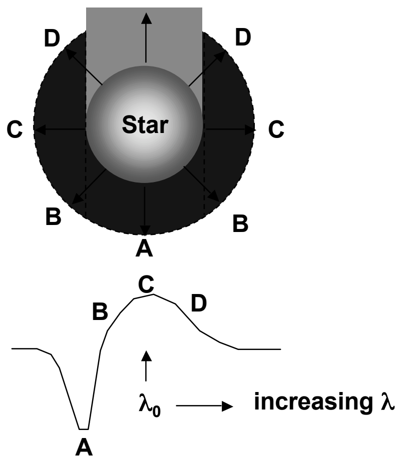
Line Blanketing

- UV opacity from metals
- Blocks flux: blanketing
- Fe, Al,…
- Temperature diagnostic



P-Cygni Line Profiles



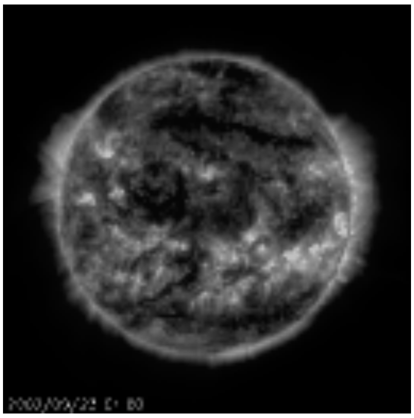


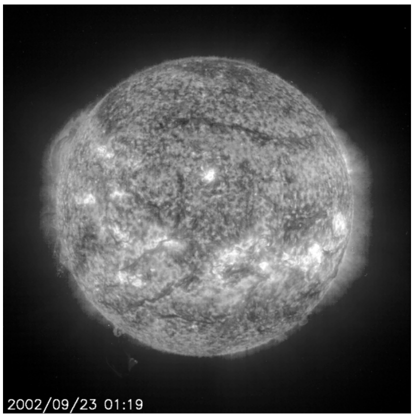
- Expanding atmosphere or stellar wind
- Wind speed from blue edge
- UV space astronomy (1960s): wind theories

Simplifications, but not simple…

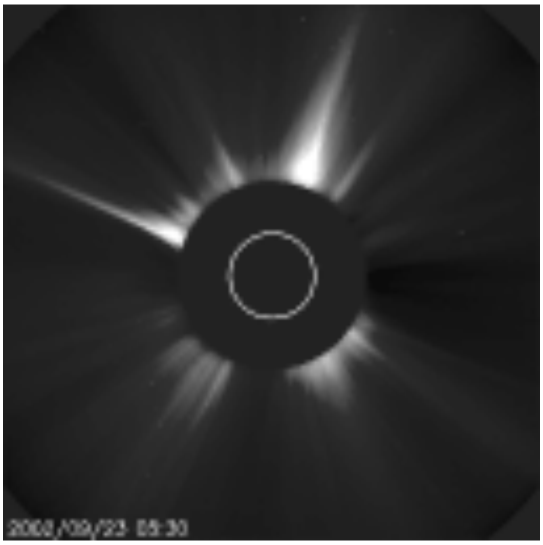
- Plane parallel: scaleheight << radius
  Sun: $h \sim 150$ km, $R \sim 10^5$ km
- Time independent
- Hydrostatic equilibrium
- Radiative equilibrium
- No magnetic fields

The Sun





The Solar Corona



What happens physically?

- Photons emitted, travel some distance, interact with atmospheric material
- Scattered, absorbed, re-emitted
- Photon interactions heat atmosphere, change level populations, ionization balance
- Hydrostatic equilibrium: density structure related to temperature structure
- Atmospheric structure depends on radiation field and vice versa
- Atmosphere model requires detailed study of radiation transfer: bulk of this course

Outline

- Basic definitions of intensities, fluxes, equation of radiation transfer (ERT)
- Opacity sources
- Local Thermodynamic Equilibrium
- Approximate solutions of ERT
- Example: pure H atmosphere
- Building a model atmosphere
- ERT for lines: residual flux in a line
- Monte Carlo: scattering, fluoresence

