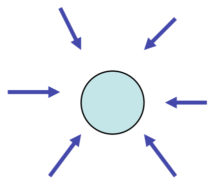
9. Examples of flows: Spherical accretion

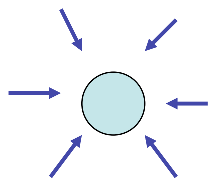
- Consider steady spherical inflow under gravity.
- Look for flows that are at rest at infinity and fall in initially subsonically.



- Gas has sonic transition somewhere and reaches the star essentially in free-fall.

- Mass conservation (4.1) ->

$$\underline{\nabla} \cdot (\rho \underline{u}) = 0$$



- or $$\int_V \underline{\nabla} \cdot (\rho \underline{u}) dV = \int_s \rho \underline{u} \cdot d\underline{S}$$

- i.e.

$$4\pi r^2 \rho u = \text{constant} = \dot{M} \qquad (9.1)$$

Mass flow rate

- Steady momentum equation

$$(\underline{u}\cdot\underline{\nabla})\underline{u} = -\frac{1}{\rho}\underline{\nabla}p - \underline{\nabla}\psi$$

- gives

Assume all gravity comes from point mass

$$u\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{GM}{r^2} \qquad (9.2)$$

- Where $\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{1}{\rho}\frac{\partial p}{\partial \rho}\frac{\partial \rho}{\partial r} = c_s^2\frac{1}{\rho}\frac{\partial \rho}{\partial r}$

- And from (9.1)

$$\frac{\partial}{\partial r}\left(r^2\rho u\right) = 0$$

$$\Rightarrow 2r\rho u + r^2 u\frac{\partial \rho}{\partial r} + r^2\rho\frac{\partial u}{\partial r} = 0$$

$$\Rightarrow -\frac{1}{\rho}\frac{\partial \rho}{\partial r} = \frac{2}{r} + \frac{1}{u}\frac{\partial u}{\partial r}$$

- And so (9.2) becomes

$$u\frac{\partial u}{\partial r} = c_s^2\left(\frac{2}{r} + \frac{1}{u}\frac{\partial u}{\partial r}\right) - \frac{GM}{r^2}$$

$$\left(u^2 - c_s^2\right)\frac{\partial(\ln u)}{\partial r} = \frac{2c_s^2}{r}\left(1 - \frac{GM}{2c_s^2 r}\right) \qquad (9.3)$$

- Hence at

$$r_s = GM/2c_s^2$$

  - Either u is a max/min or
  - $u=c_s$
- So the sonic transition must occur at r_s

Isothermal case:

- c_s constant, so T determines r_s
- Density at r_s from (9.1)

$$\rho_s = \frac{\dot{M}}{4\pi r_s^2 c_s}$$

- And Bernoulli gives:

$$\frac{1}{2}u^2 + c_s^2 \ln\rho - \frac{GM}{r} = \frac{1}{2}c_s^2 + c_s^2 \ln\rho_s - \frac{GM}{r_s}$$

$2c^2$

$$u^2 = 2c_s^2\left[\ln\left(\frac{\rho_s}{\rho}\right) - \frac{3}{2}\right] + \frac{2GM}{r}$$

- Note:

$$r \to 0, u^2 \to \frac{2GM}{r}$$ Free-fall

$$r \to \infty, u \to 0 \Rightarrow \rho_\infty = \rho_s e^{-1.5}$$

- Hence..for a given density at infinity, we know ρ_s and hence for a given M and T we know $\dot{M}$.
- So..if we set down a star in an isothermal medium with a given ρ at infinity, we can get the accretion rate onto the star.

Aside: Stellar winds

- Outwardly directed flow from stellar surface.
- Initially subsonic, passes through sonic point and is supersonic at large radii.
- Initial acceleration may be due to
  - Radiation pressure acting on dust grains
  - Line emitting atoms
  - Magnetic fields (YES!!!)
- Would give extra terms in momentum equation..

- Here just consider solution outside the initial acceleration region…flow is coasting out under influence of pressure and gravity.

- => same equations as before

- BUT boundary conditions (e.g. density or temperature) usually specified at *inner* boundary.