Recap of Nebulae Course

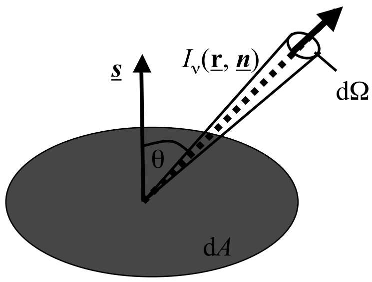
- Assume knowledge of intensities, fluxes, opacities, Saha & Boltzmann equations, equation of radiation transfer (ERT)
- Focus on approximate solutions of ERT useful in stellar atmospheres & interiors

Specific Intensity

$$dE_\nu = I_\nu \cos\theta \, dA \, dt \, d\nu \, d\Omega$$

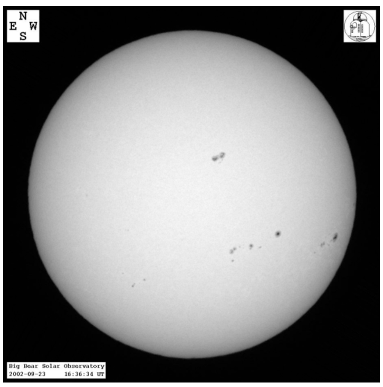
Units of I_ν: J/m^2/s/Hz/sr (ergs/cm^2/s/Hz/sr)

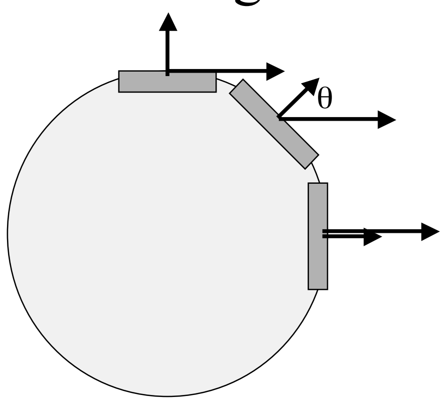
Independent of distance when no sources or sinks



s is normal to dA

Solar Limb Darkening





Assume plane parallel atmosphere locally
Measure I at different positions ("impact parameters") on solar disk => $I(\theta)$

Mean Intensity

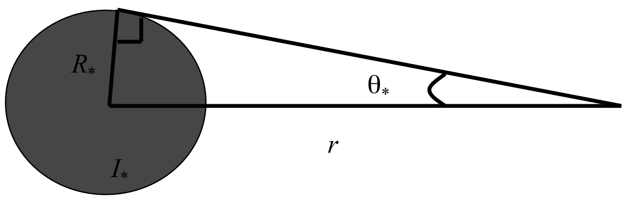
$$J_\nu = \frac{1}{4\pi}\int I_\nu \, d\Omega = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} I_\nu \sin\theta \, d\theta \, d\phi$$

Same units as I_ν, function of position

Determines heating, ionization, level populations, etc

Plane parallel atmosphere (no ϕ dependence) $\mu = \cos\theta$:

$$J_\nu(z) = \frac{1}{4\pi}\int I_\nu(z,\theta)\, 2\pi \sin\theta \, d\theta = \frac{1}{2}\int_{-1}^{1} I_\nu(z,\mu)\, d\mu$$



What is J_ν at r from a star with uniform specific intensity I_* across its surface?

$I = I_*$ for $0 < \theta < \theta_*$ $(\mu_* < \mu < 1)$

$I = 0$ for $\theta > \theta_*$ $(\mu < \mu_*)$

$$J = \frac{1}{2}\int_{\mu_*}^{1} I \, d\mu = \tfrac{1}{2} I_*(1-\mu_*)$$

$$J = I_* \tfrac{1}{2}\left(1 - \sqrt{1 - R_*^2/r^2}\right) = w I_*$$

w = dilution factor

Large r, $w = R^2/4r^2$

Monochromatic Flux

$$\mathcal{F}_\nu = \int I_\nu \cos\theta \, \mathrm{d}\Omega = \int_0^{2\pi} \int_0^{\pi} I_\nu \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

Energy passing through a surface. Units: J/s/m^2/Hz
In stellar atmospheres, outward radial direction is positive:

$$\begin{aligned}\mathcal{F}_\nu(z) &= \int_0^{2\pi} \int_0^{\pi/2} I_\nu \cos\theta \sin\theta \mathrm{d}\theta \mathrm{d}\phi + \int_0^{2\pi} \int_{\pi/2}^{\pi} I_\nu \cos\theta \sin\theta \mathrm{d}\theta \mathrm{d}\phi \\ &= \int_0^{2\pi} \int_0^{\pi/2} I_\nu \cos\theta \sin\theta \mathrm{d}\theta \mathrm{d}\phi - \int_0^{2\pi} \int_0^{\pi/2} I_\nu(\pi-\theta) \cos\theta \sin\theta \mathrm{d}\theta \mathrm{d}\phi \\ &\equiv \mathcal{F}_\nu^+(z) - \mathcal{F}_\nu^-(z)\end{aligned}$$

Where outward flux, $\mathcal{F}_\nu^+$, and inward flux, $\mathcal{F}_\nu^-$, are positive
Isotropic radiation has $\mathcal{F}_\nu^+ = \mathcal{F}_\nu^- = \pi I_\nu$ and $\mathcal{F}_\nu = 0$

Astrophysical Flux

Flux emitted by a star per unit surface area is

$$\mathcal{F}_\nu = \mathcal{F}_\nu^+ = \pi I_\nu^*$$

where I_ν^* is intensity, averaged over apparent stellar disk.
This equality is why that flux is often written as $\pi F = \mathcal{F}$,
so that $F = I^*$, with F called the ***Astrophysical Flux***.

Explains often confusing factors of π in definitions of flux:

$\mathcal{F}$ = ***Monochromatic Flux*** or just the Flux

F = ***Astrophysical Flux***

They are related by $\pi F = \mathcal{F}$.

Stellar Luminosity

Flux = energy/second per area/Hz
Luminosity = energy/second/Hz

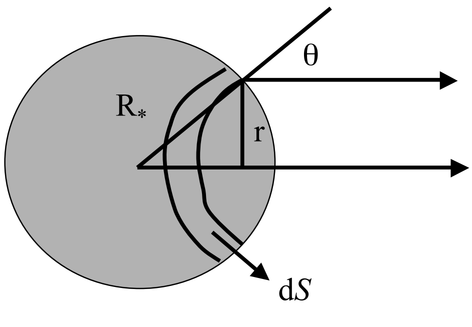
$$L_\nu = \mathcal{F}_\nu \, A_* = 4\pi \, R_*^2 \, \pi \, I_\nu$$

Assume $I_\nu = B_\nu$ and integrate to get total luminosity:

$$L = \int L_\nu \, \mathrm{d}\nu = 4\pi \, R_*^2 \, \pi \int B_\nu \, \mathrm{d}\nu = 4\pi \, R_*^2 \, \sigma \, T^4$$

Unresolved Sources

Relate energy observed to $\mathcal{F}_\nu$ at stellar surface:
Energy received per area, from annulus: $\mathrm{d}f_\nu = I_\nu \, \mathrm{d}\omega$
dω = solid angle of annulus



Annulus area dS:

$$\mathrm{d}S = 2\pi \, r \, \mathrm{d}r = 2\pi \, R_*^2 \, \mu \, \mathrm{d}\mu$$

$r = R_* \sin\theta$, $\mathrm{d}\omega = \mathrm{d}S / D^2$
Integrate over stellar disk:

$$f_\nu = 2\pi \, (R_* / D)^2 \int_0^1 I(R_*, \mu, \nu) \, \mu \, \mathrm{d}\mu$$
$$= (R_* / D)^2 \, \mathcal{F}(R_*, \nu)$$
$$= \frac{1}{4} \alpha_*^2 \, \mathcal{F}(R_*, \nu)$$

α_* = angular diameter
Unresolved => measure flux
Inverse square law. Know α_*, get absolute flux at star

Energy Density & Radiation Pressure

$$u_\nu = \frac{1}{c}\int I_\nu \, \mathrm{d}\Omega$$

$$p_\nu = \frac{1}{c}\int I_\nu \cos^2\theta \, \mathrm{d}\Omega$$

u_ν : J/m^3/Hz $\quad$ p_ν : N/m^2/Hz

Isotropic radiation: $p_\nu = u_\nu/3$

Radiation pressure analogous to gas pressure: pressure of the photon gas

Moments of the Radiation Field

First three moments of specific intensity are named *J* (zeroth moment), *H* (first), and *K* (second):

$$J_\nu = \frac{1}{4\pi}\int I_\nu \, \mathrm{d}\Omega$$

$$H_\nu = \frac{1}{4\pi}\int I_\nu \cos\theta \, \mathrm{d}\Omega$$

$$K_\nu = \frac{1}{4\pi}\int I_\nu \cos^2\theta \, \mathrm{d}\Omega$$

Define *moment operator* **M** operating on *f*:

$$\mathrm{M}^{(n)}[f] = \frac{1}{2}\int_{-1}^{1} f\, \mu^n \, \mathrm{d}\mu$$

Plane parallel atmosphere:

$$J_\nu(z) = \frac{1}{2}\int_{-1}^{1} I_\nu(z,\mu)\,\mathrm{d}\mu$$

$$H_\nu(z) = \frac{1}{2}\int_{-1}^{1} I_\nu(z,\mu)\,\mu\,\mathrm{d}\mu$$

$$K_\nu(z) = \frac{1}{2}\int_{-1}^{1} I_\nu(z,\mu)\,\mu^2\,\mathrm{d}\mu$$

Used in solving ERT

Physically: J = mean intensity; $H = \mathcal{F} / 4\pi = F / 4$

K related to radiation pressure: $$p_\nu = \frac{4\pi}{c} K_\nu$$

Photon Interactions

- Scattering: change direction (energy slightly)
- Absorption: energy added to K.E. of particles: photon thermalized
- Emission: energy taken from thermal energy of particles

Emission Coefficient

$$dE_\nu \equiv j_\nu \, dV \, dt \, d\nu \, d\Omega$$

Energy, dE_ν, added:
- stimulated emission
- spontaneous emission
- thermal emission
- energy scattered into the beam

Intensity contribution from emission along ds:

$$dI_\nu(s) = j_\nu(s) \, ds$$

Extinction Coefficient

Energy removed from beam
Defined per particle, per mass, or per volume

$$dI_\nu(s) = -I_\nu \sigma_\nu \, n \, ds$$

σ_ν = cross section per particle (m^2)
n = particle density (m^{-3})

$$dI_\nu(s) = -I_\nu \alpha_\nu \, ds$$

α_ν: units of m^{-1}

$$dI_\nu(s) = -I_\nu \kappa_\nu \rho \, ds$$

κ_ν: units $m^2 \, kg^{-1}$
ρ = density ($kg \, m^{-3}$)

Source Function

Same units as intensity: $$S_\nu \equiv \frac{j_\nu}{\alpha_\nu}$$

Multiple processes contribute to emission and extinction:

$$S_\nu^{\rm tot} = \frac{\sum j_\nu}{\sum \alpha_\nu}$$

e.g., a spectral line: $$S_\nu^{\rm tot} = \frac{j_\nu^c + j_\nu^l}{\alpha_\nu^c + \alpha_\nu^l} = \frac{S_\nu^c + \eta_\nu S_\nu^l}{1 + \eta_\nu}$$

$\eta_\nu = \alpha_\nu^l / \alpha_\nu^c$ = line-to-continuum extinction ratio;
S_ν^c, S_ν^l are continuum and line source functions

Optical Depth

$$\mathrm{d}\tau_\nu = \alpha_\nu(s)\,\mathrm{d}s = \rho(s)\kappa_\nu\,\mathrm{d}s$$

$$\tau_\nu = \int_0^s \alpha_\nu\,\mathrm{d}s = \int_0^s \rho\kappa_\nu\,\mathrm{d}s$$

Function of frequency via the opacity, and direction

Physically τ_ν is number of photon mean free paths

Equation of Radiation Transfer

ERT along a ray: $$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = S_\nu - I_\nu$$

Solution: $$I_\nu(\tau_\nu) = I_\nu(0)\,\mathrm{e}^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t_\nu)\,\mathrm{e}^{-(\tau_\nu - t_\nu)}\,\mathrm{d}t_\nu$$

Nebulae: optical depth, opacity, emissivity, source function

Goal: Determine source function!